find sign of $f'(x)$ between them:

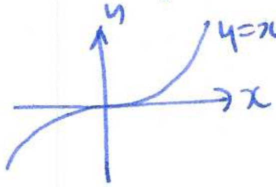
		$\pi/6$		$\pi/2$		$5\pi/6$	
$\cos(x)$	$+$		$+$		$-$		$-$
$1-2\sin x$	$+$		$-$		$-$		$+$
$f'(x)$	$+$		$-$		$+$		$-$

$\Rightarrow \frac{\pi}{6}, \frac{5\pi}{6}$ local max

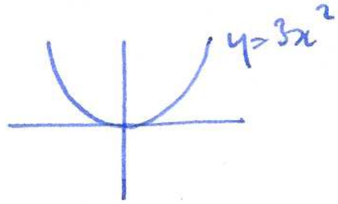
$\frac{\pi}{2}$ local min

<u>Example</u> critical point <u>not</u> a local max or min

$f(x) = x^3$

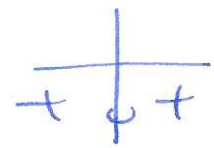


$f'(x) =$



$f'(0)=0$

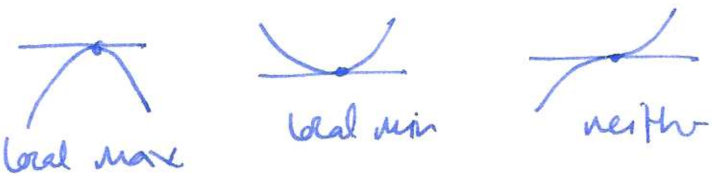
$f'(x) =$



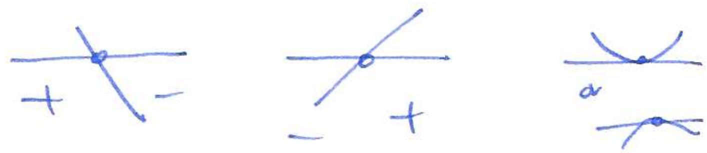
$x=0$
<u>not</u> local max or min.

<u>recall</u> critical point $f'(x)=0$

$f(x)$



$f'(x)$

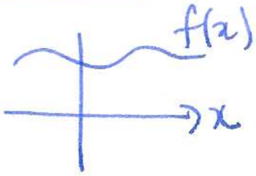


$f''(x)$ $\quad$ $-$ve $\quad$ $+$ve

§4.4 Second derivative test

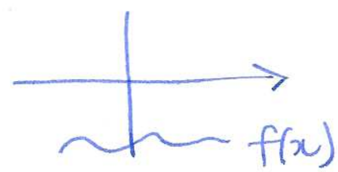
<u>recall</u> $f(x) > 0$

f positive



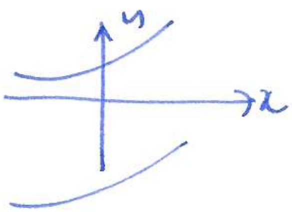
$f(x) < 0$

f negative



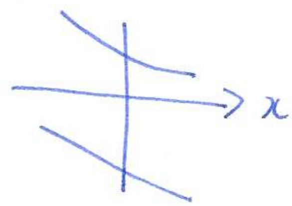
$f'(x) > 0$

f increasing



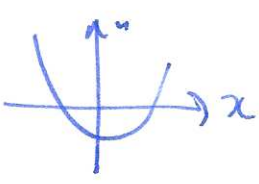
$f'(x) < 0$

f decreasing



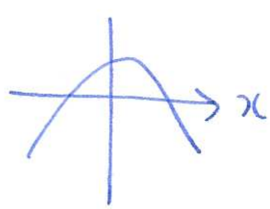
$f''(x) > 0$

concave up



$f''(x) < 0$

concave down



<u>mnemonic</u> down $\quad$ up

Q: how do the slopes change?

concave up
$\Leftrightarrow$ slopes increasing
$\Leftrightarrow$ $f''(x) > 0$

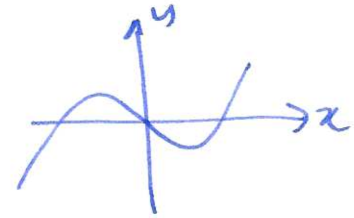
concave down
$\Leftrightarrow$ slopes decreasing
$\Leftrightarrow$ $f''(x) < 0$

<u>Defn</u> Let $f(x)$ be differentiable on an interval (a,b), then

$f(x)$ is concave up $\Leftrightarrow$ $f''(x) > 0$
$f(x)$ is concave down $\Leftrightarrow$ $f''(x) < 0$

<u>Defn</u> A <u>point of inflection</u> is where the graph changes from concave up to concave down, or vice versa.

