Math 231 Calculus 1 Fall 21 Midterm 3a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 3	
Overall	

(1) Find $\lim_{x\to 0} \frac{e^{5x}-1}{\sin(4x)}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{5e^{5x}}{\cos(4x)\cdot 4} = \frac{5}{4}$$

(2) Find $\lim_{x\to 0} \dfrac{\sin(2x^2)}{\cos(3x) - 1}$.

$$\overset{l'H}{=} \lim_{x\to 0} \frac{\cos(2x^2)\cdot 4x}{-\sin(3x)\cdot 3} = \lim_{x\to 0} -\frac{\cos(2x^2)}{3} \cdot \frac{4x}{\sin(3x)}$$

$$\overset{l'H}{=} \lim_{x\to 0} -\frac{1}{3} \cdot \frac{4}{\cos(3x)\cdot 3} = -\frac{4}{9}$$

(3) Consider the function $f(x) = 18\ln(x) - x^2$.

(a) Find all critical points of the function.

(b) Use the second derivative test to attempt to classify them

a) $f'(x) = \frac{18}{x} - 2x$

critical points, solve $f'(x) = 0$

$\frac{18}{x} - 2x = 0$

$2x^2 = 18$

$x^2 = 9$ $\quad x = \pm 3$.

function only defined for $x > 0$, so one critical pt at $x = 3$.

b) $f''(x) = \frac{-18}{x^2} - 2$

$f''(3) = \frac{-18}{9} - 2 = -4 < 0$ $\Rightarrow$ local max at $x = 3$.

(4) Consider the function $f(x) = (x^2 - 2)e^{-x}$.

(a) Find all vertical and horizontal asymptotes of the function.

(b) Find all the points of inflection.

(c) Determine the intervals where $f(x)$ is concave up and concave down.

a) no vertical asymptotes. $\lim_{x\to\infty} (x^2-2)e^{-x} = \lim_{x\to\infty} \frac{x^2-2}{e^x}$

$\overset{\text{l'H}}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$ so left ~~vertical~~ horizontal asymptote $y=0$

$\lim_{x\to-\infty} (x^2-2)e^{-x} = +\infty$, no right horizontal asymptote.

b) $f'(x) = 2x\,e^{-x} + (x^2-2)e^{-x}\cdot -1 = (-x^2+2x+2)e^{-x}$

$f''(x) = (-2x+2)e^{-x} + (-x^2+2x+2)e^{-x}\cdot -1 = (x^2-4x+\cancel{4})e^{-x}$

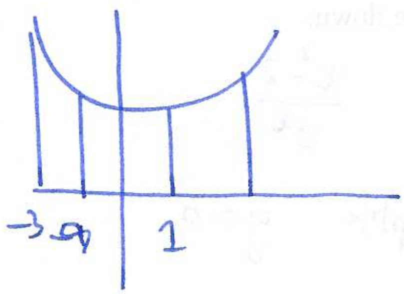
$(x-4)x\,e^{-x}$

inflection points : $f''(x)=0$, at $x=0, 4$.

concave up $(-\infty, 0) \cup (4, \infty)$

down $(0, 4)$

(5) Find the area under the graph $y = x^2 + 1$ between $x = -3$ and $x = 1$.



$$\int_{-3}^{1} x^2 + 1 \, dx = \left[\frac{1}{3} x^3 + x \right]_{-3}^{1}$$

$$= \frac{1}{3} + 1 - \left(\frac{1}{3} \cdot -27 - 3 \right)$$

$$= \frac{1}{3} + 1 + 9 + 3 = 13\frac{1}{3}$$

(6) Find the indefinite integral $\int 2e^x - 3\sqrt{x}\, dx$.

$$\int 2e^x - 3x^{1/2}\, dx = 2e^x - \frac{3x^{3/2}}{3/2} + c = 2e^x - 2x^{3/2} + c$$

(7) Find the indefinite integral $\int \frac{e^x}{\sqrt{1+3e^x}}\,dx$.

$u = \cancel{e^x}\ 1+3e^x$

$\frac{du}{dx} = 3e^x$

$\int \frac{e^x}{\sqrt{u}} \frac{dx}{du}\, du = \int \frac{e^x}{\sqrt{u}} \cdot \frac{1}{3e^x}\, du$

$= \frac{1}{3}\int u^{-1/2}\, du = \frac{1}{3} \cdot \frac{u^{1/2}}{1/2} + C = \frac{2}{3}\sqrt{1+3e^x} + C$

(8) Find the definite integral $\int_0^1 5x\cos(2x^2)dx$.

