WW 10.4 Q5.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+7)(n+9)} \leftarrow \text{alternating.}$$

converges to S.

estimate this to 10^{-3}.

$$\boxed{|s_N - s| \le a_{N+1}}$$

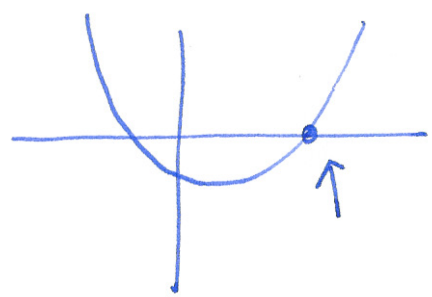
$$a_{N+1} \le \frac{1}{1000}$$

$$\frac{1}{(\underbrace{n}_{N+1}+7)(\underbrace{n}_{N+1}+9)} \le \frac{1}{1000}$$

$$(n+\overset{8}{\cancel{7}})(n+\overset{10}{\cancel{9}}) \ge 1000.$$

$$n^2 + 16n + 63 \ge 1000.$$

$$n^2 + 16n - 937 \ge 0.$$



$$\frac{-16 \pm \sqrt{16^2 + 4 \times 937}}{2}.$$

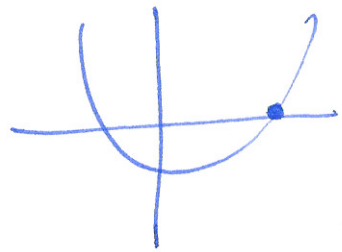
$$\approx 23.6.$$

$$\boxed{N = 24}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+7)(n+9)} = S.$$

$$a_{N+1} = \frac{1}{(N+8)(N+10)} \le \frac{1}{1000}$$

a_{24}.



$$N^2 + 18N + 80 \ge 1000$$

$$N^2 + 18N - 9\overset{2}{\cancel{8}}0 \ge 0.$$

$$\frac{-18 \pm \sqrt{18^2 + 4\times 9\overset{2}{\cancel{8}}0}}{2} = 22.6$$

$$\boxed{|s_N - s| \le a_{N+1}}$$

$$\boxed{N. \quad \cancel{N} = 23 \quad \cancel{N+1}}.$$

limit of series.

$$\boxed{a_1 - a_2 + \cdots - a_{28}}.$$

partial sum $s_N = \cancel{a_1 + a_2 +}$ $a_1 - a_2 + a_3 - \cdots \pm a_N$.

$$\left| \frac{1}{8\times10} - \frac{1}{9\times11} + \frac{1}{10\times12} - s \right| \le \frac{1}{11\times13}. \quad a_4.$$

$$a_1 \quad - \quad a_2 \quad + \quad a_3.$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+7)(n+9)} = \frac{1}{80} - \frac{1}{99} + \frac{1}{120} - \cdots = S.$$ ③

alternating series test $\Rightarrow$ this converges (to S say).

10.6 Q6: $(1-x^{12})^{-1}$. $f(x) = \dfrac{1}{1-x^{12}}$ find power series center $c=0$.

hard way $a_0 + a_1 x + a_2 x^2 + \cdots$ $a_n = \dfrac{f^{(n)}(0)}{n!}$

$a_0 = f(0) = 1$. $f'(x) = -(1-x^{12})^{-2} \cdot -12x^{11}$.

$a_1 = f'(0) = 0$ $= \dfrac{12x^{11}}{1-x^{12}}$.

$a_2 = f''(0)$

$$f(x) = (1-x^{12})^{-1}.$$

$$f''(0) = 0.$$

$$f'(x) = 12x^{11}(1-x^{12})^{-2}$$

$$f''(x) = 132x^{10}(1-x^{12})^{-2} + 12x^{11}(1-x^{12})^{-3}(-2)(-x^{11}).$$

easy way

$$\frac{1}{1-x^{12}} = 1 + x^{12} + x^{24} + x^{36} + \cdots + \underbrace{(x^{12})^n}_{x^{12n}} + \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots + x^n + \cdots$$

↑ geometric series.

radius of convergence : $1 + x^{12} + x^{24} + x^{36} + \cdots$

ratio test : $\lim\limits_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| = \lim\limits_{n\to\infty} \left| \dfrac{x^{12(n+1)}}{x^{12n}} \right|.$

$= \lim\limits_{n\to\infty} \left| x^{12} \right| < 1.$

$|x^{12}| < 1.$

$|x|^{12} < 1.$

$|x| < 1.$

$x = 1$: $1 + 1 + 1 + 1 + \cdots$ diverges.

