MTH 231 Calculus I

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office: 1S-222 office hours M: 12:20 – 2:15 W: 1:25 – 2:15

- math tutoring 1S-214?
- students w/ disabilities

Text: Calculus, w/ early transcendentals Rogawski, Adams

HW: webworks

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§1.2 Linear and quadratic functions

recall: input set (domain) $\xmapsto{\text{function}}$ output set (range)

examples: $f: \mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ = real numbers)
$x \mapsto x^2$ or $f(x) = x^2$ (description of function)

example values:
$0 \mapsto 0$
$1 \mapsto 1$
$-1 \mapsto 1$
$2 \mapsto 4$

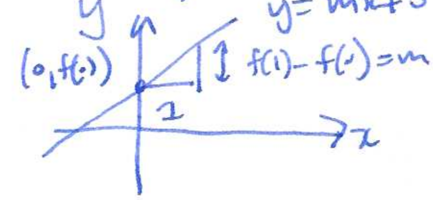
notation: $f: \mathbb{R} \to \mathbb{R}$
name ↑ ↑ inputs/domain ↑ outputs/range

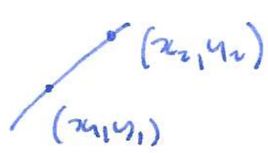
Examples
- $+: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$
  $(x, y) \mapsto x + y$
- evaluation at 0: $\left\{ \begin{array}{c} \text{functions} \\ f: \mathbb{R} \to \mathbb{R} \end{array} \right\} \to \mathbb{R}$
  $f \mapsto f(0)$

key property each input x goes to a single output value $f(x)$, not multiple values.

A linear function $f: \mathbb{R} \to \mathbb{R}$ has the form $f(x) = mx + b$ (m, b "constants", i.e. don't depend on x) The graph of a linear function is a straight line.

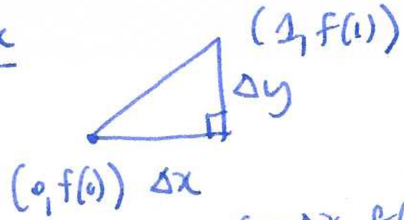
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$





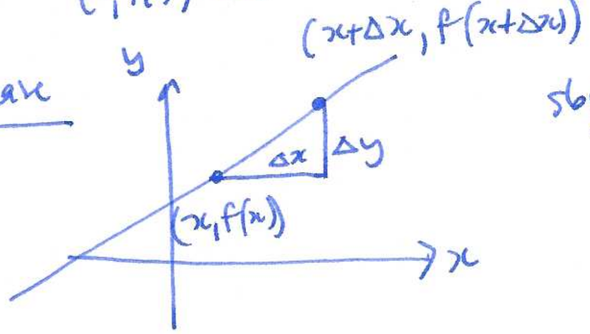
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

special case



$$\frac{\Delta y}{\Delta x} = \frac{f(1)-f(0)}{1-0} = \frac{m+b-b}{1} = m$$

general case



$$\text{slope } \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}$$

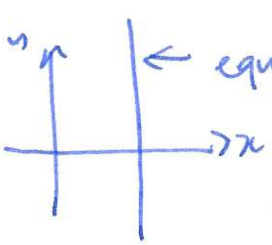
$$= \frac{m(x+\Delta x)+b-(mx+b)}{\Delta x}$$

$$= \frac{\cancel{mx}+m\Delta x+\cancel{b}-\cancel{mx}-\cancel{b}}{\Delta x}$$

$$= \frac{m\Delta x}{\Delta x} = m.$$

useful fact: a straight line has constant slope everywhere

observations:
- $|m|$ large, steep slope
- $m=0$ horizontal line
- $m>0$ increasing (from left to right)
- $m<0$ decreasing "
- vertical lines not graphs of functions



← equation of vertical line is $x=c$

equation vs function

equation ↗ relation between variables
e.g. $x=c$
$x^2+y^2=1$

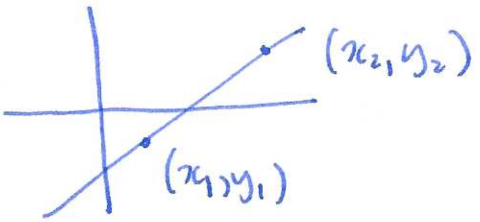
function ↑ a map from one set to another
e.g. $f: \mathbb{R} \to \mathbb{R}$.

the graph of $f: \mathbb{R} \to \mathbb{R}$ is $y=f(x)$ (an equation!) but not every equation comes from a function.

to deal with any straight line, use the general linear equation

$ax+by=c$ (at least one of a,b not zero) e.g. $x=c$, set $b=0$, $a=1$.

useful technique : find equation of line through two points



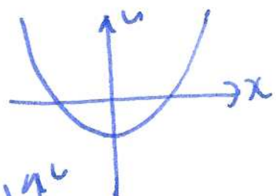
- find slope: $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- line $y - y_1 = m(x - x_1)$

point-slope formula for a line.

