$$= -x\cos x + \int \cos x \, dx$$

$$= -x\cos x + \sin x + c \qquad \text{check!}$$

Examples ① $\int \underbrace{x}_{u}\underbrace{e^x}_{v'} dx$ $\quad u = x \quad v' = e^x$, $\quad u' = 1 \quad v = e^x$ } Q: spose we chose these the other way round?

$$\int uv'dx = uv - \int u'v\,dx = xe^x - \int 1e^x dx = xe^x - e^x + c \qquad \text{check!}$$

observation $\int_a^b uv'dx = [uv]_a^b - \int_a^b uv'dx$

② $\int_1^2 \ln(x)\,dx = \int_1^2 \underbrace{1}_{v'}.\underbrace{\ln(x)}_{u}\,dx$ $\quad u = \ln(x) \quad v' = 1$, $\quad u' = \frac{1}{x} \quad v = x$

$$= [x\ln(x)]_1^2 - \int_1^2 x.\tfrac{1}{x}\,dx = 2\ln(2) - [x]_1^2 = 2\ln(2) - 1.$$

③ $\int \underbrace{x^2}_{u}\underbrace{\cos(x)}_{v'}dx = \underbrace{x^2}_{u}\underbrace{\sin(x)}_{v} - \int \underbrace{2x}_{u'} \cdot \underbrace{\sin(x)}_{v}dx$

$$= x^2\sin x - 2x(-\cos x) + \int 2(-\cos x)\,dx = x^2\sin x + 2x\cos x - 2\sin x + c \qquad \text{check!}$$

④ $\int \underbrace{e^x}_{u}\underbrace{\sin x}_{v'}\,dx = \underbrace{e^x}_{u}\underbrace{(-\cos x)}_{v} - \int \underbrace{e^x}_{u'}\underbrace{(-\cos x)}_{v}\,dx$

$$\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$$

$2\int e^x\sin x\,dx = e^x(\sin x - \cos x)$ $\qquad \int e^x \sin x\,dx = \frac{1}{2}e^x(\sin x - \cos x) + c.$ check!

§7.2 Trig integrals $\int \sin^m x \cos^n x\,dx$?

tools:

- $\cos^2 x + \sin^2 x = 1$
- $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- sub $u = \sin x \quad \frac{du}{dx} = \cos x$
- sub $u = \cos x \quad \frac{du}{dx} = -\sin x$
- parts.

Examples

- $\int \sin^2 x\,dx = \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$ (check!)

- $\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx$ sub $u = \cos x$, $\frac{du}{dx} = -\sin x$

$$= \int (1-u^2)\sin x \cdot \frac{1}{-\sin x}\,du = -\int 1-u^2\,du = -u + \frac{1}{3}u^3 + c$$

$$= -\cos x + \frac{1}{3}\cos^3 x + c \quad \text{(check!)}$$

moral: squares: double angle formula.
odd powers: do sub u = other trig function.

Example $\int \sin^4 x \cos^3 x\,dx$ try $u = \sin x$, $\frac{du}{dx} = \cos x$

$$\int \sin^4 x \cos^3 x\,dx = \int \sin^4 x \cos^2 x \cdot \cos x\,dx = \int u^4(1-u^2)\,du = \int u^4 - u^6\,du = \frac{1}{5}u^5 - \frac{1}{7}u^7 + c$$

$$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C.$$

{ [maybe do $\int \sin^2 x\,dx$ by parts first].

even powers Example $\int \sin^4 x \cos^2 x\,dx$ ← get everything in terms of either $\sin(x)$ or $\cos(x)$, then use parts.

$$\int \sin^4 x(1-\sin^2 x)\,dx = \int \sin^4 x - \sin^6 x\,dx$$

$$\int \sin^6 x\,dx = \int \underbrace{\sin^5 x}_{u}\ \underbrace{\sin x}_{v'}\,dx \qquad u = \sin^5 x \quad v' = \sin x \qquad u' = 5\sin^4 x\cos x \quad v = -\cos x$$

$$\int \sin^6 x\,dx = uv - \int u'v\,dx$$

$$= -\sin^5 x\cos x - \int 5\sin^4 x\cos x \cdot -\cos x\,dx$$

$$= -\sin^5 x\cos x + 5\int \sin^4 x(1-\sin^2 x)\,dx$$