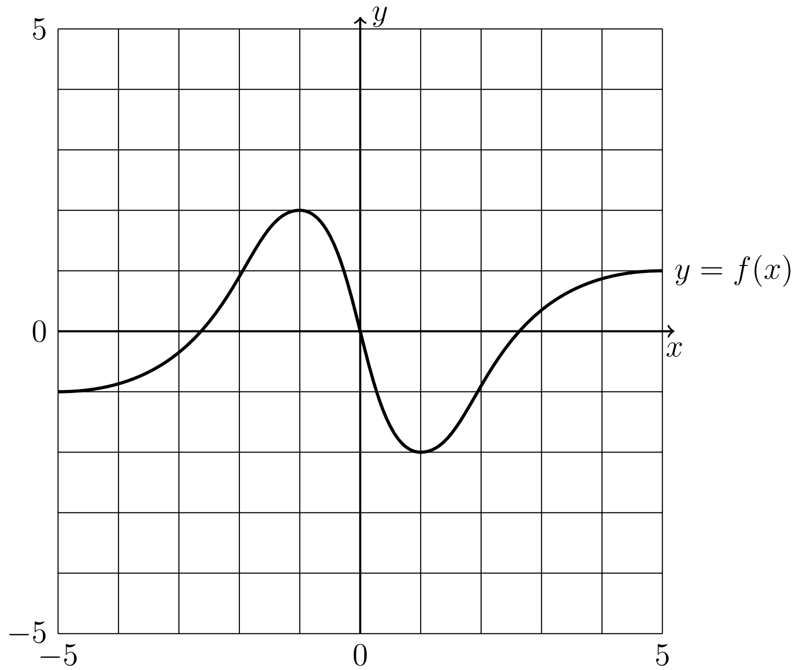
Math 231 Calculus 1 Fall 21 Sample Midterm 2

(1) Consider the function $f(x)$ defined by the following graph.

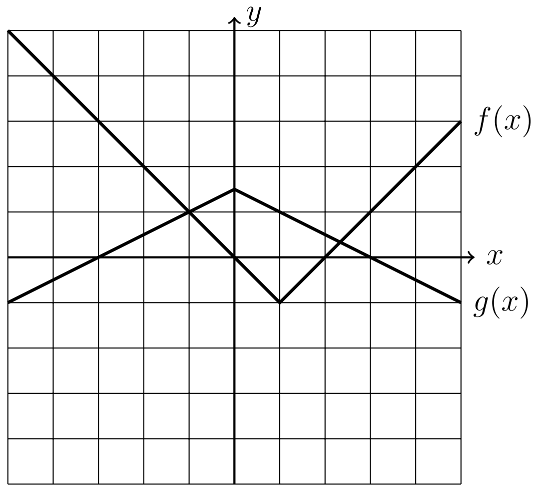


(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) Sketch a graph of $f'(x)$ on the figure.
(d) What is $\lim_{x\to\infty} f(x)$?
(e) What is $\lim_{x\to-\infty} f'(x)$?
(f) Label the approximate locations of all points of inflection.

(2) Find the derivatives of the following functions
(a) $x^4e^{-2x^4}$
(b) $\dfrac{\sqrt{1-2x}}{4-\sin(3x)}$
(c) x^{x^2}
(d) $\sqrt{\csc(\ln(x))}$
(e) $\tan^{-1}(3/\sqrt[3]{x})$
(f) $\sin^{-1}(3x-2)$

(3) Find the second derivatives of the functions above.

(4) The graphs of the functions f and g are shown below.



(a) Let $h(x) = f(x)g(x)$ Find $h'(2)$.
(b) Let $h(x) = f(g(x))$. Find $h'(-2)$.

(5) Use implicit differentiation to find the tangent line to the hyperbola $4x^2 - y^2 = 4$ at the point $(2, -2)$.

(6) Find $\frac{dy}{dx}$ for the implicit function $x^2y - x^2y^3 = \cos(xy)$.

(7) You inflate a spherical balloon at a rate of 10cm^3 per second. How fast is the area of the balloon increasing when the radius is 20cm?

(8) Use a linear approximation to estimate $\sqrt[3]{63}$. What is the percentage error?

(9) Find all the critical points for the function $f(x) = e^x(x^2 - x - 7)$. Use the first derivative test to identify them as local maxima or local minima.

(10) Find the absolute maximum and minimum of the function $f(x) = x^2 - 4x - 4$ on the interval $[-2, 2]$.

(11) Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 1$ as its only critical point.

$$
\begin{aligned}
f(1) &= 3 \\
f'(1) &= 0 \\
f'(x) &> 0 \text{ for } x < 1 \\
f'(x) &< 0 \text{ for } x > 1 \\
\lim_{x\to\infty} f'(x) &= \lim_{x\to-\infty} f'(x) = 1
\end{aligned}
$$

(12) Consider the function

$$f(x) = \frac{e^x}{9 - x^2}$$

(a) Find all vertical and horizontal asymptotes of the function.
(b) Find all critical points of the function.
(c) Determine the intervals where $f(x)$ is increasing and decreasing.
(d) Use the 2nd derivative test to attempt to identify all local maxima and minima.
(e) Sketch the function and label all relative maxima and minima.

(13) Consider the following function:

$$g(x) = x \ln x - 2x$$

(a) Find, if they exist, the coordinates of all relative maxima and minima.
(b) Determine the interval(s) where g is increasing and those where g is decreasing.
(c) Find, if they exist, the coordinates of all points of inflection.
(d) Determine the intervals where g is concave up and those where g is concave down.
(e) Sketch the curve as accurately as possible.

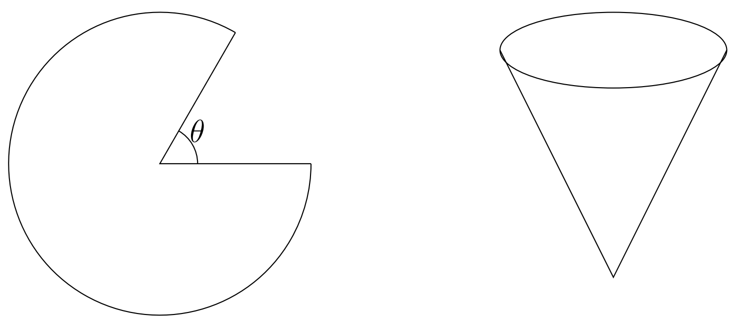
(14) A function $f(x)$ has derivative

$$f'(x) = \frac{1}{e^{x^2} + 1}.$$

Where on the interval $[1, 3]$ does it take its maximum value?

(15) You are swimming in the ocean 100m from the shore, and you wish to get to a point 200m along the beach as quickly as possible. If you can run twice times as fast as you can swim, what is your fastest route? Assume you should swim in a straight line to some point along the shore.

(16) A circular piece of paper of radius R has a sector removed of angle θ, and the remainder is folded into a cone shaped cup. Which angle θ maximizes the volume?



(17) Compute the following limits. Show all work.

(a)
$$\lim_{x\to-\infty} \frac{2-3x}{\sqrt{2x^2-3}}$$

(b)
$$\lim_{x\to 0} \frac{\sin^{-1}(2x)}{\cos^{-1}(3x)}$$

(c)
$$\lim_{x\to 0} \sin(x)\ln(x)$$

(d)
$$\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{e^x - 1} \right)$$

(e)
$$\lim_{x\to 0} \frac{3\tan x - \tan 3x}{\sin^2 2x}$$