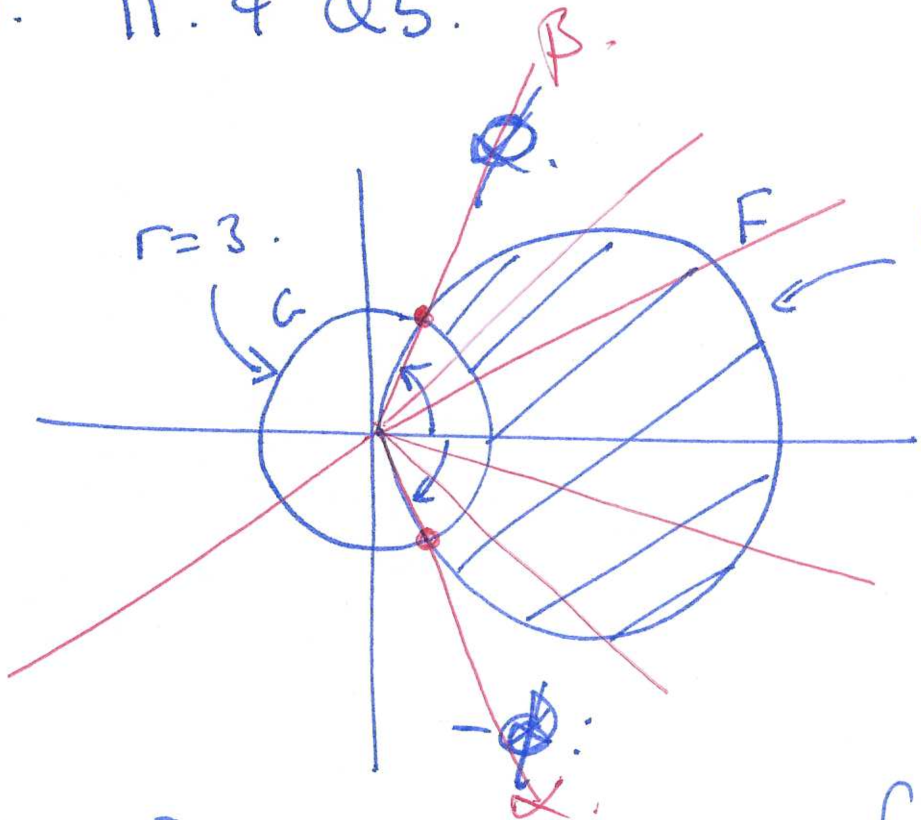
WW. 11.4 Q5.



$$\int_{\alpha}^{\beta} \frac{1}{2} (f(\theta))^2 \, d\theta.$$

$$\frac{1}{2} \int_{-\phi}^{\phi} (8\cos\theta)^2 - (3)^2 \, d\theta.$$

$3 = 8\cos\theta$

$\cos\theta = \frac{3}{8}$

$\phi = \cos^{-1}(3/8).$

$$\frac{1}{2} \int 64\cos^2\theta - 9 \, d\theta.$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1.$

$\cos^2\theta = \frac{1}{2}\cos 2\theta - \frac{1}{2}.$

$$\frac{1}{2}\int_{-\phi}^{\phi} 64\left(\frac{1}{2}\cos 2\theta - \frac{1}{2}\right) - 9 \, d\theta .$$

$$\frac{1}{2}\left[\frac{32\sin 2\theta}{2} - 32\theta - 9\theta\right]_{-\phi}^{\phi} .$$

$$\left[8\sin 2\theta - 41\theta\right]_{-\phi}^{\phi} \qquad \cancel{8\sin 2\phi - 41\phi + 8\sin 2\phi}$$

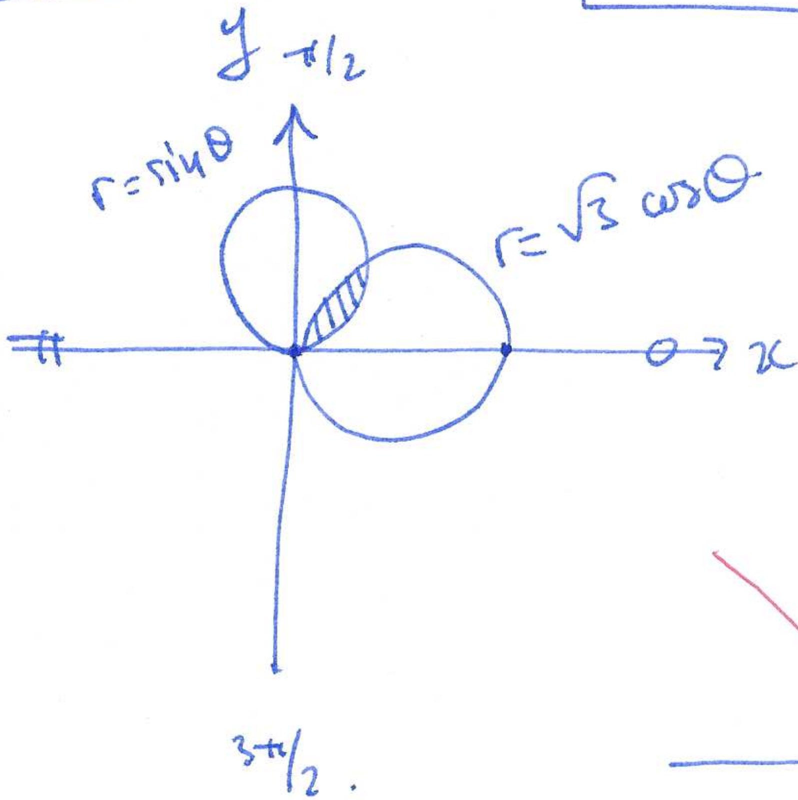
$$8\sin 2\phi - 41\phi - \left(8\sin(-2\phi) - 41(-\phi)\right).$$

