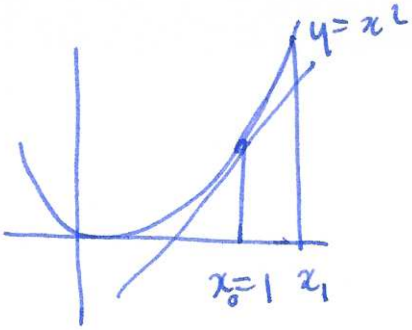


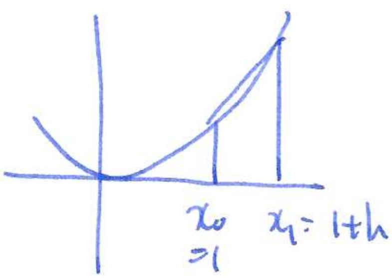
$x_1 = 2$: $\dfrac{f(2) - f(1)}{2-1} = \dfrac{4-1}{1} = 3$.

$x_1 = 1.5$: $\dfrac{f(3/2) - f(1)}{3/2 - 1} = \dfrac{9/4 - 1}{1/2} = \dfrac{5}{2} = 2.5$

$x_1 = 1.1$: $\dfrac{1.21 - 1}{0.1} = 2.1$

$x_1 = 1.01$: $\dfrac{1.0201 - 1}{0.01} = 2.01$

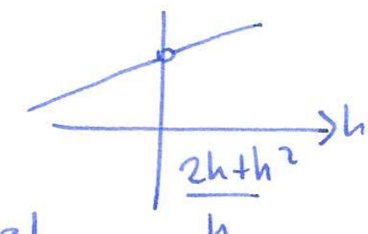
③ seems to work algebraically

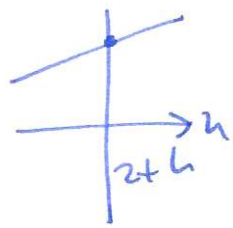


average rate of change from $\underset{x_0}{1}$ to $\underset{x_1}{1+h}$

$$= \frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+h)^2 - 1^2}{h} = \frac{1 + 2h + h^2 - 1}{h}$$

$$= \frac{2h + h^2}{h} = 2 + h$$
$(h \neq 0)$



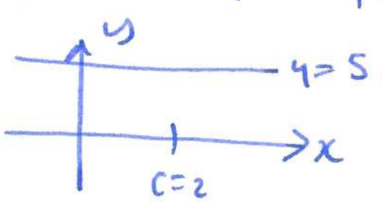


__Def^n__ Let f be a function defined on an interval containing c, but not necessarily at c. We say "the limit of $f(x)$ as x approaches c is equal to L" if $|f(x) - L|$ becomes arbitrarily small as x gets close to c

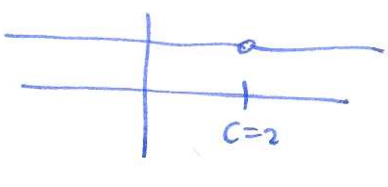
__notation__ : $\lim_{x \to c} f(x) = L$ or $f(x) \to L$ as $x \to c$.

we also say : "$f(x)$ converges to L as x tends to c".

__Examples__ a) $f(x) = 5$, $c = 2$



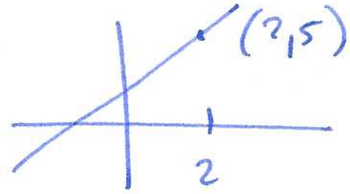
b) $f(x) = \dfrac{5(x-2)}{(x-2)}$, $c = 2$



want to show: $|f(x)-5|$ close to 0 if x close to 2

$|f(x)-5| = |5-5| = 0$ for all $x \neq 2$, so this is true.

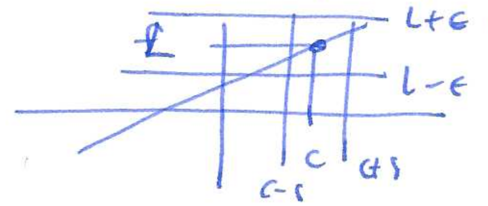
c) $\lim\limits_{x\to 2} 2x+1 = 5$



want to show: $|f(x)-5|$ close to 0 when x close to 2

$|f(x)-5| = |2x+1-5| = |2x-4| = 2|x-2|$ $\quad$ x close to 2 $\iff |x-2|$ small.