$x = -1$: $1 - 1 + 1 - 1 + \cdots$ ".

S_N: $1, 0, 1, 0, 1, \ldots$

$(-1, 1)$.

interval of convergence.

$$1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \frac{1}{1-x}$$

$$1 + (x^{12}) + (x^{12})^2 + (x^{12})^3 + \cdots + (x^{12})^n + \cdots = \frac{1}{1-x^{12}}.$$

$$x^{12n}.$$

<u>10.6 Q4:</u>

$$\sum_{x=2}^{\infty} \frac{x^n}{\ln(3n)} = 0 + 0x + \frac{x^2}{\ln(6)} + \frac{x^3}{\ln(9)} + \cdots$$

ratio test:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{\ln(3(n+1))} \frac{\ln(3n)}{x^n} \right|.$$

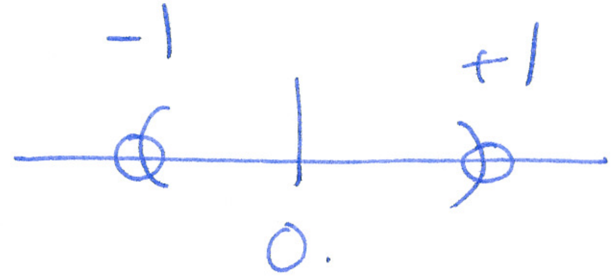
$$= \lim_{n\to\infty} |x| \frac{\ln(3n)}{\ln(3n+3)}. = \lim_{n\to\infty} |x| \cdot \cancel{\ln(3n)} \; 1.$$

$$\lim_{n\to\infty} \frac{\ln(3n)}{\ln(3n+3)}. = \lim_{n\to\infty} \frac{1/3n \cdot 3}{1/3n+3 \cdot 3}.$$

l'Hôpital.

$$= \lim_{n\to\infty} \frac{3n+3}{3n} = \lim_{n\to\infty} 1+\frac{1}{n} = 1.$$

ratio test: $|x| < 1$.



$(-1, 1)$.

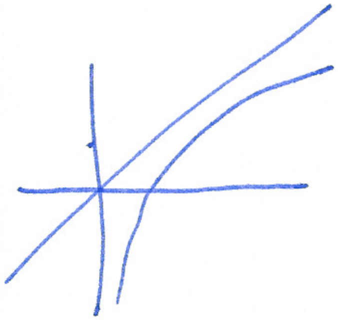
check endpoints manually.

$\underline{x=1}$. $\displaystyle\sum_{n\,\cancel{x}=2}^{\infty} \frac{1}{\ln(3n)} = \frac{1}{\ln(6)} + \frac{1}{\ln(9)} + \frac{1}{\ln(12)} + \cdots$

comparison test with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{3n} = \frac{1}{3}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\right)$.

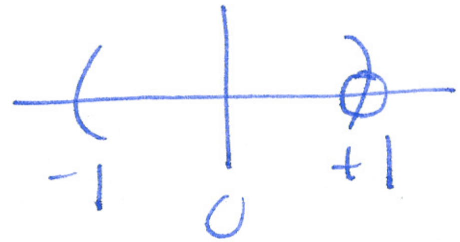
$\ln(3n) < 3n$.

harmonic series diverges.



$\ln(3n) < 3n \qquad n \geq 2.$

$$\frac{1}{\ln(3n)} > \frac{1}{3n}.$$



$\sum \frac{1}{\ln(3n)} \Leftarrow \sum \frac{1}{3n}$ diverges $(-1, +1)$

$[-1, +1)$.

ln. diverges.

check $x=-1$.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(3n)} = \frac{1}{\ln(6)} - \frac{1}{\ln(9)} + \frac{1}{\ln(12)} - \cdots$$

alternating series test: $\sum_{n=1}^{\infty} (-1)^n a_n$ converges if:

a_n positive ✓ $\quad a_n = \frac{1}{\ln(3n)} > 0$. positive.

decreasing ✓

converge to 0 ✓

$3n < 3(n+1)$,

$\ln(3n) < \ln(3n+3)$.