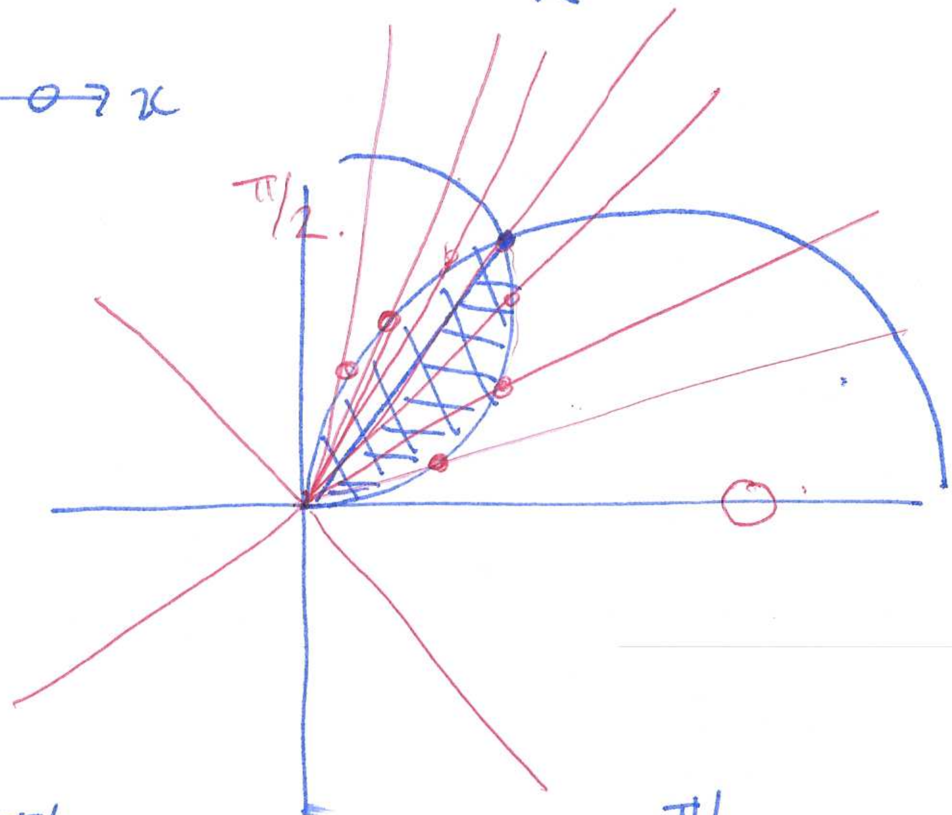
$$16\sin 2\phi - 82\phi . \qquad \phi = \cos^{-1}(3/8) .$$

11.4 Q4

$\boxed{r = \sin\theta}$ $\boxed{r = \sqrt{3}\cos\theta}$

$$\int_{\alpha}^{\beta} \frac{1}{2}\left(f(\theta)\right)^2 \, d\theta$$





$\sin\theta = \sqrt{3}\cos\theta$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$$\int_{0}^{\pi/3} \frac{1}{2}(\sin\theta)^2 \, d\theta + \int_{\pi/3}^{\pi/2} \frac{1}{2}\left(\sqrt{3}\cos\theta\right)^2 \, d\theta$$

11.3. Q3.

$$r = 9\,\mathrm{cosec}\,\theta - \sec\theta$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \frac{9}{\sin\theta} - \frac{1}{\cos\theta}$$

