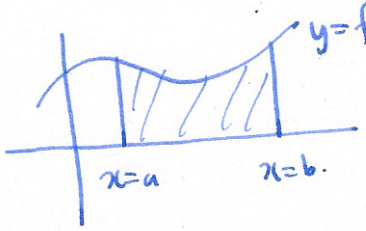
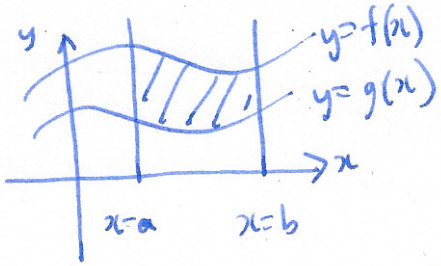
§6.1 Area between two curves

recall 

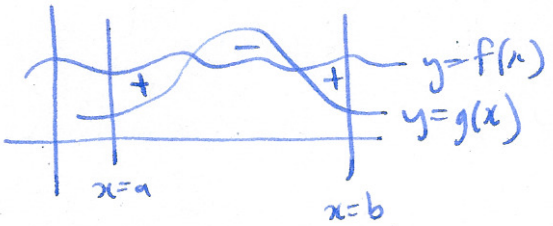
area under graph $\int_a^b f(x)\,dx$



area between two curves

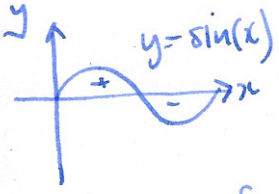
$$\int_a^b f(x)-g(x)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx .$$

warning : signs!



$$\int_a^b f(x)-g(x)\,dx$$

Q: what if you want "absolute area" / unsigned area?



$$\int_0^{2\pi} \sin(x)\,dx = 0$$

want: $\int_0^{2\pi} |\sin(x)|\,dx$ ← need to know where $\sin(x)$ is +ve or −ve

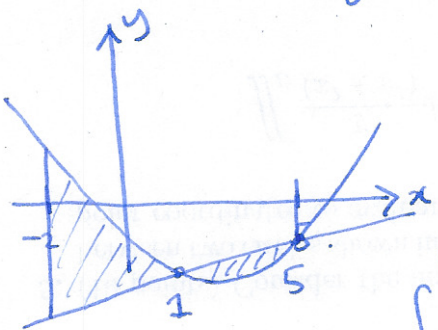
i.e. need to find zeros of $\sin(x)=0$

$$\int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx = \Big[-\cos(x)\Big]_0^{\pi} + \Big[-\cos(x)\Big]_{\pi}^{2\pi} = 2-(-2) = 4.$$

Examples

- find area between graphs $f(x) = x^2-5x-7$ and $g(x) = x-12$ on $[-2,5]$

draw picture:



intersection points: $x^2-5x-7 = x-12$

$x^2-6x+5=0$

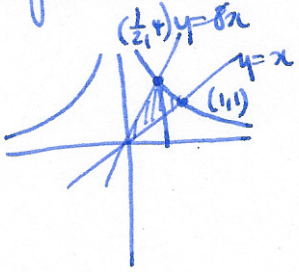
$(x-5)(x-1)$

← need to break into two integrals

$$\int_{-2}^{1} x^2-5x-7-x+12\,dx + \int_1^5 x-12-x^2+5x+7\,dx .$$

• find area of region bounded by $y = \frac{1}{x^2}$, $y = x$, $y = 8x$

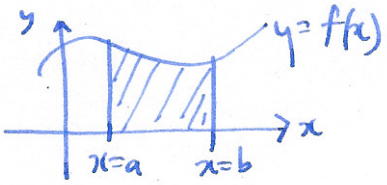
draw picture:



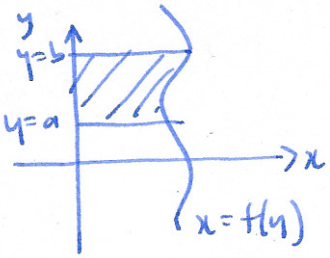
← need to break into two regions

$$\int_0^{1/2} 8x - x\,dx + \int_{1/2}^{1} \frac{1}{x^2} - x\,dx.$$

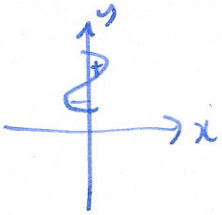
Integration along y-axis recall: $\int_a^b f(x)\,dx$



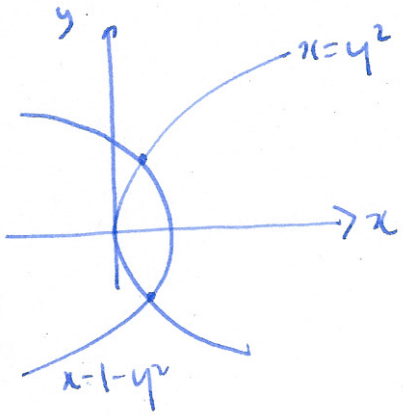
$\int_a^b f(y)\,dy$



signed area:



Example find area between curves $x=y^2$ and $x=1-y^2$



intersection points $y^2 = 1-y^2$ $2y^2 = 1$ $y = \pm\frac{1}{\sqrt{2}}$

$$\int_{-1/\sqrt{2}}^{+1/\sqrt{2}} 1 - y^2 - y^2\,dy$$

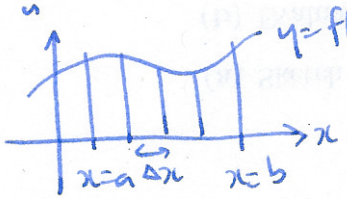
§6.2 Volume, average value

recall: volume of cylinder is Ah  A = area of base

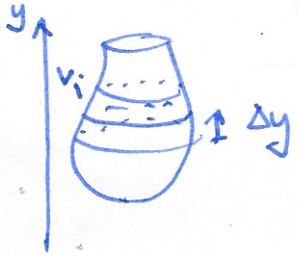
Fact: this works for cylinders of any base shape  $V = Ah$

suppose the shape is not a cylinder:  can approximate by horizontal slices

recall



← can approximate area under curve by rectangles of width Δx



← can approximate volume of object by cylinders of height Δy

volume $V = \sum V_i \approx \sum A(y_i)\,\Delta y$ volume $V = \int_a^b A(y)\,dy$