$$\int \sin^6 x\,dx = -\sin^5 x\cos x + 5\int \sin^4 x\,dx - 5\int \sin^6 x\,dx$$

$$6\int \sin^6 x\,dx = -\sin^5 x\cos x + \underbrace{5\int \sin^4 dx}_{\text{do by parts!}}$$

other trig functions

recall: $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$ $\quad u=\cos x$, $\frac{du}{dx} = -\sin x$ $= \int \frac{\sin x}{u}\cdot\frac{-1}{\sin x}\,du$

$$= -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|\cos x| + C = +\ln|\sec x| + C.$$

Fact: $\int \sec(x)\,dx = \ln|\sec x + \tan x| + C$ (check: $\frac{1}{\sec x+\tan x}\cdot(\sec x\tan x + \sec^2 x)$).

$$\int \operatorname{csc}(x)\,dx = -\ln|\operatorname{csc} x + \cot x| + C$$

other trig function powers

$\int \tan^a x \sec^b x\,dx$

use:

- $\cos^2 x + \sin^2 x = 1 \iff 1 + \tan^2 x = \sec^2 x$
- $u = \sec x$ $\quad \frac{du}{dx} = \sec x\tan x$
- $u = \tan x$ $\quad \frac{du}{dx} = \sec^2 x$.
- parts.

- **a odd Example** $\int \tan^3 x \sec^2 x\,dx = \int \tan^2 x \sec x\,(\tan x \sec x)\,dx$

$$= \int (1-\sec^2 x)\sec x\,(\tan x\sec x)\,dx \qquad u = \sec x,\ \frac{du}{dx} = \sec x\tan x$$

$$= \int (1-u^2)\,u\,du = \frac{1}{2}u^2 - \frac{1}{4}u^4 + C = \frac{1}{2}\sec^2 x - \frac{1}{4}\sec^4 x + C.$$

- **beven Example** $\int \tan^3 x \sec^2 x\,dx$ $\quad u = \tan x$, $\frac{du}{dx} = \sec^2 x$