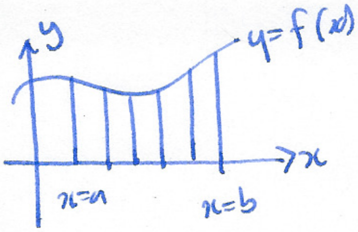


average : $\frac{1}{n}\left(f(x_1) + f(x_2) + \dots + f(x_n)\right)$

recall : $\int_a^b f(x)\,dx \approx \left(f(x_1) + \dots + f(x_n)\right)\Delta x \qquad \Delta x = \frac{b-a}{n}$

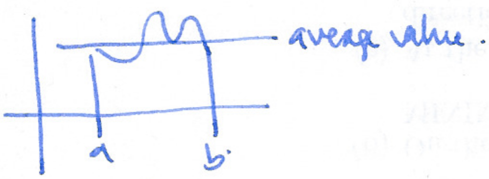
so $\int_a^b f(x)\,dx \approx$ average $f(x)$. $(b-a)$, so average value is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

<u>Example</u> find average value of $\sin(x)$ on $[0,\pi]$.

$[0,\pi]$: $\frac{1}{\pi}\int_0^\pi \sin(x)\,dx = \frac{1}{\pi}\left[-\cos(x)\right]_0^\pi = \frac{1}{\pi}\left(-(-1)+1\right) = \frac{2}{\pi}$.

<u>useful fact</u> (mean value theorem for integrals)

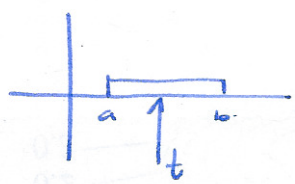
If $f(x)$ is cts on $[a,b]$, then there is a $c \in [a,b]$ s.t. $f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$



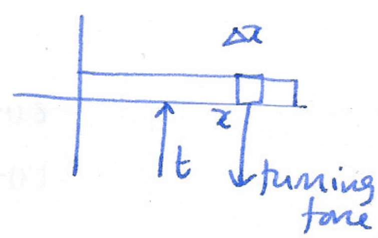
<u>Example</u> find average value of $\frac{\sin(\pi/x)}{x^2}$ on $[1,2]$

Q: which has a bigger average on $[0,\pi]$, $\sin(x)$ or $\sin^2(x)$.

<u>Application</u> center of mass



density $\rho(x)$



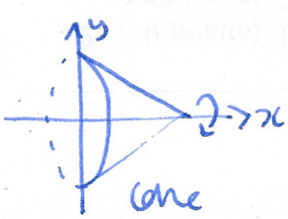
turning force at t : $(x-t)\,\Delta x\,\rho(x)$

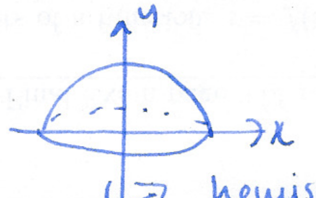
total turning force at t : $\sum_i \rho(x)(x-t)\Delta x_i \approx \int_a^b \rho(x)(x-t)\,dt$

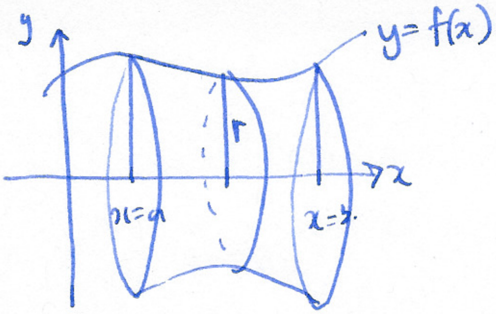
$= \int_a^b x\rho(x)\,dx - t\int_a^b \rho(x)\,dx = 0$ at balance, so $t = \frac{\int_a^b x\rho(x)\,dx}{\int_a^b \rho(x)\,dx}$

<u>§6.3 Volumes of revolution</u>

<u>Examples</u>







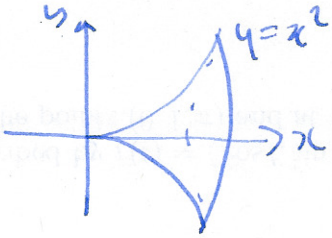
recall $V=\int_a^b A(x)\,dx$

$A(x)$ = area of vertical cross-section

$= \pi r^2 = \pi y^2 = \pi (f(x))^2$

so volume $V=\int_a^b \pi (f(x))^2\,dx$.

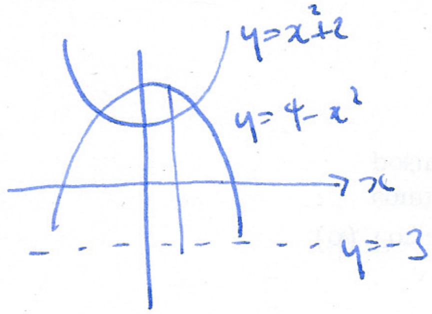
Example rotate $y=x^2$ about x-axis



$\pi\int_0^2 (x^2)^2\,dx = \pi\left[\frac{1}{5}x^5\right]_0^2 = \frac{32\pi}{5}$

Example rotate region between $f(x)=x^2+2$ about $y=-3$.

$f(x)$ $4-x^2$



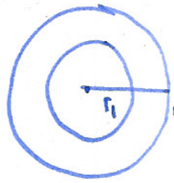
$V=\int_a^b A(x)\,dx$

a, b: find intersections

$x^2+2=4-x^2$

$2x^2=2$ $\qquad x=\pm 1$

$A(x)$ = area of cross-section



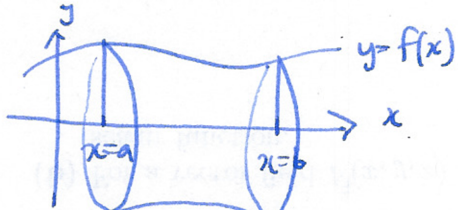
$r_1 = x^2+2+3$

$r_2 = 4-x^2+3$

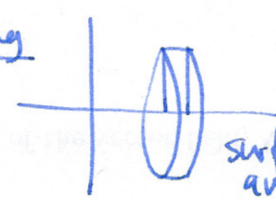
$A(x)=\pi r_2^2-\pi r_1^2 = \pi(r_2^2-r_1^2)$

$\pi\int_{-1}^1 (7-x^2)^2-(x^2+5)^2\,dx = \pi\int_{-1}^1 24-4x^2\,dx = \pi\left[24x-\frac{4}{3}x^3\right]_{-1}^1 = \left(48-\frac{8}{3}\right)\pi.$

surface area



wrong

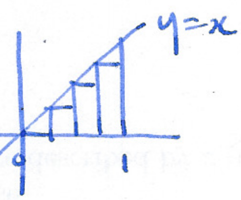


area of cylinder $= 2\pi y_i \Delta x_i$

surface area $\approx \sum 2\pi y_i \Delta x_i$

$= 2\pi\int_a^b y\,dx.$

analogy: arc length



arc length

$\neq \sum \Delta x_i$ $\cancel{\neq \sum \Delta x_i}$

$\neq \sum \Delta x_i + \Delta y_i$

$= \sum \sqrt{\Delta x_i^2 + \Delta y_i^2}$