Math 232 Calculus 2 Fall 21 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int x^2 \cos(1 - x^3)\, dx$.

$u = 1 - x^3$

$\frac{du}{dx} = -3x^2$

$$\int x^2 \cos u \frac{dx}{du}\, du = \int x^2 \cos u \frac{1}{-3x^2}\, du$$

$$= -\frac{1}{3}\int \cos u\, du = -\frac{1}{3}\sin u + c = -\frac{1}{3}\sin(1-x^3) + c$$

(2) (10 points) Find $\int \frac{e^{-x}}{e^{-x}-1}\,dx$.

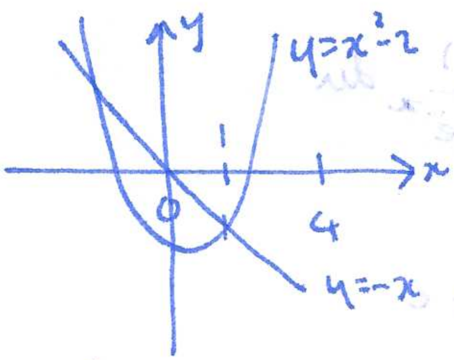
$u = e^{-x} - 1$

$\frac{du}{dx} = -e^{-x}$

$\int \frac{e^{-x}}{u} \frac{dx}{du}\,du = \int \frac{e^{-x}}{u} \frac{1}{-e^{-x}}\,du$

$= -\int \frac{1}{u}\,du = -\ln|u| + c = -\ln|e^{-x} - 1| + c$

(3) (10 points) Find the area bounded between the curves $y = -x$ and $y = x^2 - 2$ over the interval $0 \le x \le 4$.



intersections: $-x = x^2 - 2$

$x^2 + x - 2 = 0$

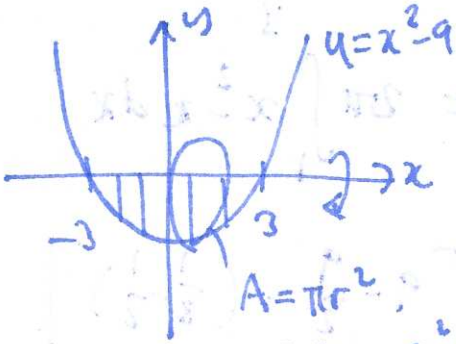
$(x+2)(x-1) = 0$

$$\int_0^1 -x-(x^2-2)\,dx + \int_1^4 x^2-2-(-x)\,dx$$

$$= \int_0^1 -x-x^2+2\,dx + \int_1^4 x^2-2+x\,dx = \left[-\frac{1}{2}x^2-\frac{1}{3}x^3+2x\right]_0^1 + \left[\frac{1}{3}x^3-2x+\frac{1}{2}x^2\right]_1^4$$

$$= -\frac{1}{2}-\frac{1}{3}+2 + \frac{64}{3}-8+8-\left(\frac{1}{3}-2+\frac{1}{2}\right) = -1-\frac{2}{3}+4+\frac{64}{3} = \frac{71}{3}$$

(4) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 9$ and the x-axis. Find the volume of revolution of this region rotated about the x-axis.



$A = \pi r^2$

$r = y = x^2 - 9$

$$V = \int_{-3}^{3} \pi (x^2-9)^2 \, dx$$

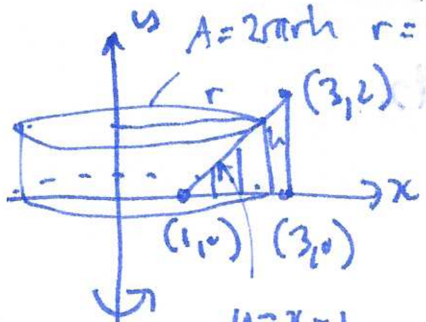
$$= \pi \int_{-3}^{3} x^4 - 18x^2 + 81 \, dx$$

$$= \pi \left[\frac{1}{5}x^5 - 6x^3 + 81x \right]_{-3}^{3} = 2\pi \left(\frac{3^5}{5} - 162 + 81 \times 3 \right) = 2\pi \left(\frac{3^5}{5} + 81 \right)$$

$$= \frac{1296\pi}{5} \approx 814.30$$

(5) (10 points) Find the volume of revolution of the triangle with vertices $(1,0)$, $(3,0)$ and $(3,2)$, rotated about the y-axis.

