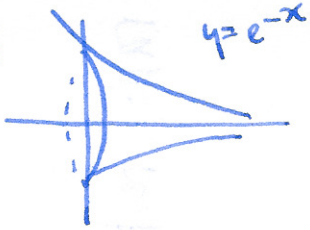
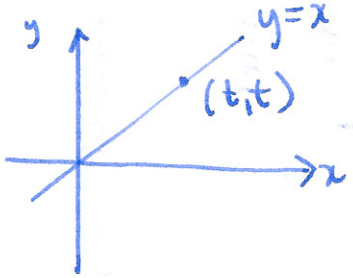
Example  $y = e^{-x}$ $\frac{dy}{dx} = -e^{-x}$ surface area $\int_0^\infty 2\pi e^{-x}\sqrt{1+(e^{-x})^2}\,dx$

sub $u = x$ $u = e^{-x}$, $\frac{du}{dx} = -e^{-x}$

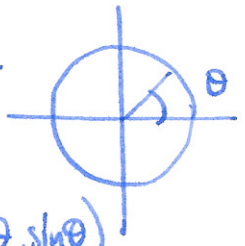
$$\int_1^0 2\pi u\sqrt{1+u^2}\,\frac{dx}{du}\,du = \int_1^\infty 2\pi\sqrt{1+u^2}\,du \quad \text{(big sub integral)}.$$

§11.1 Parametric equations



$y = x$ $x(t) = t$, $y(t) = t$

$c(t) = (x(t), y(t))$ ← parameterization

e.g. 

$c(\theta) = (\cos\theta, \sin\theta)$

problem: many different parameterizations give same curve.

eg $c(t) = (2t, 2t)$ gives $y = x$, as does $c(t) = (t^3, t^3)$.

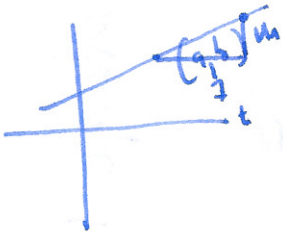
warning: $c(t) = (t^2, t^2)$ **not** the same. Or $c(t) = (\sin t, \sin t)$?

Example ① describe the parameterized curve $c(t) = (t+1, t^3+1)$ as $y = f(x)$.

$x = t+1$ $\quad t = x-1$.

$y = t^3+1$ $\quad y = (x-1)^3+1 = x^3-2x+2$

example ② find a parametric equation for the line through (a,b) w/ slope m

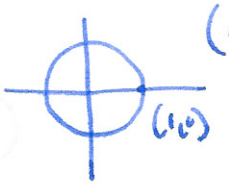


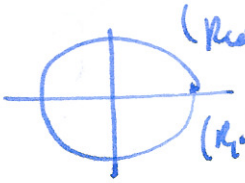
$t = 0$ (a,b)

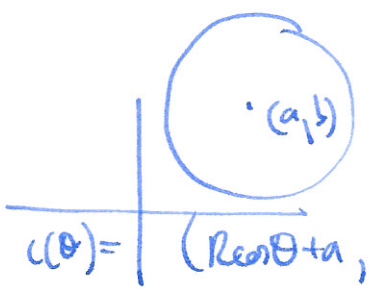
$t = 1$ $(a+1, b+m)$

$c(t) = (a+t, b+tm)$

③ circle of radius R, centered at (a,b)







$c(\theta) = (R\cos\theta + a,\ R\sin\theta + b)$