$u = 1+9t^4$

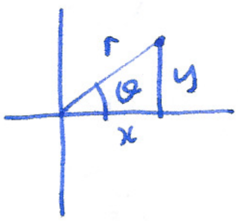
$\frac{du}{dt} = 36t^3$

$$2\pi\int_1^{10} t^3\sqrt{u}\,\frac{dt}{du}\,du = 2\pi\int_1^{10} t^3\sqrt{u}\,\frac{1}{36t^3}\,du = \frac{\pi}{18}\int_1^{10}\sqrt{u}\,du$$

$$= \frac{\pi}{18}\left[\frac{2u^{3/2}}{3}\right]_1^{10} = \frac{\pi}{27}\left(10^{3/2}-1\right) \approx 3.56$$

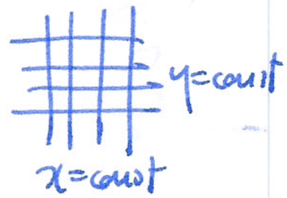
Example $c(t) = (t - \tanh t, \operatorname{sech} t)$

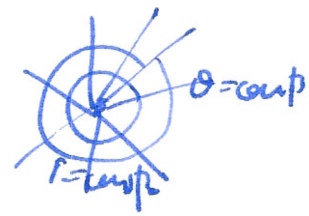
§11.3 Polar coordinates



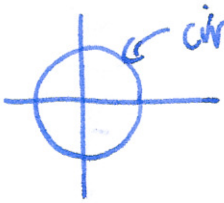
$x = r\cos\theta$ $\qquad \tan\theta = y/x$

$y = r\sin\theta$ $\qquad r^2 = x^2+y^2$



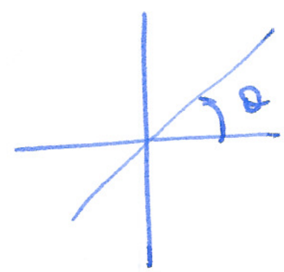


Equations in polar coordinates



circle of radius R: cartesian $x^2+y^2=R^2$

polar $r = R$

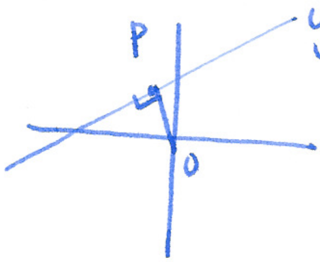


straight line through origin $y = mx$

$\theta = \tan^{-1}(m)$, $\theta = \tan^{-1}(m)+\pi$.

convention: $(-r,\theta) = (r,\theta+\pi)$, so then just need $\theta = \tan^{-1}(m)$

general line:



in polars: let P be closest point to $O = (0,0)$

and let $d(P,O) = d$

$\frac{d}{r} = \cos(\theta-\alpha)$ $\qquad r = \frac{d}{\cos(\theta-\alpha)} = d\sec(\theta-\alpha)$