Math 231 Calculus 1 Fall 21 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

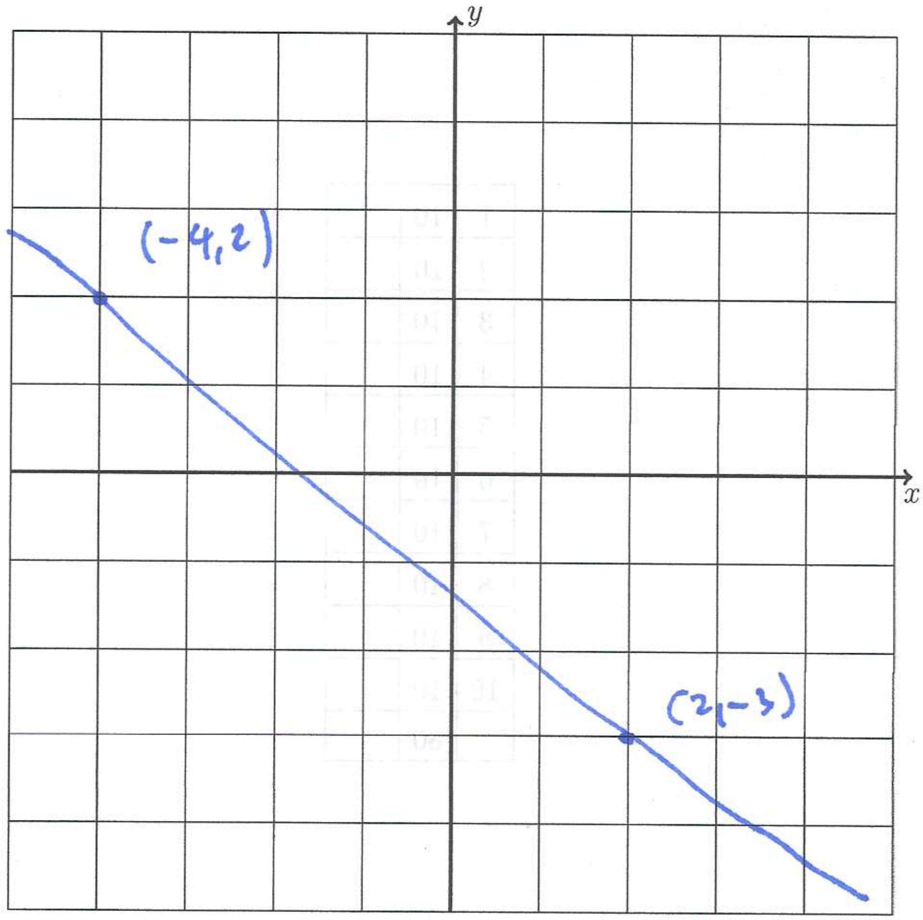
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(2, -3)$ and $(-4, 2)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

