Math 231 Calculus 1 Fall 21 Midterm 1a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3 × 5 index card of notes.

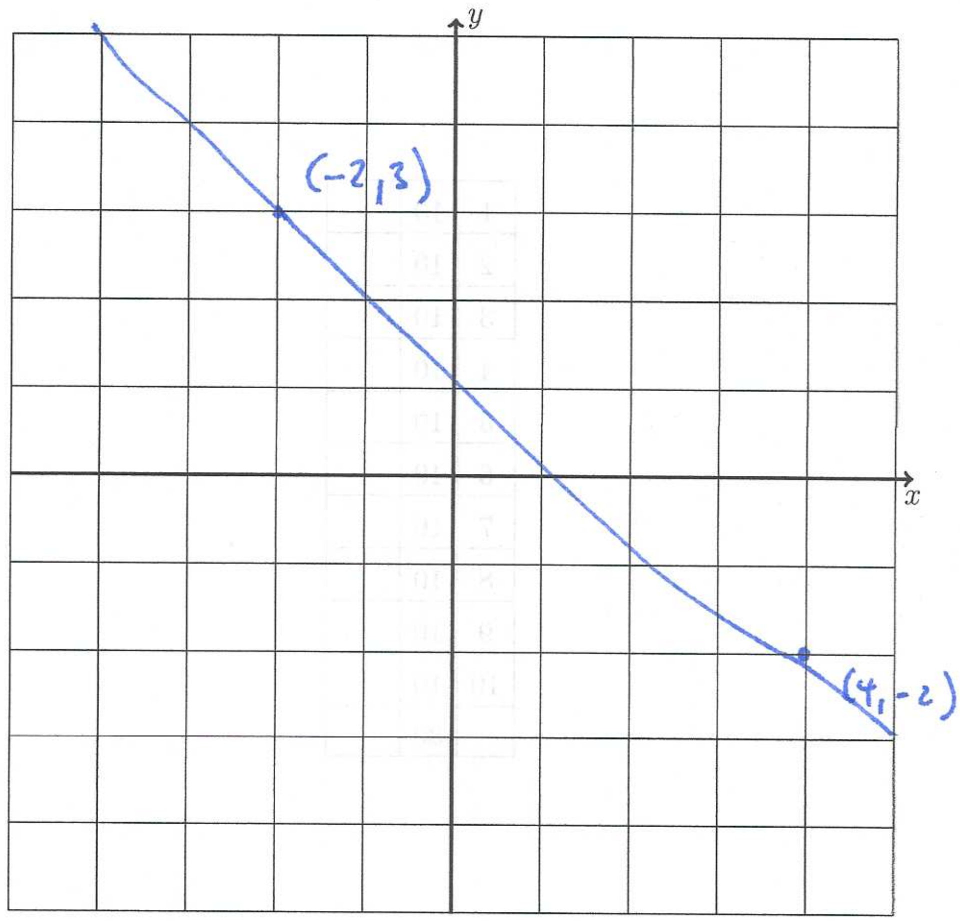
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Plot the points $(4, -2)$ and $(-2, 3)$ on the grid below, and draw the straight line through the two points. Find the equation of the straight line.

