MTH 232 Calculus 2

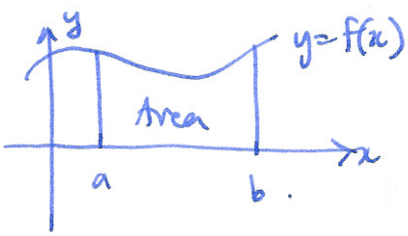
Joseph Maher joseph.maher@csi.cuny.edu
web page: http://www.math.csi.cuny.edu/~maher
office 1S-222 office hours M 12:20 – 2:15
W 2:30 – 3:20

- math tutoring: 1S-214
- students w/ disabilities

Text: Calculus, early transcendentals, Rogawski + Adams.

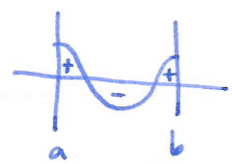
HW: webworks / Matlab projects / quizzes.

§5.2 Definite integral



intuition: $\int_a^b f(x)\,dx =$ area under the curve $y = f(x)$ between $x = a$ and $x = b$

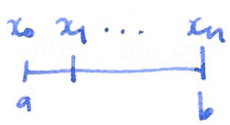
note: signed area



formal def$^{\underline{n}}$: Riemann sum $R(f,P,c) = \sum f(c_i)\Delta x_i$, $\Delta x_i = |x_i - x_{i-1}|$

f function

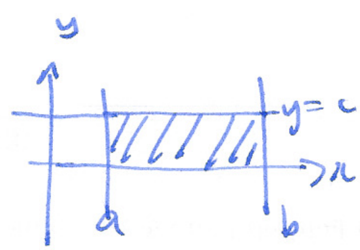
P partition of $[a,b]$



c choice of points $c_i \in [x_{i-1}, x_i]$

$$\int_a^b f(x)\,dx = \lim_{\|P\| \to 0} R(f,P,c) \qquad \|P\| = \max \Delta x_i$$

useful properties $\int_a^b c\,dx = c(b-a)$



$$\int_a^b f(x) + g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$

reversing limits: $\int_a^b f(x) = -\int_b^a f(x)\,dx$

0-length intervals : $\int_a^a f(x)\,dx = 0$

adjacent intervals : $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

§5.3 Indefinite integrals

Defn A function $F(x)$ is an anti-derivative for $f(x)$ if $F'(x) = f(x)$

General antiderivative : if $F(x)$ is an anti-derivative for $f(x)$, then any other antiderivative is of the form $F(x)+c$ for some constant c.

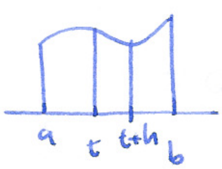
[Proof: spse f has antiderivatives F, H, then $(F-H)' = F' - H' = f - f = 0$
$\Rightarrow$ $F-H$ is ~~the~~ a constant function]

notation $\int f(x)\,dx = F(x)+c$ means $F(x)+c$ general antiderivative for $f(x)$

§5.4 Fundamental theorem of calculus I

Thm (FTC①) suppose $f(x)$ is cts on $[a,b]$ and $F(x)$ is an antiderivative for $f(x)$, i.e. $F'(x) = f(x)$. Then $\int_a^b f(x)\,dx = F(b) - F(a)$

intuition :



consider $\int_a^t f(x)\,dx$ ← function of t!

Q: what is the rate of change wrt t?

recall $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ so $\frac{d}{dt}\left(\int_a^t f(x)\,dx\right)$

$$= \lim_{h\to 0} \frac{\int_a^{t+h} f(x)\,dx - \int_a^t f(x)\,dx}{h} = \lim_{h\to 0} \frac{\int_t^{t+h} f(x)\,dx}{h} \approx \text{area of rectangle } \frac{f(t)\times h}{h} = f(t)$$

i.e. $\int_a^t f(x)\,dx$ is an antiderivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t) + c$

Q: what is the constant $t=a$: $\int_a^a f(x) = 0 = F(a) + c \Rightarrow c = -F(a)$

so $\int_a^t f(x)\,dx = F(t) - F(a)$ □.

Example $\int_2^3 \sqrt{x} + \frac{1}{x} + \sin(x)\,dx = \left[\frac{2x^{3/2}}{3} + \ln|x| - \cos(x)\right]_2^3$

$$= \frac{3(3)^{4/3}}{3} + \ln|3| - \cos(3) - \left(\frac{2(2)^{4/3}}{3} + \ln|2| - \cos(2) \right).$$

§5.5 Fundamental theorem of calculus II

Thm (FTC ②) let $f(x)$ be cts on $[a,b]$, then $A(x) = \int_a^x f(t)\,dt$ is an antiderivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx} \Leftrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

Furthermore: $A(a) = 0$.

Example $\int_0^x e^{-t^2}\,dt$ ← a function with derivative e^{-x^2}

Example what about $\int_0^{x^2} \sin(t)\,dt$ ← function of a function

to find $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$ set $A(x) = \int_0^x \sin(t)\,dt$, then $A'(x) = \sin(t)$

so $\frac{d}{dx}\int_0^{x^2} \sin(t)\,dt = \frac{d}{dx}\left(A(x^2)\right) = \underbrace{A'(x^2).(x^2)'}_{\text{chain rule}} = A'(x)^2 \cdot 2x = \sin(x^2).2x$

[aside: when people say "not every formula can be integrated" what do they mean? If $f(x)$ is cts then $A(x) = \int_0^x f(t)\,dt$ is an integral for $f(x)$, but we might not be able to write it as a formula involving basic functions.

Analogy
- $\sqrt{2} = 1.414\ldots$ is a real number, but not a fraction.
- $x^5 - x - 1$ has one real root which cannot be written as an expression involving rational numbers and fractional powers [Galois theory]

- e^{-x^2} has an integral that can't be written as a combination of elementary functions [differential Galois theory]

useful rules

$f(x)$	$f'(x)$
x^n	nx^{n-1}
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$

$\tan^{-1}(x)$	$\frac{1}{1+x^2}$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$

$f(x)$	$\int f(x)\,dx$
x^n	$\frac{x^{n+1}}{n+1}$
e^x	e^x
$\frac{1}{x}$	$\ln\lvert x\rvert$
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$