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

$$r = \frac{9}{y/r} - \frac{1}{x/r}$$

$$1 = \frac{9}{y} - \frac{1}{x}$$

$$r = \frac{9r}{y} - \frac{r}{x}.$$

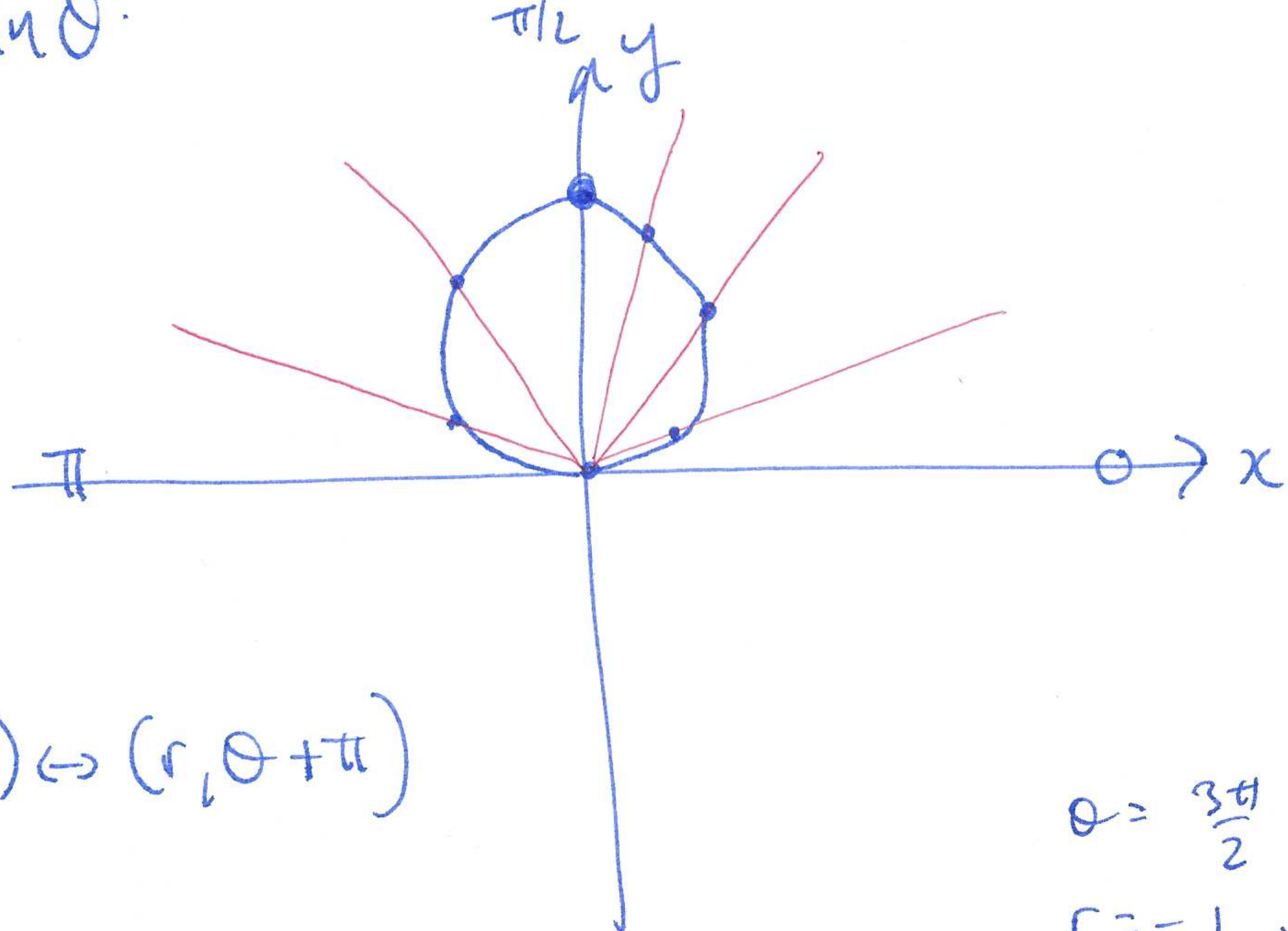
$$xy = 9x - y$$

$$xy + y = 9x$$

$$y(x+1) = 9x$$

$$\boxed{y = 9x/(x+1)}$$

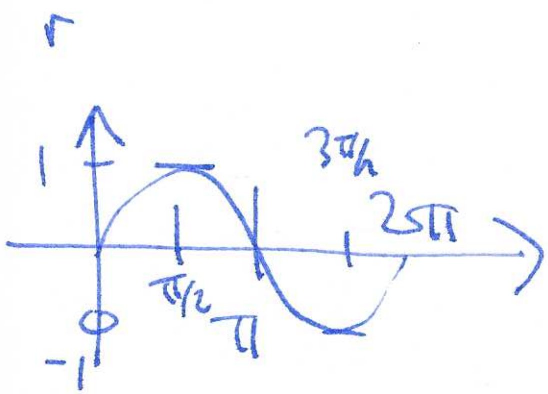
$r = \sin\theta$.



$(-r, \theta) \leftrightarrow (r, \theta + \pi)$

$\theta = \frac{3\pi}{2}$

$r = -1$.

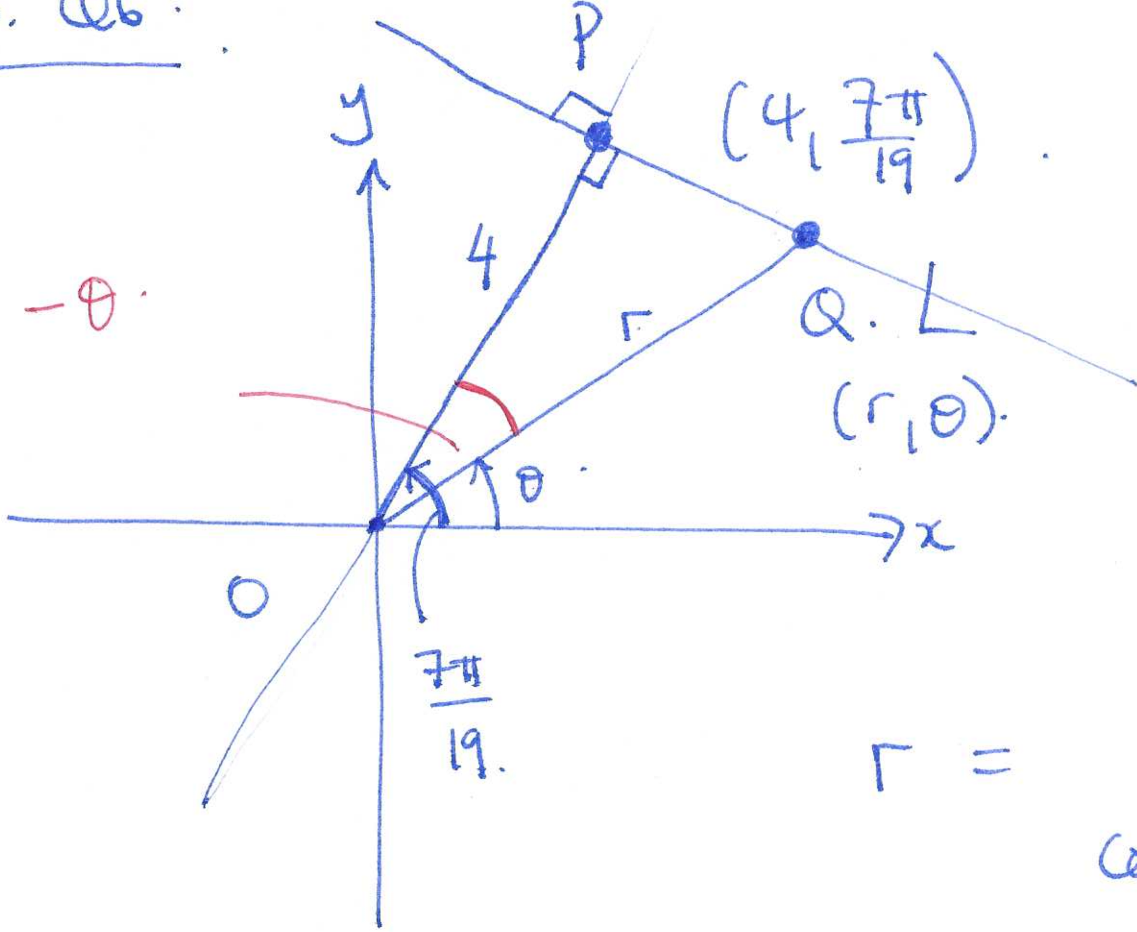


$\left(-1, \frac{3\pi}{2}\right)$

$\left(1, \frac{3\pi}{2} + \pi\right)$

$\pi/2$

11.3. Q6.



$\frac{7\pi}{19} - \theta$

$r = f(\theta)$.