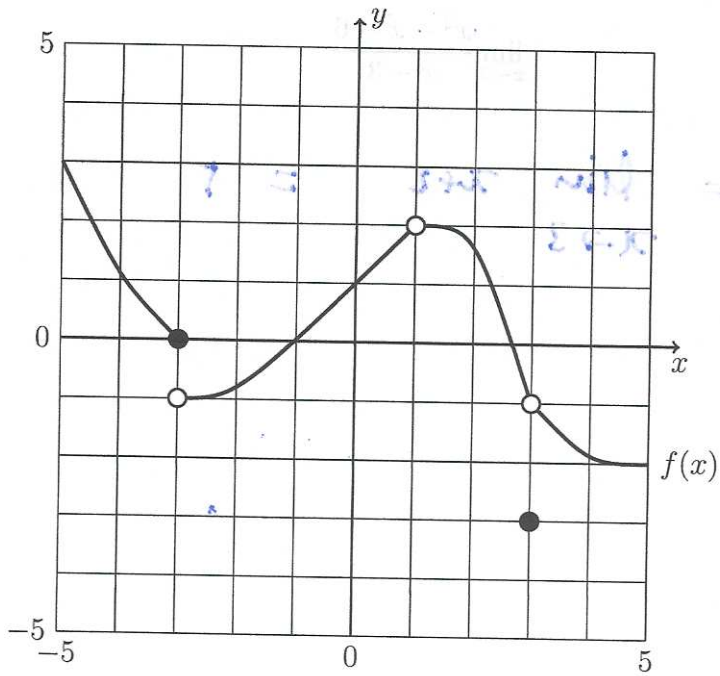
slope

$$m = \frac{2-(-3)}{-4-2} = -\frac{5}{6}$$



$$y-2 = -\frac{5}{6}(x+4)$$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x\to -3-} f(x)$ 0

(b) $\lim_{x\to -3} f(x)$ DNE

(c) $\lim_{x\to 1+} f(x)$ 2

(d) $\lim_{x\to 1} f(x)$ 2

(e) $\lim_{x\to 3-} f(x)$ −1

(f) $\lim_{x\to 3} f(x)$ −1

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 3} \frac{x^2 - x - 6}{x - 3}$$

$$= \lim_{x\to 3} \frac{(x-3)(x+2)}{x-3} = \lim_{x\to 3} x+2 = 5$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 4} \frac{2-\sqrt{x}}{4-x}$$

$$= \lim_{x\to 4} \frac{2-\sqrt{x}}{(2-\sqrt{x})(2+\sqrt{x})} = \lim_{x\to 4} \frac{1}{2+\sqrt{x}} = \frac{1}{4}$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 - 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - 1 - (x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{1} - \cancel{x^2} + \cancel{1}}{h} = \lim_{h \to 0} 2x + h = 2x$$

(6) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x-2}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{x-2-(x+h-2)}{(x+h-2)(x-2)} = \lim_{h \to 0} \frac{-h}{h(x+h-2)(x-2)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}$$

(7) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{4x^2+1}}{4x-1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4+1/x^2}}{4-1/x} = \frac{\sqrt{4}}{4} = \frac{1}{2}$$

$-x^{-1/2}$

(8) Find the first and second derivatives of $f(x) = x^2 + \cos(x) - 1/\sqrt{x}$.

$f'(x) = 2x - \sin(x) + \frac{1}{2}x^{-3/2}$

$f''(x) = 2 - \cos(x) - \frac{3}{4}x^{-5/2}$

$$\left(\frac{f}{g}\right)' = \frac{gf' - g'f}{g^2}$$

$x^{1/3}$.

(9) Find the first and second derivatives of $f(x) = \frac{x}{e^x} + \sqrt[3]{x}$.

$$f'(x) = \frac{e^x.(x)' - (e^x)'x}{(e^x)^2} + \frac{1}{3}x^{-2/3}.$$

$$f'(x) = \frac{e^x - xe^x}{e^{2x}} + \frac{1}{3}x^{-2/3}.$$

$$(fg)' = f'g + fg'$$

$$f''(x) = \frac{e^{2x}.(e^x - xe^x)' - (e^{2x})'(e^x - xe^x)}{(e^{2x})^2} - \frac{2}{9}x^{-5/3}$$

$$f''(x) = \frac{e^{2x}(\cancel{e^x} - \cancel{e^x} - xe^x) - (e^x(e^x)' + e^x(e^x)')(e^x - xe^x)}{e^{4x}} - \frac{2}{9}x^{-5/3}$$

$$f''(x) = \frac{-xe^{3x} - 2e^{2x}(e^x - xe^x)}{e^{4x}} - \frac{2}{9}x^{-5/3}.$$

$$f''(x) = \frac{-x - 2 + 2x}{e^x} - \frac{2}{9}x^{-5/3}.$$

$$= \frac{-2 + x}{e^x} - \frac{2}{9}x^{-5/3}.$$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.