<u>Note</u>: point of inflection $\Rightarrow f''(x) = 0$
$\not\Leftarrow$

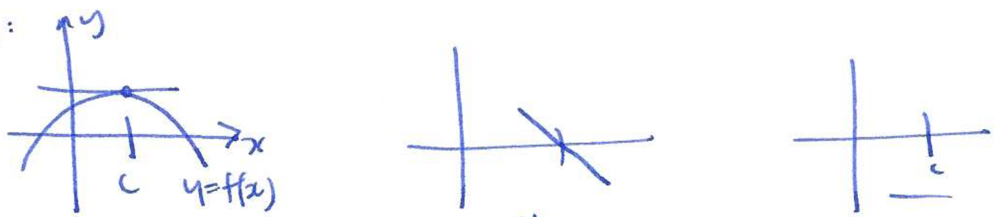


<u>Example</u> $f(x) = x^3 - x = x(x^2-1) = x(x-1)(x+1)$
$f'(x) = 3x^2 - 1$
$f''(x) = 6x$

$\left.\begin{array}{l} f''(x) > 0 \text{ for } x > 0 \\ f''(x) < 0 \text{ for } x < 0 \end{array}\right\}$ inflection pt at $x = 0$.

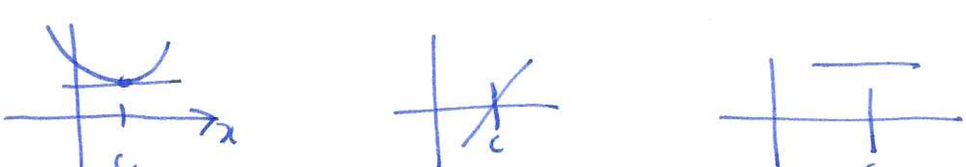
<u>Second derivative test</u>

local max:



f(x)
f'(x)
f''(x) < 0

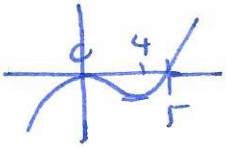
local min:



f(x)
f'(x)
f''(x) > 0.

<u>Thm</u> suppose $f(x)$ is differentiable and c is a critical pt

If $f''(c) > 0 \Rightarrow c$ is a local min
$f''(c) < 0 \Rightarrow c$ is a local max
$f''(c) = 0 \Rightarrow$ NO INFORMATION (may be local min/max/neither).



Example $f(x) = x^5 - 5x^4 = x^4(x-5)$

$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$

$f''(x) = 20x^3 - 60x^2$

critical points : $f'(x) = 0 \Rightarrow x = 0, 4$

2nd derivative test : $x = 0$ $f''(0) = 0$ no information

$x = 4$ $f''(4) = 320 > 0 \Rightarrow$ local min.

at $x = 0$ use first derivative test :

		0	
$(x-4)$	$-$		$-$
x^3	$-$		$+$
$f'(x)$	$+$		$-$

$\Rightarrow$ local max.

§4.5 L'Hôpital's rule

Thm Suppose $f(x)$ and $g(x)$ are differentiable, and $f(a) = g(a) = 0$
or $f(a) = g(a) = \pm\infty$

then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$, provided this limit exist.

Warning : ① this is not the quotient rule!

② $\frac{f(a)}{g(a)}$ must be indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, can't use l'Hopital on $\lim_{x\to 0} \frac{1}{x}$.

Examples ① $\lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \lim_{x\to 1} \frac{3x^2}{1} = 3.$

② $\lim_{x\to \frac{\pi}{2}} \frac{\cos^2 x}{\sin x - 1} = \lim_{x\to \frac{\pi}{2}} \frac{-2\cos x \sin x}{\cos x} = \lim_{x\to \frac{\pi}{2}} -2\sin x = -2$

③ $\lim_{x\to 0+} x\ln(x) = \lim_{x\to 0+} \frac{\ln(x)}{1/x} = \lim_{x\to 0+} \frac{1/x}{-x^{-2}} = \lim_{x\to 0+} -x = 0$

④ $\lim_{x\to 0} \frac{e^x - x - 1}{\cos x - 1} = \lim_{x\to 0} \frac{e^x - 1}{-\sin x} = \lim_{x\to 0} \frac{e^x}{-\cos x} = -e$