$\frac{1}{\ln(3n)} > \frac{1}{\ln(3n+3)}$

$a_n > a_{n+1}$.

$$\lim_{n\to\infty} \frac{1}{\ln(3n)} = 0.$$

$\Rightarrow$ converges.

$[-1, 1)$.

$x=1.$ $\sum_{n=2}^{\infty} \frac{1}{\ln(3n)} = \frac{1}{\ln(6)} + \frac{1}{\ln(9)} + \frac{1}{\ln(12)} + \cdots$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges.

$|s_N - s| \leq a_{N+1}.$ $\rightarrow$ 10.4. Q 4.
10.5 Q 5.

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ converges.

10.5 Q5. $\sum_{n=1}^{\infty} \frac{1}{n^{8n}}.$ Q: use the root test.

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left(\frac{1}{n^{8n}}\right)^{1/n} = \lim_{n\to\infty} \frac{1}{n^8}.$$

$$\lim_{n\to\infty} n^{\frac{1}{8}} = 0 < 1 \Rightarrow \underline{\text{converges}}. \quad \sum_{n=1}^{\infty} a_n$$

root test $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$

$L < 1 \Rightarrow$ converges

$L > 1 \Rightarrow$ diverges.

$L = 1$ no information.

ratio test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$

$L < 1$ converges

$L > 1$ diverges

$L = 1$ no information.

alternating series test $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

$\Uparrow$

if: a_n ~~positive non-negative~~. positive.

a_n decreasing $a_{n+1} < a_n$.

$\lim_{n \to \infty} a_n = 0$.

limit comparison test (positive series).

$\sum a_n, \sum b_n$ $\lim_{n \to \infty} \frac{a_n}{b_n} = L$

$0 < L < \infty$

$\sum a_n$ converges $\Leftrightarrow$ $\sum b_n$ converges.

10.3 Q6. comparison test

$$\sum_{n=2}^{\infty} \frac{n^{55/9}}{n^7 - n} \quad \overset{\text{large } n}{\sim} \quad \frac{n^{55/9}}{n^7} = n^{-8/9}.$$

$$\sum_{n=2}^{\infty} n^{-8/9} \quad \text{p-series.}$$

$$\frac{n^{55/9}}{n^7 - n} \qquad \frac{1}{n^{8/9}}. \qquad \sum_{n=2}^{\infty} \frac{1}{n^{8/9}}. \quad \text{diverges.}$$

$$\sum \frac{n^{55/9}}{n^7 - n} > \sum \frac{n^{55/9}}{n^7}.$$

rule $\sum_{n=1}^{\infty} \frac{1}{n^p}$. converges if $p > 1$. diverges $p \le 1$.

integral test.

diverges. $\Leftarrow$ diverges

$$\sum_{n=2}^{\infty} \frac{n^{55/9}}{n^7 - n}$$ comparison $$\frac{n^{55/9}}{n^7} = \frac{1}{n^{8/9}}.$$

$$\sum_{n=2}^{\infty} \frac{n^{55/9}}{n^7 + n}.$$ $$\frac{n^{55/9}}{n^7+n} < \frac{n^{55/9}}{n^7} = \frac{1}{n^{8/9}}.$$

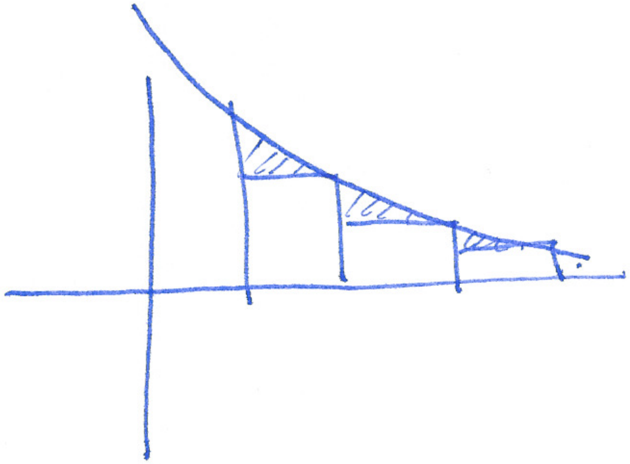
nothing $\Leftarrow$ diverges

$\uparrow$

try limit comparison test.

