Math 231 Calculus 1 Fall 21 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

(a) $-3x^4 + 4\sqrt[3]{x^5} - \cot(2x)$

(b) $f(x) = \dfrac{x - x^2}{\ln(3x - 1)}$

(c) $f(x) = e^{-4x}\sin(3 - 2x)$

(d) $f(x) = \sqrt[3]{e^{-\cos(4x)} + 3}$

(2) Evaluate the following integrals.

(a) $\displaystyle\int \frac{1}{x^3} - 3\sin(x) + e^x \; dx$

(b) $\displaystyle\int \frac{(2 - x)^2}{\sqrt{x^3}} \; dx$

(c) $\displaystyle\int_0^{\pi/4} \sin(4x)\cos^3(4x) \; dx$

(d) $\displaystyle\int \frac{1}{9 + x^2} \; dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE). Justify your answers.

(a) Find $\displaystyle\lim_{x \to 4} \frac{x^2 - x - 12}{x - 4}$.

(b) Find $\displaystyle\lim_{x \to 0} \frac{1 - e^{2x}}{\sin(5x)}$.

(c) Find $\displaystyle\lim_{x \to 0+} x^{\tan(x)}$.

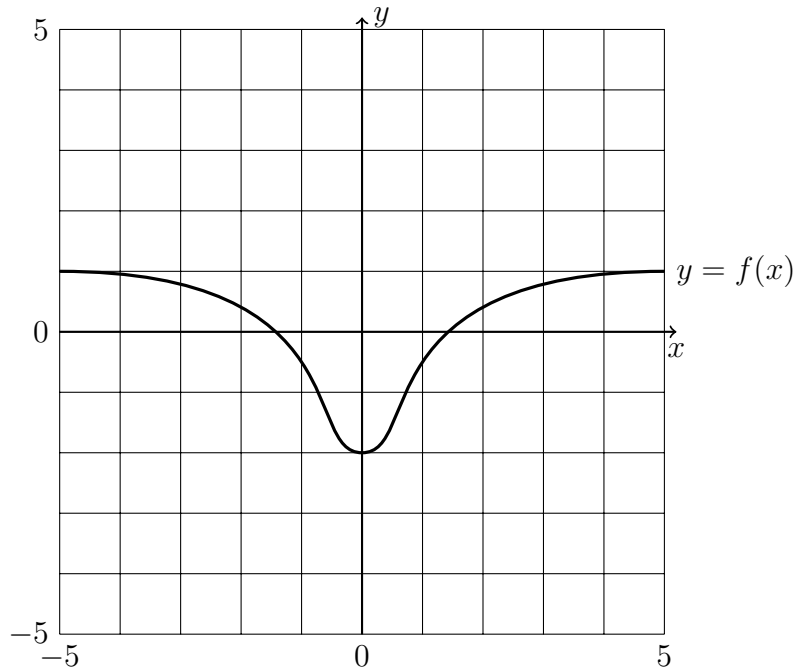(d) Find $\displaystyle\lim_{x \to 0} \frac{1}{x^2} - \frac{1}{\sin^2(x)}$.

(4) Consider $f(x) = 3x - x^3$.

(a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.

(b) Give the interval(s) for which f is increasing.

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(c) Give the intervals for which f is concave up, and for which it is concave down.

(d) Decide which critical points are maxima, minima, or neither.

(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.

(a) Label all regions where $f(x) < 0$.

(b) Label all regions where $f'(x) > 0$.

(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{2}{x - 2}$.

(a) Sketch the graph of $f(x)$ showing any asymptotes.

(b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.

(c) Sketch the tangent line at $x = -1$ on your graph.

(7) Let $f(x) = \frac{1}{\sqrt{x}}$. Find the derivative *using the limit definition of the derivate.* Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^2y - xy^2 + 4y = 10$ at the point $(-2, 1)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.
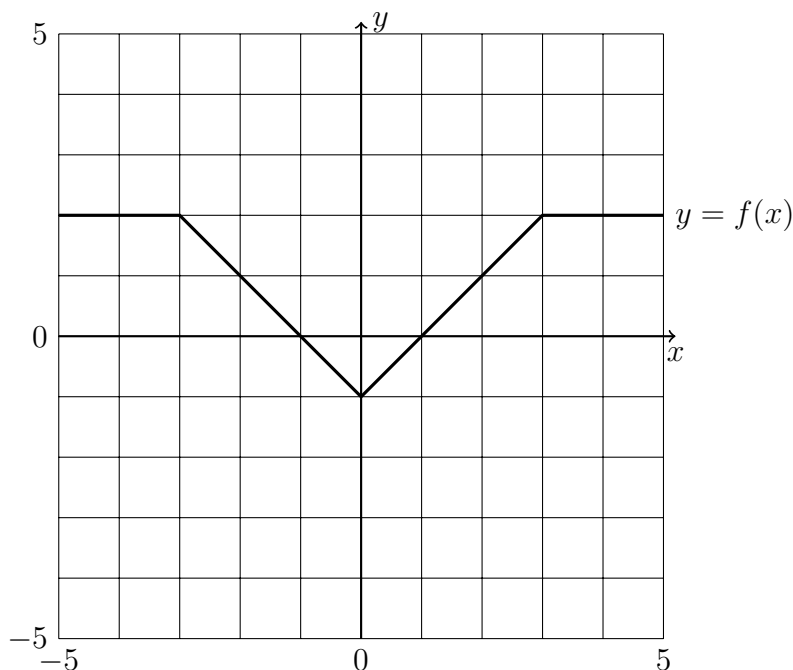
(10) A region in the plane is bounded by the x-axis, the graph $y = 10 - x^2$, and the lines $x = -1$ and $x = 3$.
  (a) Sketch the region (shading it in) and label the boundaries.
  (b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $6\text{in}^3/\text{s}$. How fast is the surface area growing when $r = 4\text{in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the surface area is $A = 4\pi r^2$.)

(12) Use linear approximation to estimate $\sqrt[3]{60}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) What's the closest point on the ellipse $x^2 + 4y^2 = 16$ to the point $(7, 2)$?