Math 231 Calculus 1 Fall 21 Midterm 3b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x \to 0} \dfrac{e^{4x} - 1}{\sin(3x)}$.

l'H
$= \lim_{x \to 0} \dfrac{4e^{4x}}{\cos(3x) \cdot 3} = \dfrac{4}{3}$

(2) Find $\lim_{x\to 0} \dfrac{\cos(5x)-1}{\sin(3x^2)}$.

$$\overset{\text{l'H}}{=} \lim_{x\to 0} \frac{-5\sin(5x)}{\cos(3x^2)\cdot 6x} = \lim_{x\to 0} \frac{-5}{\cos(3x^2)} \cdot \frac{\sin(5x)}{6x}$$

$$\overset{\text{l'H}}{=} \lim_{x\to 0} -5 \cdot \frac{\cos(5x)\cdot 5}{6} = \frac{-25}{6}$$

(3) Consider the function $f(x) = 8\ln(x) - x^2$.
(a) Find all critical points of the function.
(b) Use the second derivative test to attempt to classify them

a) $f'(x) = \frac{8}{x} - 2x$ critical points: solve $f'(x) = 0$ $\frac{8}{x} - 2x = 0$

$x^2 = 4$

$x = \pm 2$

function only defined for $x > 0$

are critical point at $x = 2$

b) $f''(x) = -\frac{8}{x^2} - 2$ $f''(2) = -2 - 2 = -4 < 0 \Rightarrow$ local max

(4) Consider the function $f(x) = (2 - x^2)e^{-x}$.
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all the points of inflection.
(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) no vertical asymptotes, horizontal asymptotes $\lim_{x\to\infty} (2-x^2)e^{-x} = \lim_{x\to\infty} \frac{2-x^2}{e^{+x}}$

$\overset{l'H}{=} \lim_{x\to\infty} \frac{-2x}{e^x} = \lim_{x\to\infty} \frac{-2}{e^x} = 0$, right horizontal asymptote $y = 0$

left horizontal asymptote: $\lim_{x\to-\infty} (2-x^2)e^{-x} = +\infty$, no left asymptote.

b) $f'(x) = -2xe^{-x} + (2-x^2)\cdot -e^{-x} = (x^2-2x-2)e^{-x}$

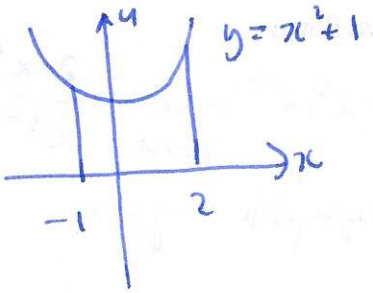
$f''(x) = (2x-2)e^{-x} + (x^2-2x-2)e^{-x}\cdot -1 = (-x^2+4x)e^{-x}$

inflection points $f''(x) = 0$: $-x(x-4)e^{-x} = 0$, $x = 0, +4$

c)
concave up $(0,4)$

down $(-\infty,0) \cup (4,\infty)$

(5) Find the area under the graph $y = x^2 + 1$ between $x = -1$ and $x = 2$.



$$\int_{-1}^{2} x^2 + 1 \, dx = \left[\frac{1}{3}x^3 + x \right]_{-1}^{2}$$

$$= \frac{8}{3} + 2 - \left(-\frac{1}{3} - 1 \right)$$

$$= \frac{8}{3} + 2 + \frac{1}{3} + 1 = \frac{12}{3} = 6$$

(6) Find the indefinite integral $\int 3e^x - 2\sqrt{x}\, dx$.

$\int 3e^x - 2x^{1/2}\, dx = 3e^x - 2\frac{x^{3/2}}{3/2} + C = 3e^x - \frac{4}{3}x^{3/2} + C$

(7) Find the indefinite integral $\int \frac{e^x}{\sqrt{1+2e^x}}\,dx$.

$u = 1 + 2e^x$

$\frac{du}{dx} = 2e^x$

$\int \frac{e^x}{\sqrt{u}} \frac{dx}{du}\,du = \int e^x\, u^{-1/2}\, \frac{1}{2e^x}\,du$

$= \frac{1}{2}\int u^{-1/2}\,du = \frac{1}{2}\,\frac{u^{1/2}}{1/2} + C = u^{1/2} + C = \sqrt{1+2e^x} + C$

(8) Find the definite integral $\int_0^1 4x\sin(3x^2)dx$.

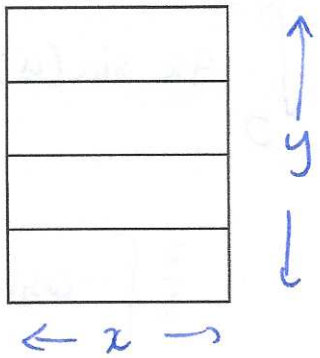
$u = 3x^2$

$\frac{du}{dx} = 6x$

$$\int_0^3 4x \sin(u) \frac{dx}{du} du = \int_0^3 4x \sin(u) \frac{1}{6x} du$$

$$= \frac{2}{3}\int_0^3 \sin(u)du = \frac{2}{3}\left[-\cos(u)\right]_0^3 = \frac{2}{3}\left(-\cos(3)+1\right).$$

(9) You wish to build a bookshelf with 4 shelves, as illustrated below. If you have 40 feet of wooden planks, what are the dimensions of the bookshelf of largest area you can construct?



$A = xy$

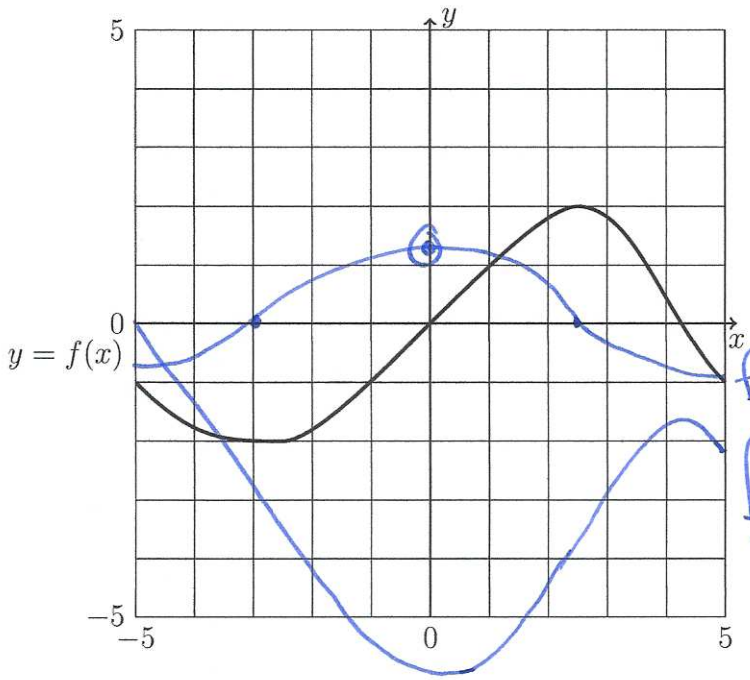
$L = 5x + 2y = 40 \quad y = \frac{40 - 5x}{2}$

$A = \frac{1}{2}x(40 - 5x) = 20x - \frac{5}{2}x^2$

$A' = 20 - 5x$ critical point $20 - 5x = 0 \quad x = 4$

$y = 10$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. ⊙ approx $x=0$

(c) Sketch the graph of $\int_{-5}^{x} f(t)\,dt$.