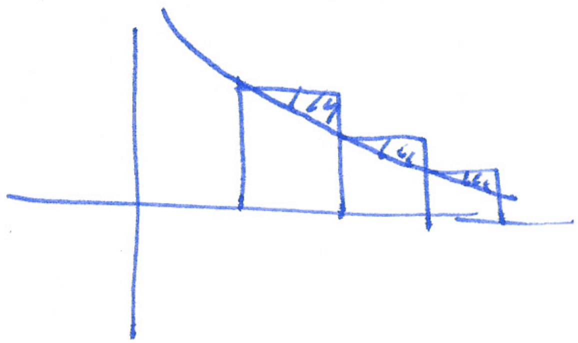
$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty.} \frac{n^{55/9}}{n^7+n} \cdot \frac{n^7}{n^{55/9}} = \lim_{n\to\infty.} \frac{n^7}{n^7+n}$$

$$\lim_{n\to\infty.} \frac{1}{1+1/n^6} = 1. < \infty$$
$$0 <$$



$\sum a_n \qquad a_n = f(n)$

$f(x)$ positive cts decreasing



$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}.$$

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots \qquad ?$$

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots \qquad \frac{\pi^4}{90}.$$

$$\overline{\frac{1}{2^6} + \cdots} \qquad ? \qquad \frac{\pi^6}{945}.$$

$$\sum_{n=2}^{\infty} \frac{n^{55/9}}{n^7 - n}$$ try ratio test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)^{55/9}}{(n+1)^7 - (n+1)} \cdot \frac{n^7 - n}{n^{55/9}}.$$

$$= \lim_{n\to\infty} \frac{\cancel{n^{55/9}}\left(1+\frac{1}{n}\right)\cancel{n^7}\left(1-\frac{1}{n^6}\right)}{\cancel{n^7}\left(1+ \text{—}\right)\cancel{n^{55/9}}} = 1.$$

no information

$$\sum_{n=1}^{\infty} \frac{n^{1/7}}{n^3+n}.$$ for large n: $\dfrac{n^{1/7}}{n^3} = \dfrac{1}{n^{3-1/7}}$

$$\frac{1}{n^{20/7}}.$$

$p > 1$

guess: converges.

comparison

$$\sum \frac{1}{n^p}.$$

$$\sum \frac{n^{1/7}}{n^3+n} < \sum \frac{n^{1/7}}{n^3}.$$

converges. $\Leftarrow$ converges by p-series.

$$\frac{n^a}{n^b.} = n^{a-b.} = \frac{1}{n^{b-a}}.$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ converges if $p > 1$
diverges if $p \leq 1$.

$$\sum_{n=1}^{\infty} n^p$$ converges if $p < -1$
diverges if $p \geq -1$.

$$\cancel{\sum n^7} \qquad \sum_{n=1}^{\infty} \frac{n^{1/7} \cos^2(3n+2)}{n^3+n} \qquad 0 \le \cos^2(3n+2) \le 1.$$

