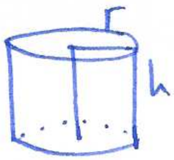


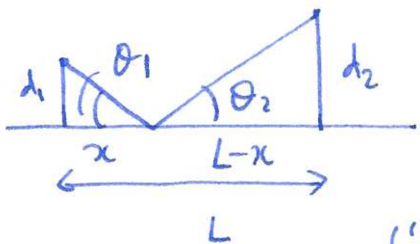
$V = \pi r^2 h = 1 \iff h = \frac{1}{\pi r^2}$

$A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r \cdot \frac{1}{\pi r^2} = 2\pi r^2 + \frac{2}{r}$

$A'(r) = 4\pi r - \frac{2}{r^2}$ solve $A'(r) = 0$: $r^3 = \frac{1}{2\pi}$ $r = \sqrt[3]{1/2\pi}$.

Example a ball bounces off of a wall. Show that the path which minimizes distance has equal angle of incidence and reflection

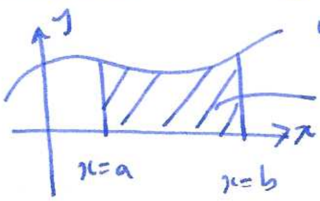


$f(x) = \text{distance} = \sqrt{d_1^2 + x^2} + \sqrt{d_2^2 + (L-x)^2}$

$f'(x) = \frac{1}{2}(d_1^2 + x^2)^{-1/2} \cdot 2x + \frac{1}{2}(d_2^2 + (L-x)^2)^{-1/2} \cdot 2(L-x)(-1)$

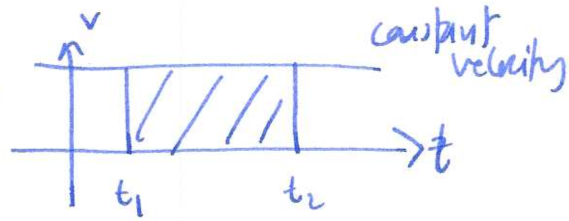
$f'(x) = 0$: $\frac{x}{\sqrt{d_1^2 + x^2}} = \frac{L-x}{\sqrt{d_2^2 + (L-x)^2}} \iff \cos\theta_1 = \cos\theta_2 \iff \theta_1 = \theta_2$

§5.1 Approximating area



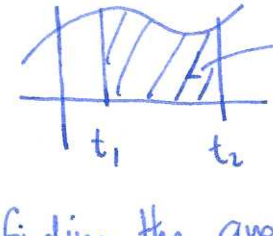
area under the graph

Example



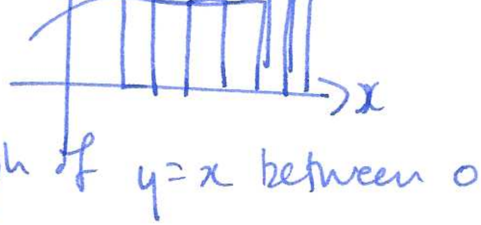
distance travelled = velocity × time
= area under the graph

non-constant velocity :

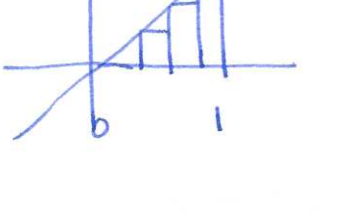


distance travelled = area under the graph

finding the area : approximate by rectangles



Example



find area under graph of $y = x$ between 0 and 1
(answer $= \frac{1}{2}$)