**Math 232 Calculus 2 Fall 21 Midterm 1a**

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a $3 \times 5$ index card of notes.

| | | |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| 10 | 10 | |
| | 80 | |

| | |
|---|---|
| Midterm 1 | |
| Overall | |

(1) (10 points) Find $\int x^3 \sin(1 - x^4)\, dx$.

$u = 1 - x^4$

$\frac{du}{dx} = -4x^3$

$\int x^3 \sin(u) \frac{dx}{du}\, du$

$\int x^3 \sin(u) \frac{1}{-4x^3}\, du$

$-\frac{1}{4} \int \sin(u)\, du$

$\frac{1}{4} \cos(u) + c$

$\frac{1}{4} \cos(1 - x^4) + c$

(2) (10 points) Find $\int \frac{e^{-x}}{1+e^{-x}}\, dx$.

$u = 1+e^{-x}$

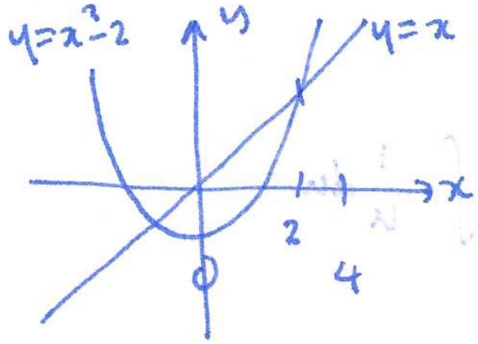
$\frac{du}{dx} = -e^{-x}$

$\int \frac{e^{-x}}{u} \cdot \frac{dx}{du}\, du = \int \frac{e^{-x}}{u} \cdot \frac{1}{-e^{-x}}\, du = -\int \frac{1}{u}\, du$

$= -\ln|u| + c$

$= -\ln|1+e^{-x}| + c$

(3) (10 points) Find the area bounded between the curves $y = x$ and $y = x^2 - 2$ over the interval $0 \leq x \leq 4$.




find intersections: $x = x^2 - 2$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

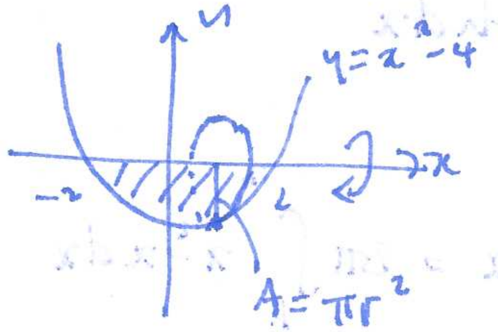
$$\int_0^2 x - (x^2 - 2)\,dx + \int_2^4 x^2 - 2 - x\,dx$$

$$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 + 2x\right]_0^2 + \left[\frac{1}{3}x^3 - 2x - \frac{1}{2}x^2\right]_2^4$$

$$2 - \frac{8}{3} + 4 + \frac{64}{3} - 8 - 8 - \left(\frac{8}{3} - 4 - 2\right)$$

$$12 - 16 - \frac{16}{3} + \frac{64}{3} = \frac{48 - 12}{3} = 12$$

(4) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 4$ and the $x$-axis. Find the volume of revolution of this region rotated about the $x$-axis.




$A = \pi r^2$

$r = y$

$$V = \int_{-2}^{2} \pi y^2 \, dx = \pi \int_{-2}^{2} (x^2-4)^2 \, dx$$

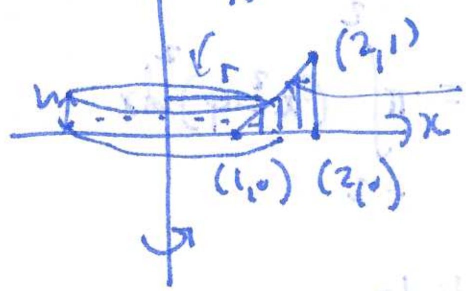
$$= \pi \int_{-2}^{2} x^4 - 8x^2 + 16 \, dx$$

$$= \pi \left[ \frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x \right]_{-2}^{2} = 2\pi \left( \frac{32}{5} - \frac{64}{3} + 32 \right)$$

$$= \frac{512\pi}{15} \approx 107.23$$

(5) (10 points) Find the volume of revolution of the triangle with vertices $(1,0)$, $(2,0)$ and $(2,1)$, rotated about the $y$-axis.