Precise defn. Let $f(x)$ be defined on an interval containing c, but not nec at c. We say $\lim\limits_{x\to c} f(x) = L$ if for all $\epsilon > 0$ there is a $\delta > 0$ s.t. if $|c-x| < \delta$ then $|f(x)-L| < \epsilon$.



useful facts

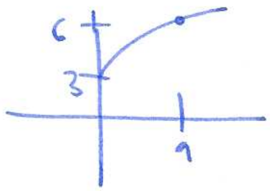
Thm for any constants k, c $\quad$ $\lim\limits_{x\to c} k = k$

$\lim\limits_{x\to c} x = c$

investigating limits: try
- drawing a picture
- calculating close values
- algebra

Example $\lim\limits_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$ $\quad$ problem: can't plug in $x=9$, get $0/0$

- draw picture



look like $f(9) = 6$

- calculate:

x	$\frac{x-9}{\sqrt{x}-3}$
8.9	5.983
9.1	6.016
8.99	5.998
6.01	6.002

- algebra: difference of two squares

$x-9 = (\sqrt{x})^2 - 3^2 = (\sqrt{x}-3)(\sqrt{x}+3)$

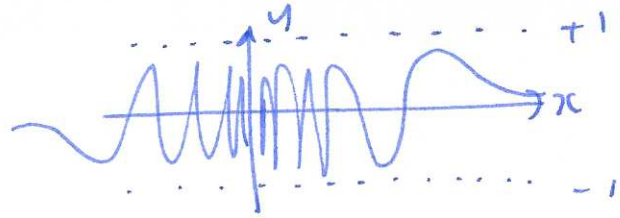
$\dfrac{x-9}{\sqrt{x}-3} = \dfrac{(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \sqrt{x}+3$ $\quad (x \neq 9)$

$\lim\limits_{x\to 9} \dfrac{x-9}{\sqrt{x}-3} = \lim\limits_{x\to 9} \sqrt{x}+3 = 6.$

bad example : no limit

$f(x) = \sin(\frac{1}{x})$

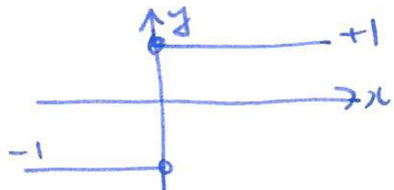
no limit at $x = 0$



note : $f(\frac{1}{2\pi n}) = \sin(2\pi n) = 0$

$f(\frac{1}{2\pi n + \frac{\pi}{2}}) = \sin(2\pi n + \frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$.

One-sided limits example $f(x) = \frac{x}{|x|}$ $\quad f(x) = \begin{cases} +1 & x > 0 \\ -1 & x < 0 \end{cases}$



sometimes useful to distinguish left limit | right limit | two sided limit

notation $\lim_{x \to 0+} f(x)$ means right limit (only consider $x > 0$)

$\lim_{x \to 0-} f(x)$ means left limit (only consider $x < 0$)

note: in order for the (two sided) limit $\lim_{x \to c} f(x)$ to exist, the left limit must equal the right limit: $\lim_{x \to c+} f(x) = \lim_{x \to c-} f(x)$.

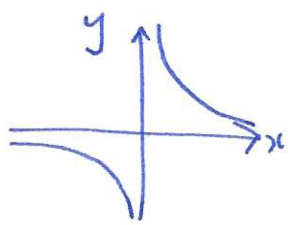
example $f(x) = \frac{x}{|x|}$ $\quad \left.\begin{array}{l} \lim_{x \to 0+} f(x) = 1 \\ \lim_{x \to 0-} f(x) = -1 \end{array}\right\}$ $1 \neq -1$ so $\lim_{x \to 0} f(x)$ DNE.

Infinite limits

we say $\lim_{x \to c} f(x) = +\infty$ if $f(x)$ becomes arbitrarily large and positive as $x \to c$

$\lim_{x \to c} f(x) = -\infty$ if $f(x)$ " .. negative as $x \to c$

Example $f(x) = \frac{1}{x}$



$\left.\begin{array}{l} \lim_{x \to 0+} \frac{1}{x} = +\infty \\ \lim_{x \to 0-} \frac{1}{x} = -\infty \end{array}\right\}$ different! so $\lim_{x \to 0} \frac{1}{x}$ DNE.