$A = 2\pi r h \quad r = x \quad h = y$



$y = x - 1$

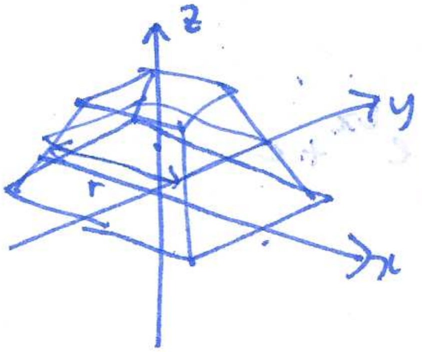
$$\int_1^3 2\pi x(x-1)\,dx = 2\pi\int_1^3 x^2 - x\,dx$$

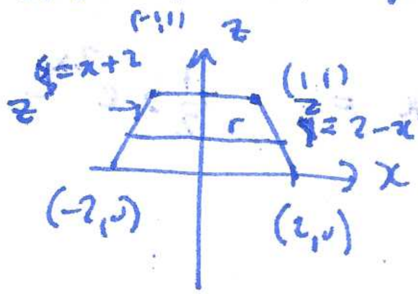
$$= 2\pi\left[\frac{1}{3}x^3 - \frac{1}{2}x^2\right]_1^3 = 2\pi\left[9 - \frac{9}{2} - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$= 2\pi\left(\frac{9}{2} + \frac{1}{6}\right) = \pi\left(9 + \frac{1}{3}\right) = \frac{28\pi}{3}$$

(6) (10 points) Find the volume of the truncated pyramid whose base is the square in the xy-plane with vertices $(\pm 2, \pm 2, 0)$ and whose top is the square in the plane $z = 1$ with vertices $(\pm 1, \pm 1, 1)$.

cross-section at height z is a square $A = r^2$





$$r = (2-z) - (z-2) = 4 - 2z$$

$$V = \int_0^1 (4-2z)^2 \, dz = \int_0^1 16 - 16z + 4z^2 \, dz$$

$$= \left[16z - 8z^2 + \frac{4z^3}{3} \right]_0^1 = 16 - 8 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = x \qquad v' = e^{3x}$

(7) (10 points) Find $\int xe^{3x}\,dx$.

$u' = 1 \qquad v = \frac{1}{3}e^{3x}$

$$\int xe^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}\,dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$$

$\int uv'\,dx = uv - \int u'v\,dx$

$u = e^{-x}$ $\quad v' = \sin(2x)$

$u' = -e^{-x}$ $\quad v = -\frac{1}{2}\cos(2x)$

(8) (10 points) Find $\int_0^\pi \underset{u}{e^{-x}} \underset{v'}{\sin(2x)}\, dx$.

$$\int_0^\pi e^{-x}\sin 2x\,dx = \left[-\frac{1}{2}e^{-x}\cos 2x\right]_0^\pi - \int_0^\pi -e^{-x}\cdot -\frac{1}{2}\cos(2x)\,dx$$

$u = e^{-x}$ $\quad v' = \cos 2x$

$u' = -e^{-x}$ $\quad v = \frac{1}{2}\sin 2x$

$$\int_0^\pi e^{-x}\sin 2x\,dx = -\frac{1}{2}\left(e^{-\pi} - 1\right) - \frac{1}{2}\int_0^\pi \underset{u}{e^{-x}} \underset{v'}{\cos 2x}\,dx$$

$$\int_0^\pi e^{-x}\sin 2x\,dx = \frac{1}{2}\left(1 - e^{-\pi}\right) - \frac{1}{2}\left[e^{-x}\frac{1}{2}\sin 2x\right]_0^\pi + \frac{1}{2}\int_0^\pi -e^{-x}\frac{1}{2}\sin 2x\,dx$$

0

$$\frac{5}{4}\int_0^\pi e^{-x}\sin 2x\,dx = \frac{1}{2}\left(1 - e^{-\pi}\right)$$

$$\int_0^\pi e^{-x}\sin 2x\,dx = \frac{2}{5}\left(1 - e^{-\pi}\right)$$

(9) Find $\displaystyle\int \cos^3 x \, dx$. $= \displaystyle\int \sin^{0} \cos^2 x \cos x \, dx = \int (1-\sin^2 x)\cos x \, dx$

$= \displaystyle\int \cos x \, dx - \int \sin^2 x \cos x \, dx = \sin x - \int u^2 \cos x \frac{dx}{du} \, du$

$du = \sin x$

$\dfrac{du}{dx} = \cos x$ $\quad = \sin x - \displaystyle\int u^2 \frac{\cos x}{\cos x} \, du$

$= \sin x - \frac{1}{3}u^3 + c = \sin x - \frac{1}{3}\sin^3 x + c$

(10) Find $\int \cos(5x)\cos(2x)\,dx$.

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$\frac{1}{2}\int \cos(7x) + \underset{\cos(3x)}{\underset{\shortparallel}{\cos(-3x)}}\,dx = \frac{1}{14}\sin 7x + \frac{1}{6}\sin 3x + c$