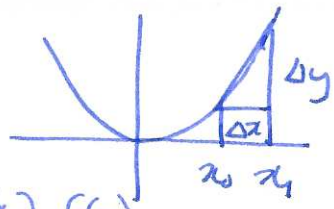
§3.4 Rates of change



recall: average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$

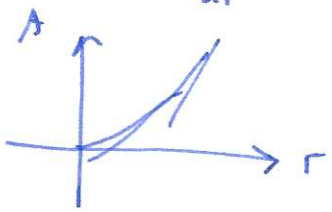
instantaneous rate of change $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_1 \to x_0} \frac{f(x_1)-f(x)}{x_1-x_0}$

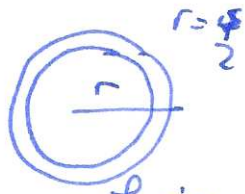
observation: if Δx is small, we can use average rate of change to approximate actual rate of change, and vice versa.

example: area of circle is πr^2

calculate rate of change of area wrt radius: $\frac{dA}{dr} = 2\pi r$

e.g. $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi$ $\left.\frac{dA}{dr}\right|_{r=5} = 10\pi$





area of ring $\simeq 4\pi\Delta r$.

for small h, $f'(x_0) \simeq \frac{f(x_0+h)-f(x)}{h}$

or $f(x_0+h) \simeq f(x_0) + h f'(x_0)$ ← linear approximation formula

example stopping distance in feet given by $F(s) = 1.1s + 0.05s^2$ (s speed in mph) calculate stopping distance when $s=30$ $F(30) = 78$.

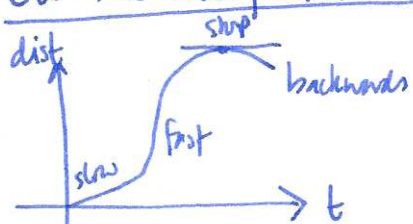
$F'(s) = 1.1 + 0.1s$ $F'(30) = 1.1 + 0.1 \cdot 30 = 4.1$ ft/mph

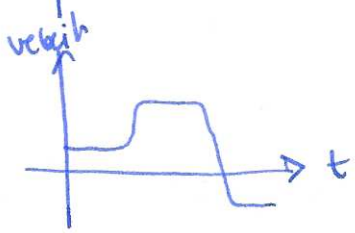
estimate stopping distance at $s=31$ (using above info):

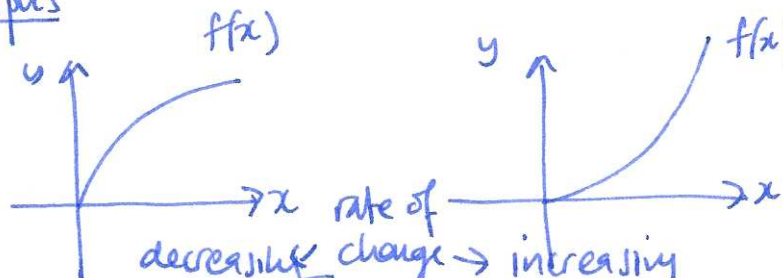
$F(s+h) \simeq F(s) + h F'(s)$

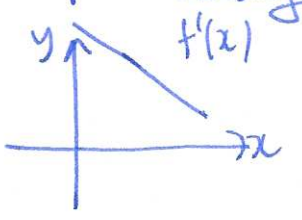
$F(31) \simeq F(30) + 1 \cdot F'(30) = 78 + 1 \cdot 4.1 = 82.1$

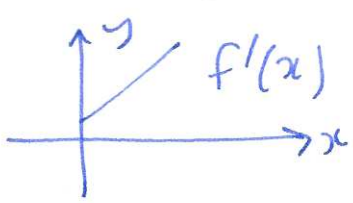
On the interpretation of graphs



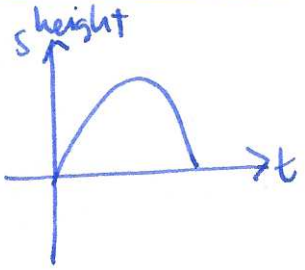








Motion under gravity



$s_0 = s(0) =$ height at $t=0$

$v_0 = v(0) =$ velocity at $t=0$

$s''(t) = v'(t) = a(t) = -g$ $\quad g = 9.8\ m/s^2$ (constant) $32\ ft/s^2$

$s'(t) = v(t) = -gt + v_0$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

Q: when is the maximum height?

A: when $s'(t) = v(t) = 0$: $\quad v_0 - gt = 0 \quad t = \frac{v_0}{g}$, so $s\left(\frac{v_0}{g}\right) =$

$$-\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\frac{v_0}{g} + \underset{0}{s_0} = \frac{1}{2}\frac{v_0^2}{g}$$

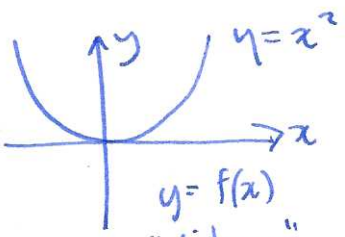
Example throw a stone upwards at 10 m/s from height 2m, what is max height?

$s(t) = 2 + 10t - \frac{1}{2}gt^2$

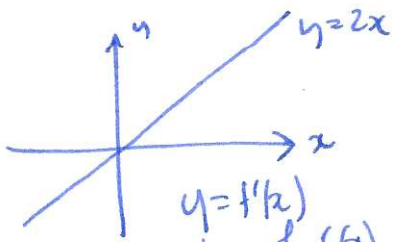
$v(t) = 10 - gt$ $\quad v(t) = 0 \Rightarrow t = \frac{10}{g} \approx 1 \quad s(1) = 2 + 10 - 5 = 7m.$

- how fast is it going when it hits the ground?
- if I can throw a stone 10m high, how fast can I throw it?

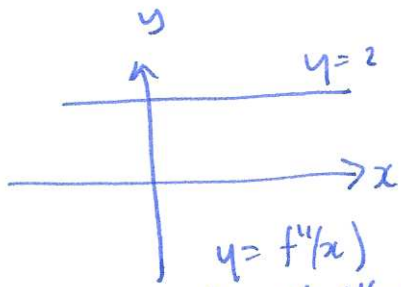
§3.5 Higher derivatives



$y = f(x)$
"distance"



$y = f'(x)$
slope of $f(x)$ "velocity"



$y = f''(x)$
slope of $f'(x)$
"acceleration"

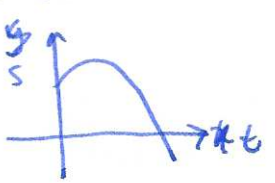
Example $f(x) = xe^x$

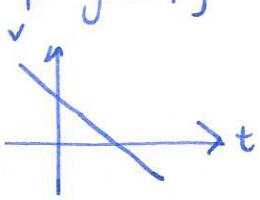
$f'(x) = xe^x + e^x$

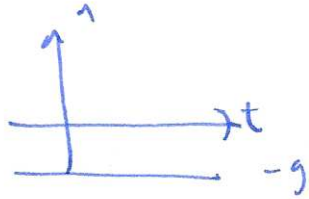
$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$. etc.

Example accelation due to gravity







$-g$ const.