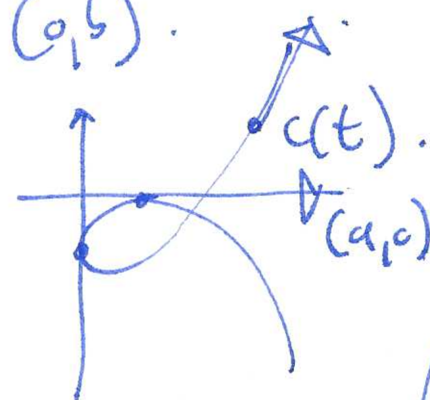
$$\frac{4}{r} = \cos\left(\frac{7\pi}{19} - \theta\right).$$

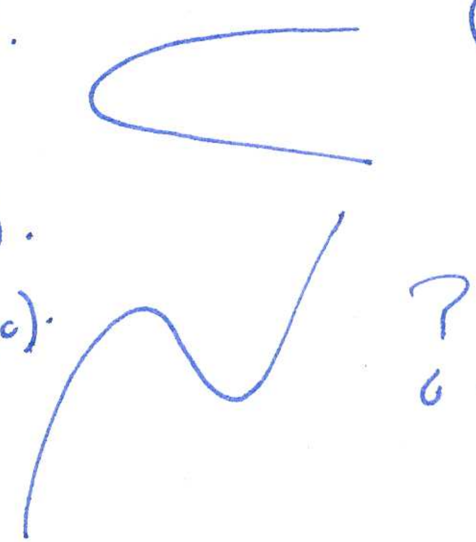
$$r = \frac{4}{\cos\left(\frac{7\pi}{19} - \theta\right)}.$$

$$r = 4\sec\left(\frac{7\pi}{19} - \theta\right).$$

Q20. $x(t) = t^2 - t + 2$

$y(t) = t^3 - 3t$.





$c(t) = (x(t), y(t))$.

$c'(t) = (x'(t), y'(t))$.

$= (2t-1, 3t^2-3)$.

$2t - 1 = 0 \quad t = \frac{1}{2}. \quad \left(\frac{1}{4} - \frac{1}{2} + 2, \frac{1}{8} - \frac{3}{2}\right) \leftarrow$ vertical tangents

$3t^2 - 3 = 0 \quad t = +1 \quad (2, -2).$

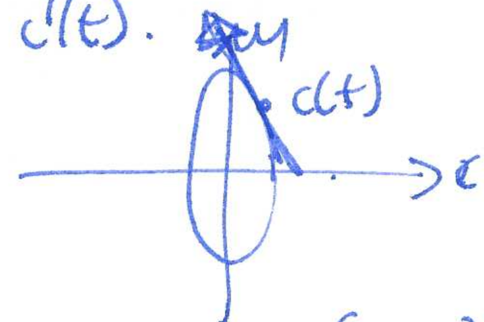
$t^2 = 1$ $\quad$ } horizontal tangents

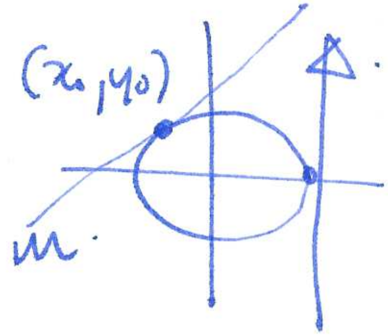
$t = -1. \quad (4, 2)$

$t = \pm 1.$

Q19 a) $x = \cos t$

$y = 3\sin t$





$c(t) = (x(t), y(t))$

$c'(t) = (x'(t), y'(t)) = (-\sin t, 3\cos t)$. m.

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = 3\cos t \big/ -\sin t = \boxed{-3\cot t} \quad m.$$

$$\frac{dy}{dx} = \frac{3x}{-y/3} = -9x/y$$

$y - y_0 = m(x - x_0)$.

$y - 3\sin t = m(x - \cos t)$.

$$\boxed{y - 3\sin t = -3\cot(t)\,(x - \cos t)}$$. tangent line at t $(\cos t, 3\sin t)$.
$x_0 \quad y_0$.

$t=0$.

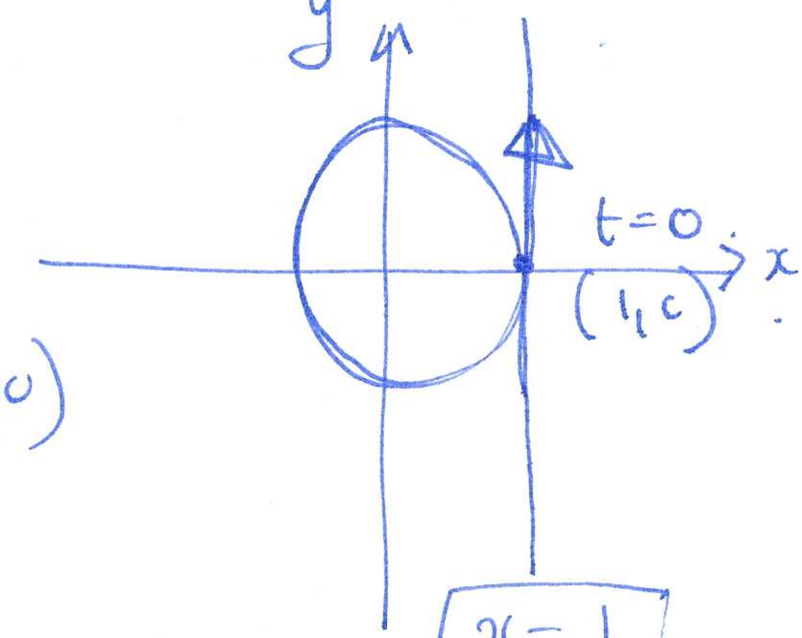
$$y - y_0 = -\frac{9x_0}{y_0}(x - x_0).$$

~~b) $x = \sqrt{t}$, $y = 4t - 1$.~~

$$c(0) = (\cos(0), 3\sin(0)) = (1,0)$$

$$\boxed{c'(0) = (-\sin t, 3\cos t)}$$

$$= (0, 3).$$



$\boxed{x = 1}$

tangent line.

$$c(t) = (\cos(t), 3\sin(t))$$

Q19 b). $x = \sqrt{t} \quad y = \sqrt{t-1}. \quad t = 2.$

$c(t) = (x(t), y(t))$

$c'(t) = (x'(t), y'(t)) \qquad \frac{dx}{dt} = \frac{1}{2} t^{-1/2} \quad \frac{dy}{dt} = \frac{1}{2}(t-1)^{-1/2}.$

$t = 2$

$c(2)$

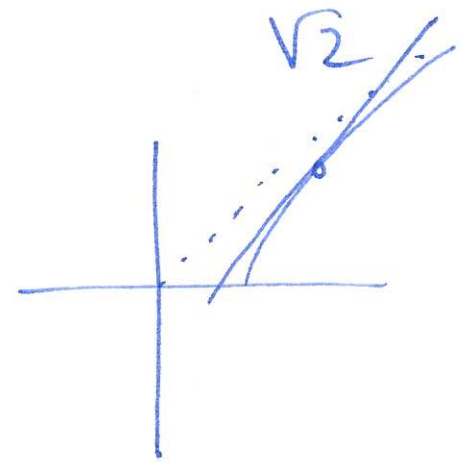
$c(2) = (\sqrt{2}, 1)$. point

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}.$

$c'(2) = \left(\underset{\frac{dx}{dt}}{\frac{1}{2\sqrt{2}}}, \underset{\frac{dy}{dt}}{\frac{1}{2}} \right)$ slope $\frac{1/2}{1/2\sqrt{2}}$

