Math 231 Calculus 1 Fall 21 Midterm 2b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 2	
Overall	

(1) (10 points) Find the derivative of the following functions.

(a) $f(x) = x^2 \sin(x)$.

$f'(x) = 2x\sin(x) + x^2 \cdot \cos(x)$

(b) $f(x) = \dfrac{\ln(x)}{e^x}$.

$f'(x) = \dfrac{e^x \frac{1}{x} - e^x \ln(x)}{(e^x)^2}$

(2) (10 points) Find the derivative of the function $f(x) = \tan^{-1}\left(3 + \sqrt{x}\right)$.

$$f(x) = \frac{1}{1+(3+\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2}$$

(3) (10 points) Find the second derivative of the function $f(x) = \sqrt{3x - 2}$.

$$f'(x) = \frac{1}{2}(3x-2)^{-1/2} \cdot 3$$

$$f''(x) = -\frac{1}{4}(3x-2)^{-3/2} \cdot 3 . 3 = -\frac{9}{4}(3x-2)^{-3/2}$$

(4) (10 points) Use implicit differentiation to find the tangent line to the curve given by the equation $2x^2 + y^3 = xy + 12$ at the point $(-1, 2)$.

$4x + 3y^2 y' = y + xy'$

$x = -1$
$y = 2$: $\quad -4 + 12y' = 2 - y'$

$13y' = 6 \qquad y' = 6/13.$

$y - 2 = \frac{6}{13}(x+1)$

(5) Find the following limit: $\lim_{x\to 0} \frac{\sin^2(3x)}{e^{2x^2}-1}$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2\sin(3x).\cos(3x).3}{e^{2x^2}.4x} = \lim_{x\to 0} \frac{3\sin 6x}{4xe^{2x^2}}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{3\cos 6x.6}{4e^{2x^2}+4xe^{2x^2}.4x} = \frac{18}{4} = \frac{9}{2}$$

(6) (10 points) An oil tanker is leaking oil and forming a circular oil slick. If the area of the oil slick is growing at a rate of $10\text{m}^2/\text{minute}$, how fast is the radius growing when the radius is 6m? (The area of a circle is $A = \pi r^2$.)

$A(t) = \pi r(t)^2$

$\frac{dA}{dt} = \pi\, 2r \frac{dr}{dt}$ $\qquad \frac{dA}{dt} = 10 \quad r = 6 :$ $\qquad 10 = 12\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{10}{12\pi} = \frac{5}{6\pi}$

(7) (10 points) Use linear approximation to estimate $\sqrt{99}$. What is the percentage error in your approximation?

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(99) \approx f(100) + f'(100)(99-100)$

$= 10 + \frac{1}{20}(-1) = 9.95$

percentage error $= \frac{|9.95-\sqrt{99}|}{\sqrt{99}} \times 100 \approx 0.00126\%$

(8) Find the critical points for the function $f(x) = 12x - x^3$ and use the first derivative test to classify them.

$f'(x) = 12 - 3x^2$

$= -3(x-2)(x+2)$

critical points, solve $f'(x) = 0$:

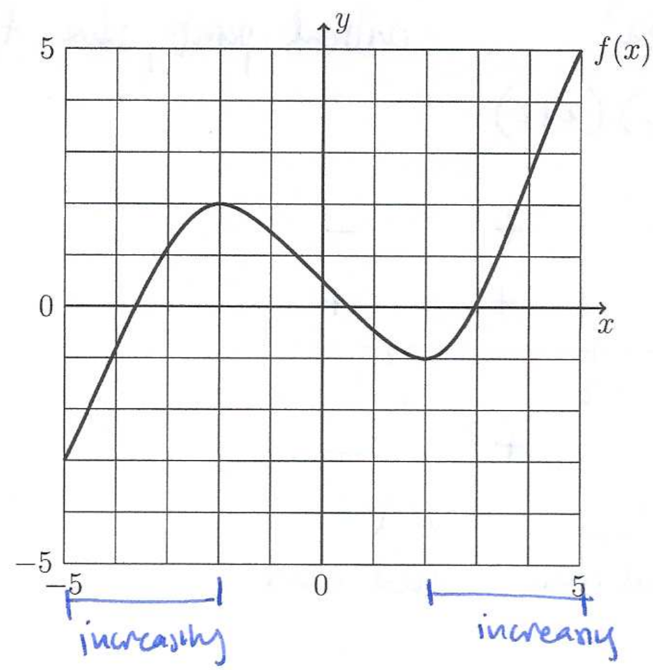
$12 - 3x^2 = 0$

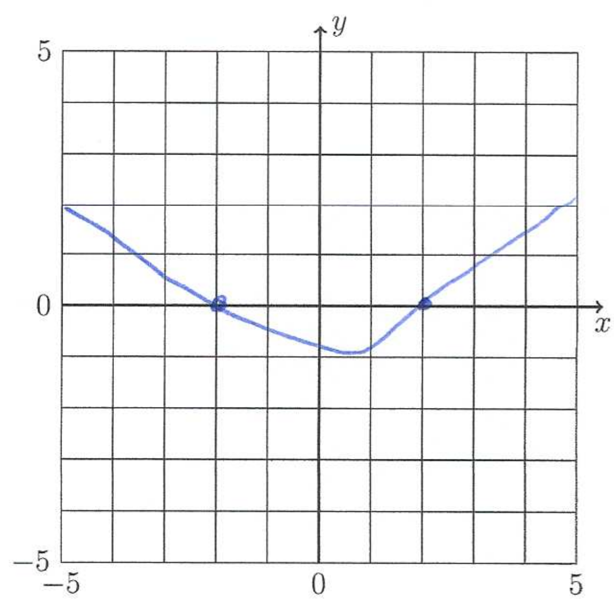
$x^2 = 4$

$x = \pm 2$.

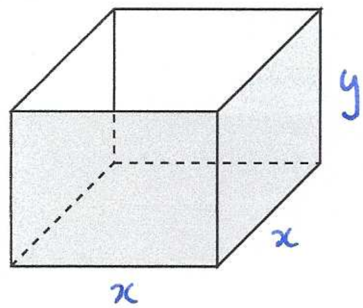
	$x < -2$	-2	$-2 < x < 2$	$+2$	$x > 2$
$-3(x-2)$	$+$		$+$		$-$
$(x+2)$	$-$		$+$		$+$
$f'(x)$	$-$		$+$		$-$
		$\cup$ local min		$\cap$ local max	

(9) (10 points) The graph of the function $f(x)$ is shown below. On the top set of axes mark where $f(x)$ is increasing. On the lower set of axes sketch $f'(x)$.





(10) A container consists of a square base and four vertical sides, but without a top side. If the total volume of the container is 1m^3, what is the smallest possible area of the container?



$V = x^2 y = 1$

$A = x^2 + 4xy$

$y = \frac{1}{x^2}$

$A = x^2 + \frac{4x}{x^2} = x^2 + \frac{4}{x}$

$\frac{dA}{dx} = 2x - \frac{4}{x^2}$ critical pt: $\frac{dA}{dx} = 0$: $2x - \frac{4}{x^2} = 0$

$x^3 = 2$

$x = \sqrt[3]{2}$

$y = \frac{1}{(\sqrt[3]{2})^2} = \frac{1}{\sqrt[3]{4}}$

$A = (\sqrt[3]{2})^2 + \frac{4\sqrt[3]{2}}{\sqrt[3]{4}} = \sqrt[3]{4} + \frac{4}{\sqrt[3]{2}}$