$A = 2\pi r h$




$r = x$

$h = y$

$$\int_1^2 2\pi r h \, dx$$

$y = x - 1$

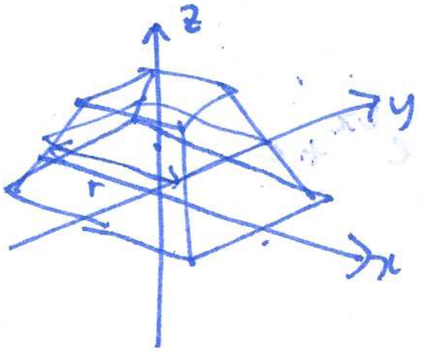
$$= 2\pi \int_1^2 x(x-1)\,dx = 2\pi \int_1^2 x^2 - x \, dx$$

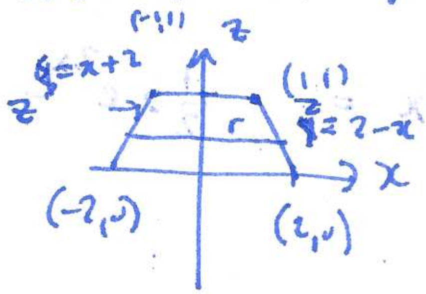
$$= 2\pi \left[ \frac{1}{3}x^3 - \frac{1}{2}x^2 \right]_1^2 = 2\pi \left( \frac{8}{3} - 2 - \left( \frac{1}{3} - \frac{1}{2} \right) \right) = 2\pi \left( \frac{2}{3} - \frac{1}{3} + \frac{1}{2} \right)$$

$$= \frac{10\pi}{6} = \frac{5\pi}{3}$$

(6) (10 points) Find the volume of the truncated pyramid whose base is the square in the $xy$-plane with vertices $(\pm 2, \pm 2, 0)$ and whose top is the square in the plane $z = 1$ with vertices $(\pm 1, \pm 1, 1)$.

cross-section at height $z$ is a square $A = r^2$




$$r = (2-z) - (z-2) = 4 - 2z$$

$$V = \int_0^1 (4-2z)^2\,dz = \int_0^1 16 - 16z + 4z^2\,dz$$

$$= \left[16z - 8z^2 + \frac{4z^3}{3}\right]_0^1 = 16 - 8 + \frac{4}{3} = 8 + \frac{4}{3} = \frac{28}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$



(7) (10 points) Find $\int \underset{u}{x}\,\underset{v'}{e^{2x}}\, dx$.

$u = x \qquad v' = e^{2x}$

$u' = 1 \qquad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = e^{-2x}$ $\quad v' = \cos(x)$

(8) (10 points) Find $\int_0^{\pi} \underset{u}{e^{-2x}} \underset{v'}{\cos(x)}\, dx$.

$u' = -2e^{-2x}$ $\quad v = \sin x$

$$\int_0^{\pi} e^{-2x} \cos x\, dx = \left[e^{-2x} \sin x\right]_0^{\pi} - \int_0^{\pi} -2e^{-2x} \sin x\, dx$$

$0$

$$\int_0^{\pi} e^{-2x} \cos x\, dx = 2 \int_0^{\pi} \underset{u}{e^{-2x}} \underset{v'}{\sin x}\, dx = 2\left[e^{-2x} \cdot -\cos x\right]_0^{\pi} - 2\int_0^{\pi} -2e^{-2x} \cdot -\cos x\, dx$$

$u = e^{-2x}$ $\quad v' = \sin x$

$u' = -2e^{-2x}$ $\quad v = -\cos x$

$$\int_0^{\pi} e^{-2x} \cos(x)\, dx = 2(e^{-2\pi} + 1) - 4\int_0^{\pi} e^{-2x} \cos x\, dx$$

$$5\int_0^{\pi} e^{-2x} \cos x\, dx = 2e^{-2\pi} + 2 \qquad \int_0^{\pi} e^{-2x} \cos x\, dx = \frac{2}{5}(e^{-2\pi} + 1).$$

(9) Find $\int \sin^3 x \, dx$. $= \int \sin^2 x \sin x \, dx = \int (1-\cos^2 x) \sin x \, dx$

$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx = -\cos x - \int u^2 \sin x \frac{dx}{du} du$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$= -\cos x - \int u^2 \sin x \frac{1}{-\sin x} dx = -\cos x + \int u^2 dx = -\cos x + \frac{1}{3} u^3 + c$

$= -\cos x - \frac{1}{3} \cos^3 x + c$

(10) Find $\int \sin(5x)\cos(2x)\, dx$.

$\sin(5x)$: A, $\cos(2x)$: B

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$= \frac{1}{2}\int \sin 7x + \sin 3x \, dx = -\frac{1}{14}\cos 7x - \frac{1}{6}\cos 3x + c$