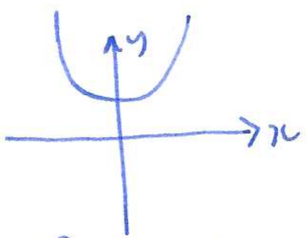
Quadratic functions

given by $f(x) = ax^2 + bx + c$ (a, b, c constants, do not depend on x)

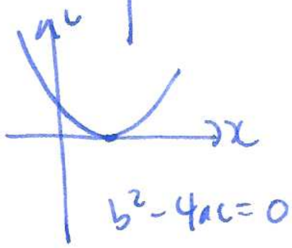
graphs are parabolas



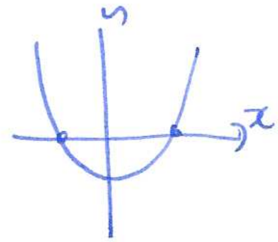
at most two distinct real solutions to $f(x) = 0$, given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$



$b^2 - 4ac < 0$



$b^2 - 4ac = 0$



$b^2 - 4ac > 0$

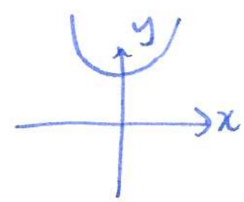
useful techniques

- factorization: $ax^2 + bx + c = a(x - r_1)(x - r_2)$, then r_1, r_2 are solutions roots.
- complete the square: any quadratic function can be written as $a(x+b)^2 + c$ example:

$x^2 + 2x + 3$

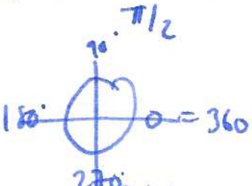
$(x+1)^2 + 2$ no solutions!

$x^2 + 2x + 1 + 2$



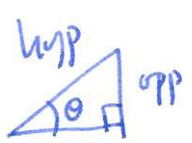
§1.4 Trig functions

- angles vs radians: radians min π



0 2π.

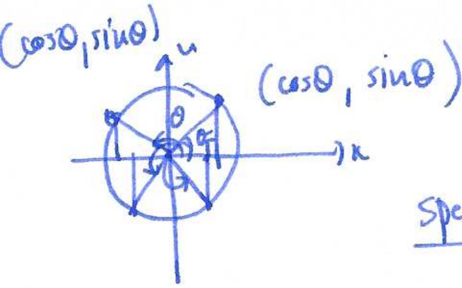
- angle in radians = distance travelled around unit circle.
- right angled triangles



$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

can extend this functions to be defined for all $\theta \in \mathbb{R}$.



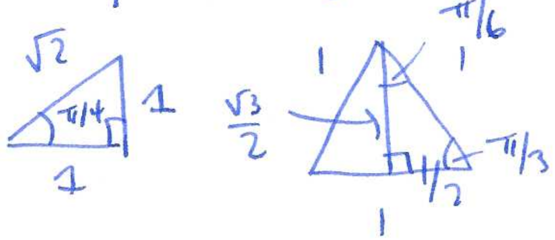
<u>useful facts</u> :

$\sin(-\theta) = -\sin\theta$ (odd function)
$\cos(-\theta) = \cos\theta$ (even function)

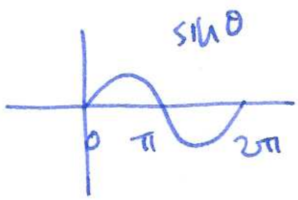
<u>special values</u> :

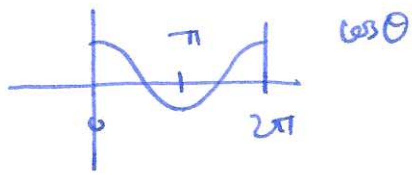
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

from special triangles :



- graphs of $\sin\theta$, $\cos\theta$ are periodic with period 2π





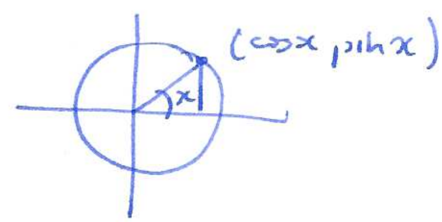
<u>other trig functions</u>

$$\tan(x) = \frac{\text{opp}}{\text{adj}} = \frac{\sin(x)}{\cos(x)}$$

$$\operatorname{cosec}(x) = \frac{1}{\sin x}$$
$$\sec(x) = \frac{1}{\cos x}$$
$$\cot(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

<u>Trig identities</u>

<u>Pythagorean identity</u> $\cos^2 x + \sin^2 x = 1$



$$\frac{\cos^2 x + \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \leftrightarrow \cot^2 x + 1 = \operatorname{cosec}^2 x$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \leftrightarrow 1 + \tan^2 x = \sec^2 x$$

<u>Double angle</u>

$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

<u>Addition</u> :

$\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$