$y - y_0 = \overset{\sqrt{2}}{m}(x - \underset{\sqrt{2}}{x_0})$

$\;\;\;\;\;\;\;\;\; 1$



$y - 1 = \sqrt{2}(x - \sqrt{2}).$

Q12 . a)

$1 \cdot 2343434\overline{43} \ldots$

$a + ar + ar^2 + \cdots = \frac{a}{1-r}$.

$$1 + \frac{2}{10} + \frac{3}{100} + \frac{4}{1000} + \frac{3}{10^4} + \frac{4}{10^5} + \cdots$$

$$\frac{34}{10^{\cancel{4}3}} + \frac{34}{10^5} + \frac{34}{10^7} + \cdots$$

$$1 + \frac{2}{10} + \frac{34/10^3}{1 - \frac{1}{100}} = \frac{12}{10} + \frac{34}{10^3}\,\frac{100}{99}$$

$$= \frac{12}{10} + \frac{34}{990}.$$

$\sqrt{2} = 1.41\ldots$

$\pi = 3.14159\ldots \quad 3 + \frac{1}{10} + \frac{4}{100} + \frac{1}{1000} + \cdots$

Q14 e). $f(x) = \dfrac{\sin(3x^2)}{x^2}$

$\cancel{\sin(\theta + 2\pi)}$
$\cancel{= \sin(\theta)}$.

$\cancel{\dfrac{1}{(x' - 2\pi)^3}}$.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\frac{\sin(3x^2)}{x^2} = \frac{3x^2 - \frac{(3x^2)^3}{3!} + \frac{(3x^2)^5}{5!} - \cdots}{x^2}$$

$$\frac{\sin(3x^2)}{x^2} = 3 - \frac{27x^4}{3!} + \frac{3^5x^8}{5!} - \cdots$$

Q11 m). $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{2^n+1}}$ $\qquad a_n = \dfrac{1}{\sqrt{2^n+1}}$.

try: alternating series test $\displaystyle\sum_{n=1}^{\infty} (-1)^n a_n$

converges if a_n

- positive ✓ for all n.
- decreasing $a_{n+1} < a_n$ ✓
- $a_n \to 0$.

$n+1 > n$.

$2^{n+1} > 2^n$.

$2^{n+1}+1 > 2^n$.

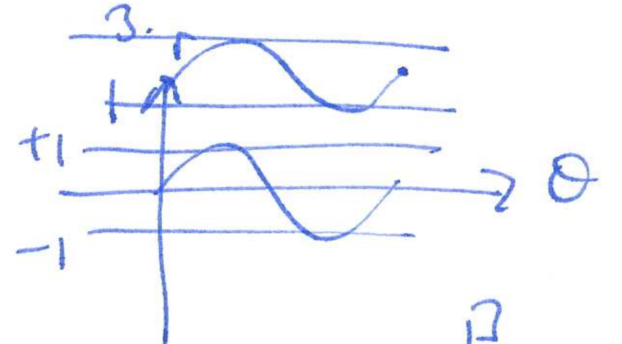
$\sqrt{2^{n+1}+1} > \sqrt{2^n+1}$.

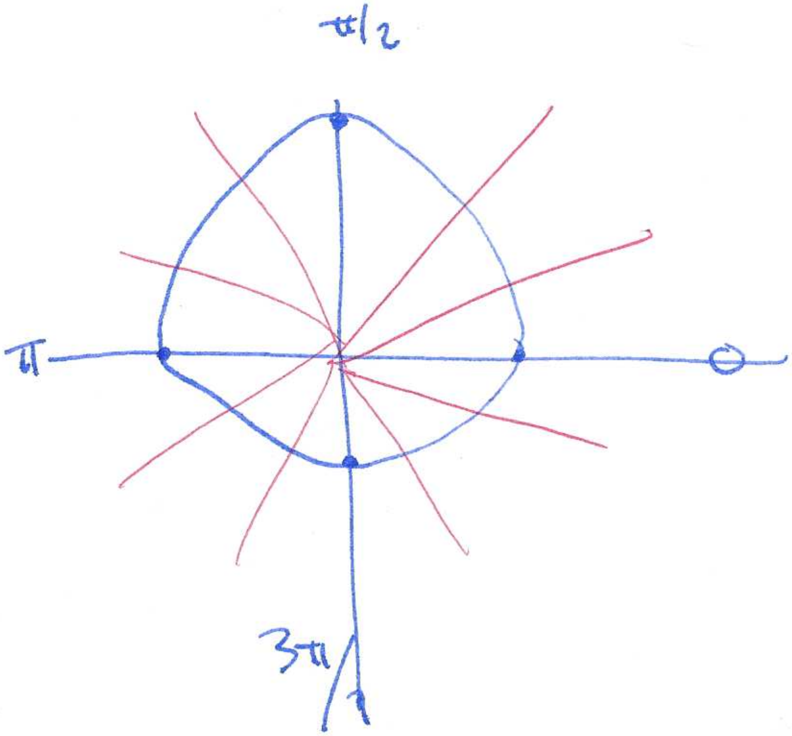
$\dfrac{1}{\sqrt{2^{n+1}+1}} < \dfrac{1}{\sqrt{2^n+1}} \Rightarrow a_{n+1} < a_n$

$$a_n = \frac{1}{\sqrt{2^n+1}} \longrightarrow 0 \text{ as } n \to \infty. \quad \checkmark.$$

alternating series test applies, so $\sum (-1)^n a_n$ converges.

Q22 $\quad r = 2+\sin\theta$.





a) area $\quad \frac{1}{2}\int_\alpha^\beta (f(\theta))^2 \, d\theta$.

b) arc length $\quad \int_\alpha^\beta \sqrt{(f')^2+f^2}\, d\theta$.

