Math 232 Calculus 2 Spring 20 Sample midterm 2

(1) Find $\int \cos^3 3x \ dx$.

(2) Find $\int \cos 6x \sin 5x \ dx$.

(3) Find $\int \frac{x}{\sqrt{16x^2+1}} dx$.

(4) Find $\int \frac{x^2-5x-2}{(x-1)^2(x+3)} dx$.

(5) Find $\int_0^1 x^2 \ln x^4 \ dx$.

(6) Find $\int_0^\infty \frac{1}{x^2+9} \ dx$.

(7) Can you find the degree three Taylor polynomial centered at $x=0$ for the function $f(x)=\sqrt{x^3}$, why or why not? Find the degree three Taylor polynomial for this function centered at $x=1$. Find an error bound for the approximation for $\sqrt{8}$.

(8) Does the sequence $a_n = \frac{2^n}{n!}$ converge or diverge?

(9) Does the series $\sum_{n=2}^{\infty} e^{-n}$ converge or diverge? If it converges, find the exact value.

(10) Does the series $\sum_{n=1}^{\infty} \frac{1}{4n^2+8n+3}$ converge or diverge? If it converges, find the exact value.

(11) Does the series $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$ converge or diverge?

(12) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^4}$ converge or diverge?

(13) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge or diverge?

(14) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n \sin n}{n^3+1}$ converge or diverge?

(15) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converge or diverge?

(16) For which values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ converge?

(17) Find the first three terms for the power series for $\cos(\sqrt{x})$ centered at $x = 1$.

(18) Find the first three terms of the power series centered at 0 for $x^2 e^{-x^2}$.

(19) Bonus question: Consider the function

$$f(x) = \begin{cases} e^{-1/x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}.$$

Show that this function is continuous, i.e.

$$\lim_{x \to 0} e^{-1/x} = 0.$$

More generally, show that

$$\lim_{x \to 0} \frac{e^{-1/x}}{x^n} = 0.$$

Show that the n-th derivative of f is given by

$$f^{(n)}(x) = \begin{cases} \frac{p_n(x)}{x^{2n}} e^{-1/x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases},$$

where $p_n(x)$ is a polynomial of degree $n-1$.

(Hint: you can do this by induction, or show the recursive formula $p_{n+1}(x) = x^2 p_n'(x) - (2nx - 1)p_n(x)$.)

Deduce that this function is smooth, i.e. infinitely differentiable, but not analytic, i.e. not equal to its Taylor series.