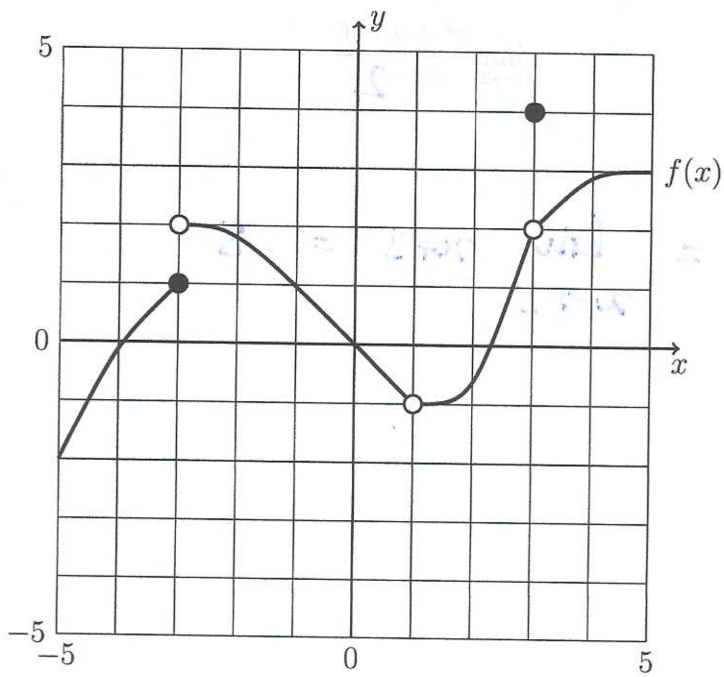
slope

$$m = \frac{3-(-2)}{-2-4} = -\frac{5}{6}$$



$$y - 3 = -\frac{5}{6}(x+2)$$

(2) (10 points) The graph of $y = f(x)$ is shown below. Evaluate each limit, or write DNE if the limit does not exist. No justifications are necessary.



(a) $\lim_{x \to -3^-} f(x)$ 1

(b) $\lim_{x \to -3} f(x)$ DNE

(c) $\lim_{x \to 1^+} f(x)$ −1

(d) $\lim_{x \to 1} f(x)$ −1

(e) $\lim_{x \to 3^-} f(x)$ 2

(f) $\lim_{x \to 3} f(x)$ 2

(3) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x\to 2} \frac{x^2+x-6}{x-\cancel{3}\,2}$$

$$= \lim_{x\to 2} \frac{(x-2)(x+3)}{(x-2)} = \lim_{x\to 2} x+3 = 5$$

(4) (10 points) Evaluate the limit algebraically. For an infinite limit, write $+\infty$ or $-\infty$. If a limit does not exist (DNE), you must justify why this is the case.

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x}$$

$$= \lim_{x \to 9} \frac{3-\sqrt{x}}{(3-\sqrt{x})(3+\sqrt{x})} = \lim_{x \to 9} \frac{1}{3+\sqrt{x}} = \frac{1}{6}$$

(5) (10 points) Use the limit definition of the derivative to differentiate $f(x) = x^2 + 2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + \cancel{2} - x^2 - \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h \to 0} 2x + h = 2x$$

(6) (10 points) Use the limit definition of the derivative to differentiate $f(x) = \dfrac{1}{x-2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x-2-(x+h-2)}{(x+h-2)(x-2)} = \lim_{h\to 0} \frac{-h}{h(x+h-2)(x-2)}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}$$

(7) (10 points) Find the following limit.

$$\lim_{x\to\infty} \frac{\sqrt{3x^2-1}}{3x+1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{3 - 1/x^2}}{3 + 1/x} = \frac{\sqrt{3}}{3}$$

$x^{-1/2}$

(8) Find the first and second derivatives of $f(x) = x^3 + \sin(x) + 1/\sqrt{x}$.

$f'(x) = 3x^2 + \cos(x) - \frac{1}{2}x^{-3/2}$

$f''(x) = 6x - \sin(x) + \frac{3}{4}x^{-5/2}$

$x^{1/3}$

(9) Find the first and second derivatives of $f(x) = \frac{e^x}{x} - \sqrt[3]{x}$.

$\left(\frac{f}{g}\right)' = \frac{gf' - g'f}{g^2}$

$$f'(x) = \frac{x(e^x)' - (x)'e^x}{x^2} - \frac{1}{3}x^{-4/3}.$$

$$f'(x) = \frac{xe^x - e^x}{x^2} - \frac{1}{3}x^{-4/3}$$

$$f''(x) = \frac{x^2(xe^x - e^x)' - (x^2)'(xe^x - e^x)}{x^4} + \frac{4}{9}x^{-7/3}$$

$$f''(x) = \frac{x^2(xe^x + \cancel{e^x} - \cancel{e^x}) - 2x(xe^x - e^x)}{x^4} + \frac{4}{9}x^{-7/3}.$$

$$f''(x) = \frac{x^2e^x - 2xe^x + 2e^x}{x^3} + \frac{4}{9}x^{-7/3}$$

(10) (10 points) The graph of $f(x)$ is given in the top picture. Sketch the graph of $f'(x)$ in the bottom picture.