$u = 2x^2$

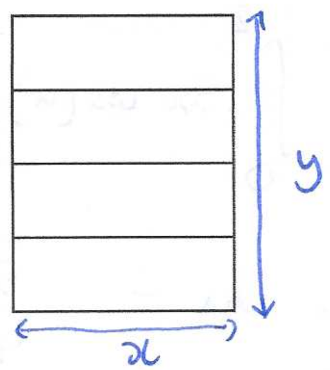
$\frac{du}{dx} = 4x$

$= \int_0^2 5x\cos(u)\frac{dx}{du}\,du$

$= \int_0^2 5x\cos(u)\frac{1}{4x}\,du = \frac{5}{4}\int_0^2 \cos(u)\,du$

$= \frac{5}{4}\left[\sin(u)\right]_0^2 = \frac{5}{4}\sin(2)$

(9) You wish to build a bookshelf with 4 shelves, as illustrated below. If you have 20 feet of wooden planks, what are the dimensions of the bookshelf of largest area you can construct?



area $A = xy$

length $L = 5x + 2y = 20$

$$y = \frac{20 - 5x}{2}$$

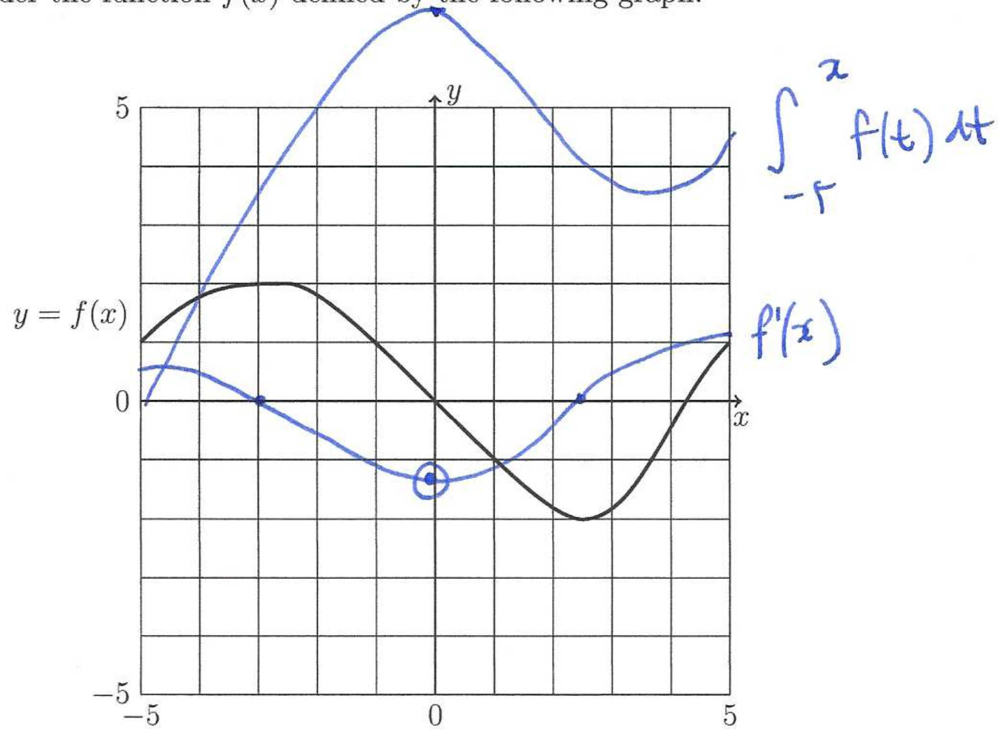
$$A = \frac{1}{2}x\,(20 - 5x) = 10x - \frac{5}{2}x^2$$

$A' = 10 - 5x$ critical point solve $A' = 0$: $10 - 5x = 0$

$x = 2$

$y = 5$

(10) Consider the function $f(x)$ defined by the following graph.



(a) Sketch a graph of $f'(x)$ on the figure.

(b) Label the points of inflection of $f(x)$. ⊙ might at $x = 0$

(c) Sketch the graph of $\displaystyle\int_{-5}^{x} f(t)\, dt$.