$r = \overset{1}{2} + \sin\theta$, f $\qquad f' = \cos\theta$.

area $= \frac{1}{2}\int_0^{2\pi} (\overset{1}{2} + \sin\theta)^2 \, d\theta$.

double angle formula.

$= \frac{1}{2}\int_0^{2\pi} \overset{1}{4} + \overset{2}{\cancel{4}} \sin\theta + \sin^2\theta \, d\theta$.

b). length $\int_0^{2\pi} \sqrt{f'^2 + f^2} \, d\theta$.

$$\int_0^{2\pi} \sqrt{\cos^2\theta + \underset{1}{\cancel{4}} + \underset{2}{\cancel{4}}\sin\theta + \sin^2\theta} \, d\theta$$

$$\int_0^{2\pi} \sqrt{\underset{2}{\cancel{5}} + \underset{2}{\cancel{4}}\sin\theta} \, d\theta.$$

$$\sqrt{2}\int_0^{2\pi}\sqrt{1+\sin\theta}\,d\theta.$$

$\theta = 2t$

$\frac{d\theta}{dt} = 2.$

$$\sqrt{2}\int_0^{2\pi}\sqrt{1+\sin 2t}\,\frac{d\theta}{dt}\,dt.$$

$1+\sin 2t.$

$1+2\sin t\cos t.$

$(\sin t+\cos t)^2.$

$\underbrace{\sin^2 t + 2\sin t\cos t + \cos^2 t}$

$1 + 2\sin t\cos t.$

$$2\sqrt{2}\int_0^{2\pi}\sqrt{1+\sin 2t}\,dt.$$

$$2\sqrt{2}\int_0^{2\pi}\sqrt{(\sin t+\cos t)^2}\,dt$$

$$2\sqrt{2}\int_0^{2\pi}|\sin t+\cos t|\,dt.$$ $\leftarrow$ we can integrate this.

$\int \frac{P(x)}{Q(x)}\,dx$ ← long division, partial fractions.

$\sin^2 t + \cos^2 t = 1$.

$\tan^2 t + 1 = \sec^2 t$.

any polys.

$\cancel{\int F(s}$ $\int \boxed{\frac{P(\sin\theta, \cos\theta)}{Q(\sin\theta, \cos\theta)}}\,d\theta$

half angle formula: can write $\sin\theta$, $\cos\theta$ etc.
in terms of $\cancel{t}$ $\tan(\frac{1}{2}\theta)$. $\sin 2t$ $\cos 2t$.

$\sin 2t = 2\sin t \cos t$

$\boxed{\cos 2t} = \cos^2 t - \sin^2 t$.

$= \boxed{1 - \tan^2 t}$

$\boxed{\sin 2t =} \frac{2\sin t}{\cos t} \cdot \cos^2 t$.

$= \frac{2\tan t}{\sec^2 t} = \boxed{\frac{2\tan t}{1+\tan^2 t}}$.

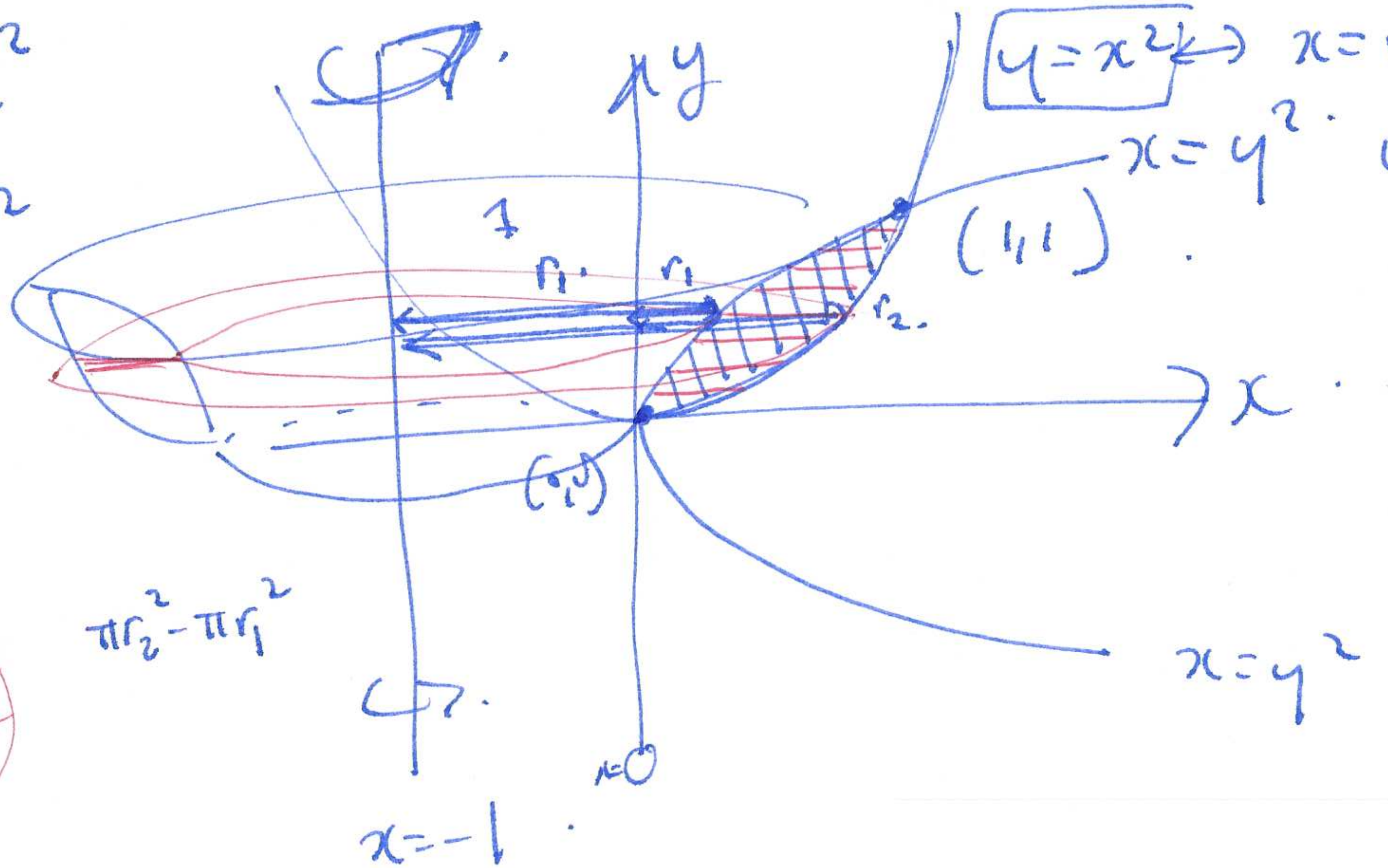
$\int \frac{R(\tan\theta/2)}{S(\tan\theta/2)}\,d\theta$,

