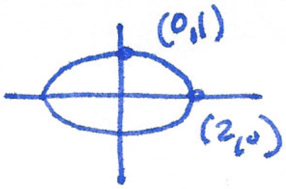
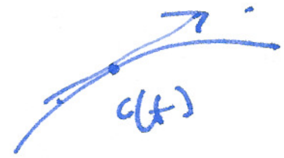
(3) ellipse



$c(\theta) = (2\cos\theta, \sin\theta)$

Q: how do we find the tangent line?



A: $\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

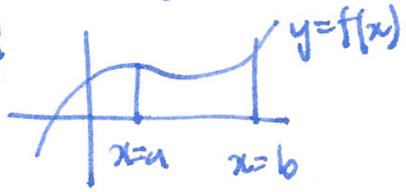
why: $c(t) = (x(t), y(t))$ chain rule: $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

<u>Example</u> $c(t) = (\cos t, \sin 2t)$

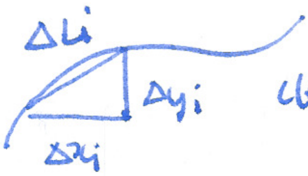
$$\left.\begin{array}{ll} x(t) = \cos t & y(t) = \sin 2t \\ \frac{dx}{dt} = -\sin t & \frac{dy}{dt} = 2\cos 2t \end{array}\right\} \frac{dy}{dx} = \frac{2\cos 2t}{-\sin t}$$

<u>§11.2 Arc length and speed</u>

<u>recall</u>



$$\text{arc length} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$



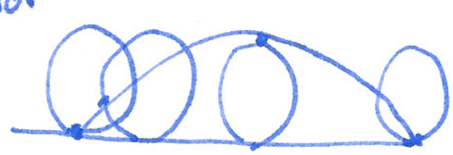
$c(t) = (x(t), y(t))$

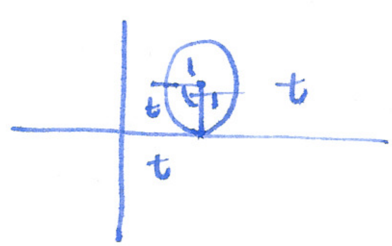
$$\text{arc length} = \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

$$= \sum_{i=1}^{n} \sqrt{\left(\frac{\Delta x_i}{\Delta t_i}\right)^2 + \left(\frac{\Delta y_i}{\Delta t_i}\right)^2}\,\Delta t_i$$

$$= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

<u>Example</u> cycloid





$x(t) = t - \sin t \qquad \frac{dx}{dt} = 1 - \cos t$

$y(t) = 1 - \cos t \qquad \frac{dy}{dt} = \sin t$

arc length: $\int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\,dt = \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}\,dt$

$$= \int_0^{2\pi} \sqrt{2-2\cos t}\, dt$$

recall $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

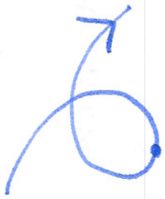
$2\sin^2\theta = 1 - \cos 2\theta$

$2\sin^2\left(\frac{t}{2}\right) = 1 - \cos t$

$$= \int_0^{2\pi} \sqrt{4\sin^2\left(\frac{t}{2}\right)}\, dt$$

$$= \int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt = 4\int_0^{\pi} \sin\frac{t}{2}\, dt = 4\left[-2\cos\frac{t}{2}\right]_0^{\pi} = 8(0+1) = 8$$

<u>speed</u>



velocity vector $c'(t) = (x'(t), y'(t))$

speed = length of velocity vector

$$\|c'(t)\| = \sqrt{(x'(t))^2 + (y'(t))^2}.$$
$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

<u>Example</u> ① $c(t) = (t, t^2)$ $x(t) = t$ $\frac{dx}{dt} = 1$

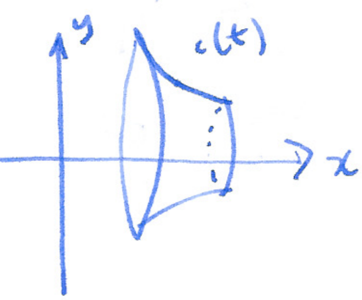
$y(t) = t^2$ $\frac{dy}{dt} = 2t$ speed $= \sqrt{1+4t^2}$

② $c(t) = (R\cos(\omega t), R\sin(\omega t))$

$c'(t) = (-\omega R\sin\omega t, \omega R\cos\omega t)$

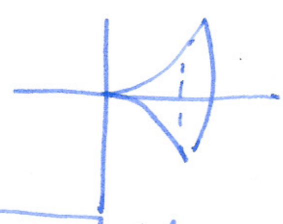
$\|c'(t)\| = \sqrt{R^2\omega^2\sin^2\omega t + \omega^2 R^2\cos^2\omega t} = R|\omega|.$

<u>surface area for parameterized curves of revolution</u>



$$\text{surface area} = 2\pi\int_a^b y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

example $c(t) = (t, t^3)$
$c'(t) = (1, 3t^2)$



$$\text{surface area} = 2\pi\int_0^1 t^3\sqrt{1^2 + (3t^2)^2}\, dt = 2\pi\int_0^1 t^3\sqrt{1+9t^4}\, dt$$