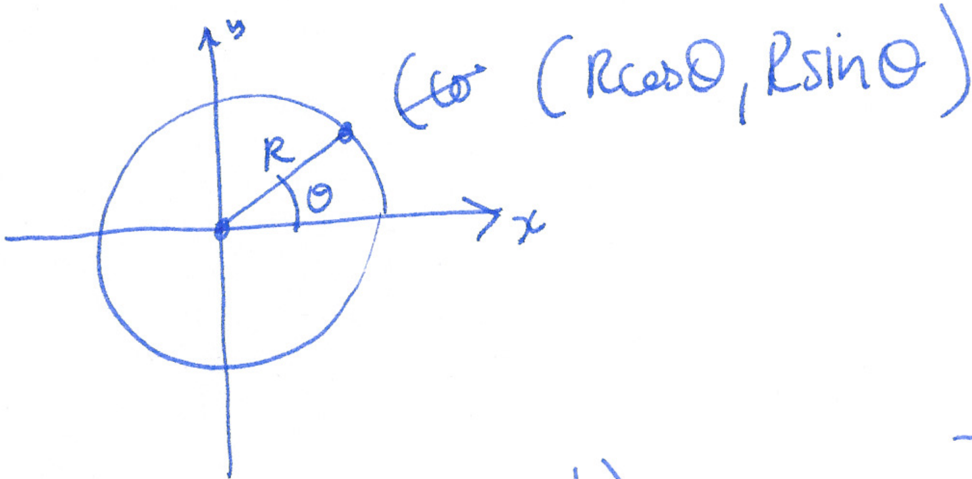
comparison test:

$$\sum \frac{n^{1/7} \cos^2(3n+2)}{n^3+n} < \sum \frac{n^{1/7}}{n^3}.$$

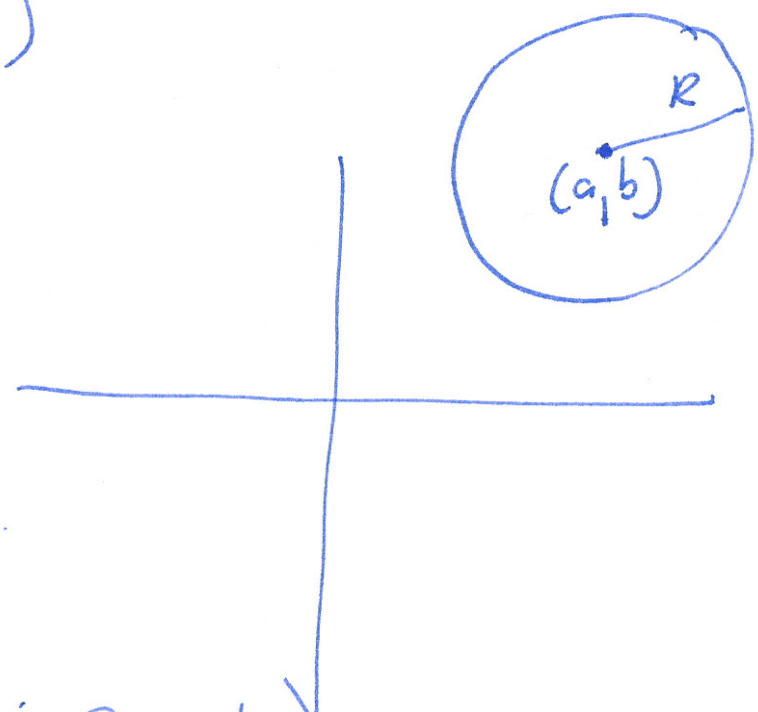
converges. $\Leftarrow$ converges

parameterizations.

$$c(\theta) = (R\cos\theta, R\sin\theta).$$

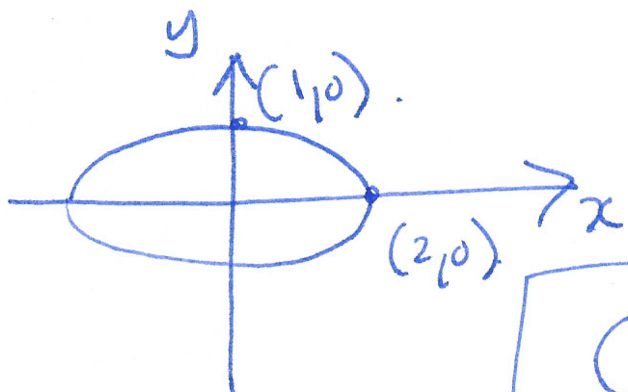


centered at (a,b).



$$c(\theta) = (R\cos\theta + a, R\sin\theta + b).$$

ellipse.



$$c(\theta) = (2\cos\theta, \sin\theta).$$

$$\boxed{\underline{Q}: c(\theta) = (\cos\theta, \sin 2\theta).}$$