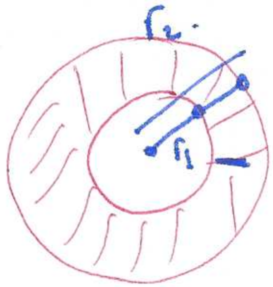
Q5. $y = x^2$

$x = y^2$

$x = -1$

$\boxed{y = x^2} \leftrightarrow x = \sqrt{y}$





$\pi r_2^2 - \pi r_1^2$

discs

$$V = \int_0^1 \pi r_2^2 - \pi r_1^2 \, dy$$

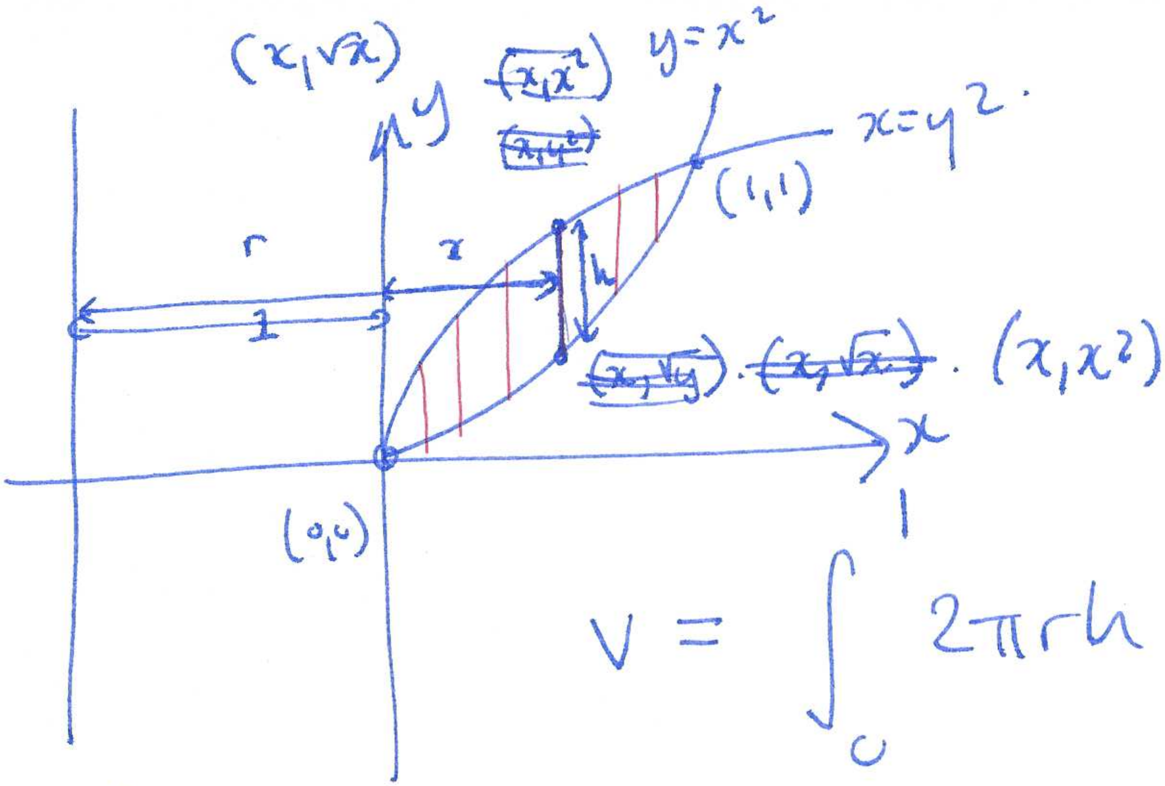
$$V = \pi \int_0^1 (\sqrt{y}+1)^2 - (y^2+1)^2 \, dy = \pi \int_0^1 (\sqrt{y}+1)^2 - (y^2+1)^2 \, dy$$

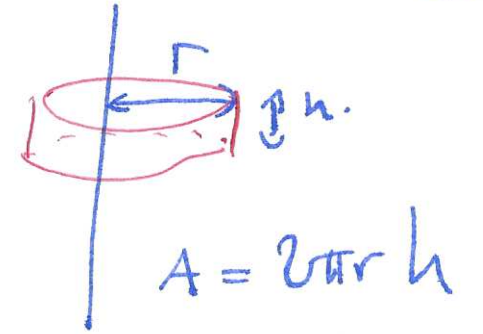
$$V = \pi \int_0^1 y + 2\sqrt{y} + \cancel{1} - y^4 - 2y^2 - \cancel{1} \, dy$$

$$V = \pi \left[\frac{1}{2} y^2 + \frac{4}{3} y^{3/2} - \frac{1}{5} y^5 - \frac{2}{3} y^3 \right]_0^1 .$$

$$V = \pi \left(\frac{1}{2} + \frac{4}{3} - \frac{1}{5} - \frac{2}{3} \right) = \pi \left(\frac{1}{2} + \frac{2}{3} - \frac{1}{5} \right)$$

$$= \pi \, \frac{15 + 20 - 6}{30} = \pi \frac{29}{30} .$$



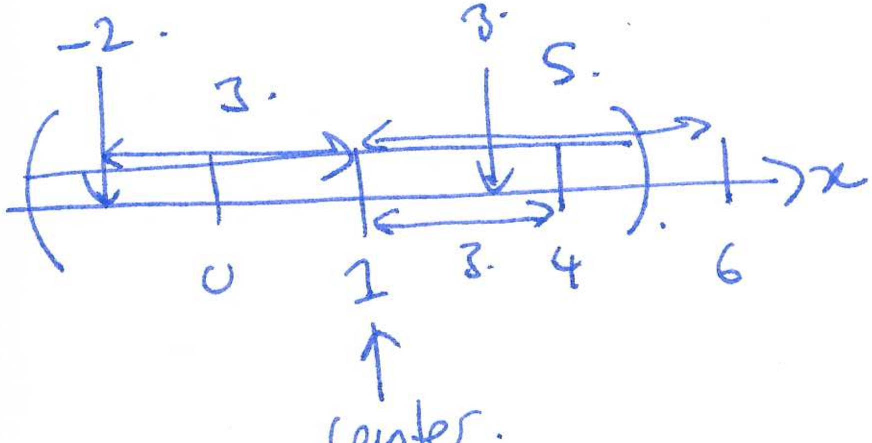


$$V = \int_0^1 2\pi r h \, dx.$$

$$V = \int_0^1 2\pi \overbrace{(x+1)}^{r} \overbrace{(\sqrt{x} - x)}^{h} \, dx.$$

$$V = 2\pi \int_0^1 x^{3/2} - x^2 + x^{1/2} - x \, dx.$$

Q13 $\sum_{n=1}^{\infty} c_n (x-1)^n$ converges when $x=4$
diverges when $x=6$.



