(40)

⑤ $\lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{x} = \lim_{x\to 0} \frac{x-\sin x}{x\sin x} = \lim_{x\to 0} \frac{1-\cos x}{\sin x + x\cos x} = \lim_{x\to 0} \frac{\sin x}{\cos x + \cos x - x\sin x} = 0$

⑥ $\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{x\ln x}$ note: e^x continuous, so $= e^{\lim_{x\to 0^+} x\ln x} = e^0 = 1$

Comparing growth rates of functions

Q: which grows faster $(\ln(x))^2$ or $\sqrt{x}$?

$\lim_{x\to\infty} \frac{\sqrt{x}}{(\ln(x))^2} = \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{2\ln(x)\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{x}}{4\ln(x)} = \lim_{x\to\infty} \frac{\frac{1}{2}x^{-1/2}}{4/x}$

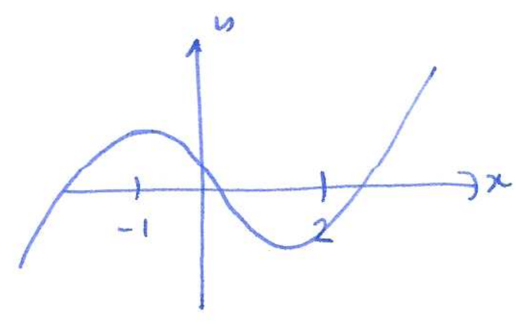
$= \lim_{x\to\infty} \frac{\sqrt{x}}{8} = \infty$, so $\sqrt{x}$ grows faster.

Thm e^x grows faster than any polynomial.

Proof: $\lim_{x\to\infty} \frac{e^x}{x^n} = \lim_{x\to\infty} \frac{e^x}{nx^{n-1}} = \cdots = \lim_{x\to\infty} \frac{e^x}{n!} = \infty$. □.

§4.6 Graph sketching and asymptotes

Example sketch graph of $\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 3 = f(x)$



helpful info: · critical points $f'(x) = x^2 - x - 2$
$(x-2)(x+1)$
· sign of $f'(x)$
· sign of $f''(x)$ $f''(x) = 2x - 1$

critical points at $x = -1, 2$ $f''(-1) = -3 < 0$ local max
$f''(2) = 3 > 0$ local min

Example $f(x) = (4x - x^2)e^x$ critical pts: $x = 1 \pm \sqrt{5}$
$f'(x) = (4 + 2x - x^2)e^x$
$f''(x) = (6 - x^2)e^x$

$1+\sqrt{5}$ local max $1-\sqrt{5}$ local min.