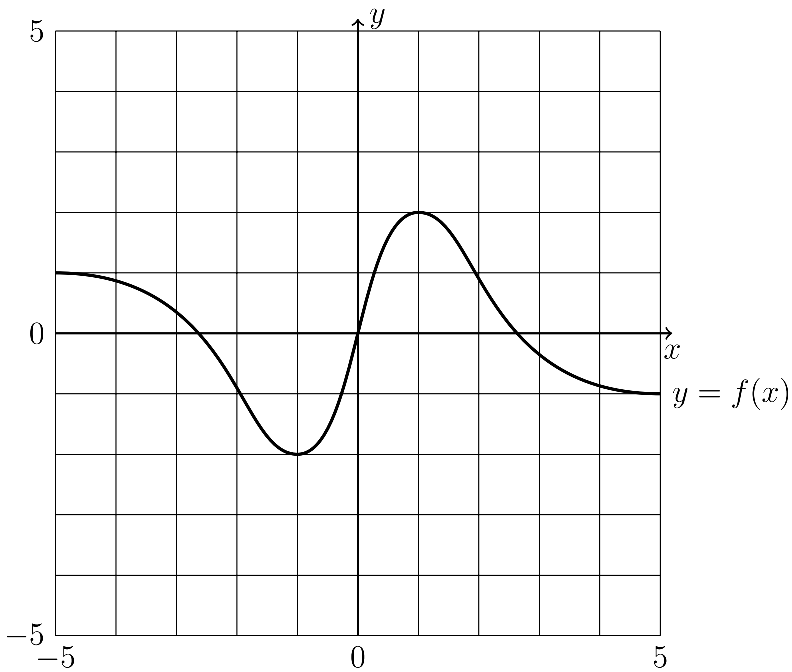
Math 231 Calculus 1 Fall 21 Sample Midterm 3

(1) Consider the function $f(x)$ defined by the following graph.



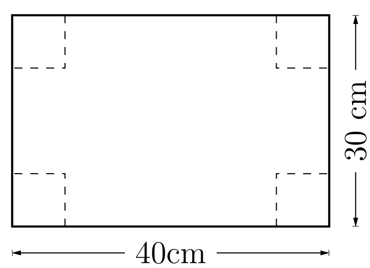
(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) What is $\lim_{x\to\infty} f(x)$?
(d) What is $\lim_{x\to-\infty} f'(x)$?
(e) What is $\lim_{x\to\infty} f''(x)$?
(f) Sketch a graph of $f'(x)$ on the figure.
(g) Sketch a graph of $\int_{-5}^{x} f(t)dt$ on the figure.
(h) Label the approximate locations of all points of inflection.

(2) Consider the function

$$f(x) = e^{4-x^2}$$

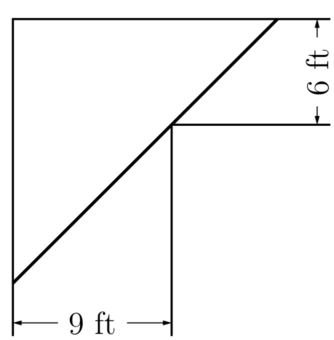
(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.
(d) Find the inflection points.
(e) Determine the intervals where $f(x)$ is concave up and concave down.
(f) Use the 2nd derivative test to attempt to identify all local maxima and minima.
(g) Sketch the function and label all relative maxima and minima.

(3) We have a piece of cardboard that is 40cm by 30cm and we are going to cut out the corners and fold up the sides to form a box. Determine the height of the box that will give a maximum volume.



(4) Find the point on the line $y = 2x - 3$ which is closest to the point $(7, 2)$.

(5) A piece of pipe is being carried down a hallway that is 9 feet wide. At the end of the hallway there is a right-angled turn and the hallway narrows down to 6 feet wide. What is the longest pipe (always keeping it horizontal) that can be carried around the turn in the hallway?



(6) Compute the following limits. Show all work.

(a)
$$\lim_{x\to 4} \frac{3x^2-11x-4}{2x^2-3x-20}$$

(b)
$$\lim_{x\to 0} \frac{\tan 3x}{2x}$$

(c)
$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$$

(d)
$$\lim_{x\to 8} \frac{2x^2-31x+120}{\sqrt{x+1}-3}$$

(e)
$$\lim_{x\to 5} \frac{3x^2-75}{1/5-1/x}$$

(f)
$$\lim_{x\to 0} \frac{\tan^{-1}(8x)}{\sin^{-1}(2x)}$$

(g)
$$\lim_{x\to 0} \frac{x^2 e^{x/2}}{\tan^2(3x/4)}$$

(h)
$$\lim_{x\to\infty} \frac{4x^3+6x^2-9x+1}{2x^3-5x^2+8}$$

(i)
$$\lim_{x\to\infty} \frac{e^{x^2}}{x^2+1}$$

(j)
$$\lim_{x\to 1/2^-} \frac{\tan \pi x}{\ln(1-2x)}$$

(7) Approximate the area under the graph of $y = e^{-x}$ between 0 and 2 using four rectangles. Use the left hand endpoints to find the heights of the rectangles. Can you say whether this is an under- or over-estimate?

(8) Evaluate the following

(a)
$$\int \frac{1+3x-2x^2}{\sqrt{x}} dx$$

(b)
$$\int_{-2}^{2} |x| \, dx$$

(c)
$$\int_{1}^{27} \frac{2}{\sqrt[3]{x}} dx$$

(d)
$$\int_{0}^{2} e^{-4x} dx$$

(e)
$$\int_{0}^{x} \frac{1}{t+2} dt$$

(f)
$$\int \frac{1}{1+4x^2} dx$$

(g)
$$\int \frac{x}{1+4x^2} dx$$

(h)
$$\int \sin(4x) dx$$

(i)

$$\int x\cos(1+x^2)dx$$

(j)

$$\int \frac{\sin(x)}{e^{\cos x}}dx$$

(k)

$$\int \frac{\cos(x)}{\sin^2(x)}dx$$

(9) A particle starting at the origin at time $t = 0$ moves along the x-axis with velocity $v(t) = (t+2)^{-5}$. Will the particle ever reach $x = 1$?