$3 \leq R \leq 5$.

$(1-3, 1+3)$.
$(-2, 4)$.

a) $\sum_{n=1}^{\infty} c_n 2^n = \sum_{n=1}^{\infty} c_n (3-1)^n \leftrightarrow x=3$.
converges.

b) $\sum_{n=1}^{\infty} c_n (-3)^n = \sum_{n=1}^{\infty} c_n (-2-1)^n \leftrightarrow x=-2$

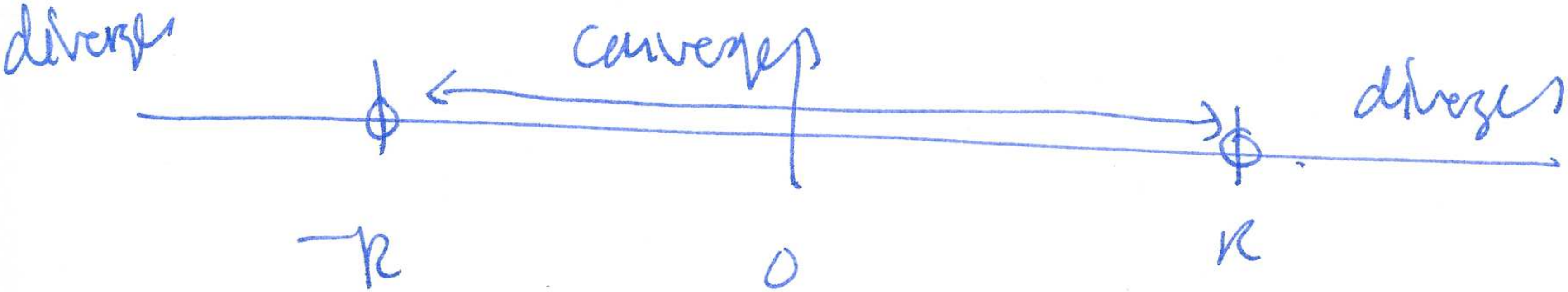
don't know if it converges or diverges.

power series $\sum_{n=1}^{\infty} a_n x^n$ radius $R < \infty$.

converges in $(-R, R)$

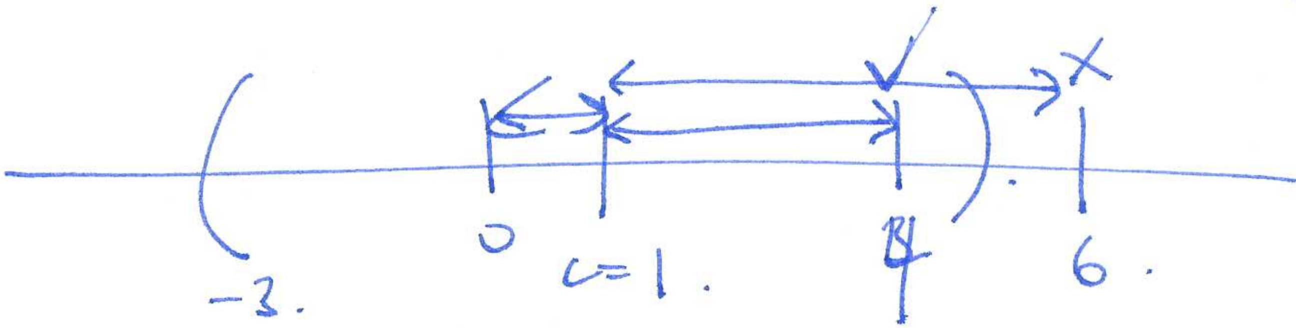
diverges for $|x| > R$

don't know for $\pm R$



$$\sum_{n=1}^{\infty} a_n(-1)^n = \sum_{n=1}^{\infty} (0-1)^n \qquad x=0.$$

converges.



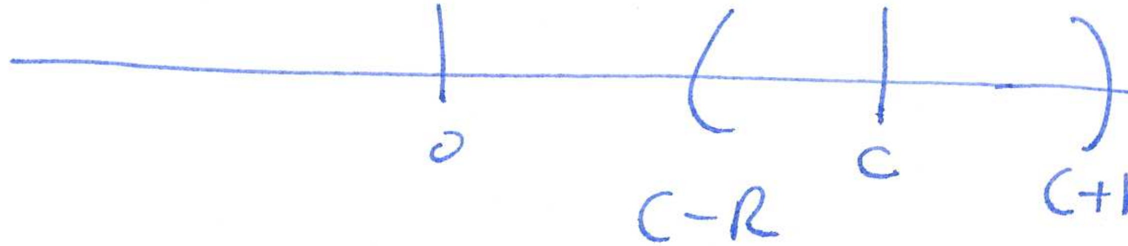
$3 \le R \le 5.$

$\cancel{(1-3,\ 1+3}$ $\qquad (1-R,\ 1+R).$

power series has a center c.

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$

center.



radius of convergence R $\qquad (c-R,\ c+R).$

Q18.

a) $$\cos(x) = \cancel{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\cos(0.3) = 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!} \leftarrow \begin{matrix} n = 4 \\ \boxed{n = 5} \end{matrix}$$

error bound: $$\text{error} \le \frac{K|x-c|^{n+1}}{(n+1)!}$$ $c = 0$, $x = 0.3$.

K is an upper bound for $|f^{(n+1)}(x)|$ on $[0, 0.3]$.

$K = 1$. $$\text{error} \le \frac{1 . |0.3|^6}{6!}$$