Math 232 Calculus 2 Spring 20 Midterm 1b

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int \cos x \sqrt{\sin x}\, dx$.

$u = \sin x$

$\frac{du}{dx} = \cos x$

$$\int \cos x \cdot \sqrt{u}\, \frac{dx}{du}\, du = \int \cos x \cdot u^{1/2} \cdot \frac{1}{\cos x}\, du$$

$$= \int u^{1/2}\, du = \frac{2}{3} u^{3/2} + c = \frac{2}{3}(\sin x)^{3/2} + c$$

(2) (10 points) Find $\int x(x-1)^{10}\,dx$.

$u = x-1$

$\frac{du}{dx} = 1$

$$\int (u+1)\,u^{10}\,\frac{dx}{du}\,du = \int u^{11} + u^{10}\,du = \frac{1}{12}u^{12} + \frac{1}{11}u^{11} + c$$

$$= \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + c$$

(3) (10 points) Find $\int x^2 e^{2-x^3}\, dx$.

$u = 2 - x^3$

$\frac{du}{dx} = -3x^2$

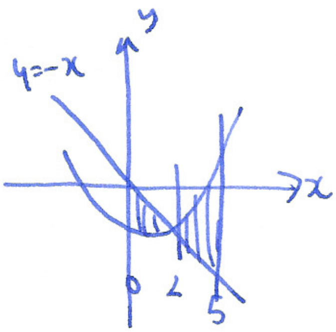
$$\int x^2 e^u \frac{dx}{du}\, du = \int x^2 e^u \frac{1}{-3x^2}\, du = \int -\frac{1}{3} e^u\, du$$

$$= -\frac{1}{3} e^u + c = -\frac{1}{3} e^{2-x^3} + c$$

(4) (10 points) Find the area bounded between the curves $y = -x$ and $y = x^2 - 6$ over the interval $0 \leq x \leq 5$.

intersections: $-x = x^2 - 6$

$x^2 + x - 6 = (x+3)(x-2)$

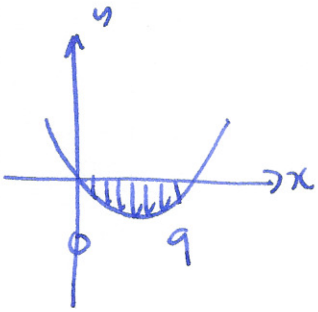


$$\int_0^2 -x - x^2 + 6 \, dx + \int_2^5 x^2 - 6 + x \, dx$$

$$= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x\right]_0^2 + \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x\right]_2^5$$

$$= -\frac{8}{3} - 2 + 12 + \frac{125}{3} + \frac{25}{2} - 30 - \frac{8}{3} - 2 + 12 = -\frac{16}{3} - 10 + \frac{125}{3} + \frac{25}{2} = \frac{233}{6}$$

(5) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 9x$ and the x-axis. Find the volume of revolution of this region rotated about the x-axis.

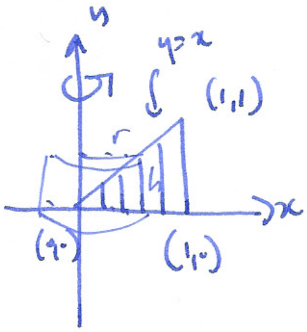


$$A = \int_0^9 \pi r^2 dx = \int_0^9 \pi y^2 dx = \int_0^9 \pi (x^2 - 9x)^2 dx$$

$$x^4 - 18x^3 + 81x^2$$

$$= \pi \left[\frac{1}{5}x^5 - \frac{9}{2}x^4 + 27x^3 \right]_0^9 = \pi \left(\frac{9^5}{5} - \frac{9^5}{2} + \frac{9^5}{3} \right) = 9^5 \pi \frac{1}{30}$$

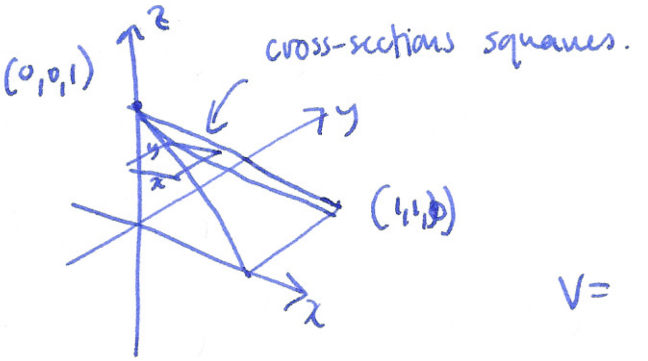
(6) (10 points) Use shells to find the volume of revolution of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$, rotated about the y-axis.

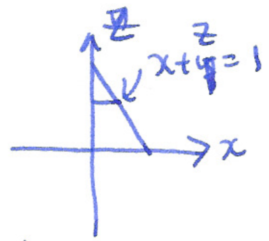


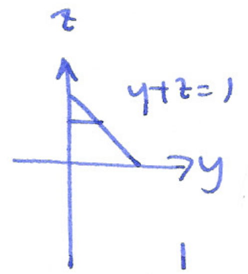
$$A = \int_0^1 2\pi r h \, dx = \int_0^1 2\pi x y \, dx = \int_0^1 2\pi x^2 \, dx$$

$$= \left[\frac{2}{3} \pi x^3 \right]_0^1 = \frac{2\pi}{3}.$$

(7) (10 points) Find the volume of the pyramid whose base is the unit square in the first quadrant with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(1,1,0)$, and whose top vertex is $(0,0,1)$.







$$V = \int_0^1 A(z)\,dz = \int_0^1 xy\,dz = \int_0^1 (1-z)^2\,dz$$

$$= \int_0^1 1-2z+z^2\,dz = \left[z - z^2 + \frac{1}{3}z^3\right]_0^1 = 1-1+\frac{1}{3} = \frac{1}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

$u = e^{-2x}$ $\quad v' = \cos x$

$u' = -2e^{-2x}$ $\quad v = \sin x$

(8) (10 points) Find $\int_0^{\pi} \underbrace{e^{-2x}}_{u} \underbrace{\cos(x)}_{v'}\, dx$.

$$\int_0^{\pi} e^{-2x}\cos(x)\,dx = \left[e^{-2x}\sin x\right]_0^{\pi} - \int_0^{\pi} -2e^{-2x}\sin x\,dx$$

0

$$\int_0^{\pi} e^{-2x}\cos x\,dx = \int_0^{\pi} \underbrace{2e^{-2x}}_{u} \underbrace{\sin x}_{v'}\,dx = \underbrace{\left[2e^{-2x}\cdot -\cos x\right]_0^{\pi}}_{\shortparallel} - \int_0^{\pi} -4e^{-2x}\cdot -\cos x\,dx$$

$u = 2e^{-2x}$ $\quad v' = \sin x$ $\quad 2e^{-2\pi} + 2$

$u' = -4e^{-2x}$ $\quad v = -\cos x$

$$\int_0^{\pi} e^{-2x}\cos x\,dx = 2e^{-2\pi} + 2 - 4\int_0^{\pi} e^{-2x}\cos x\,dx$$

$$\int_0^{\pi} e^{-2x}\cos x\,dx = \frac{2}{5}\left(e^{-2\pi}+1\right)$$

(9) Find $\displaystyle\int \cos^2 x \sin x \, dx$.

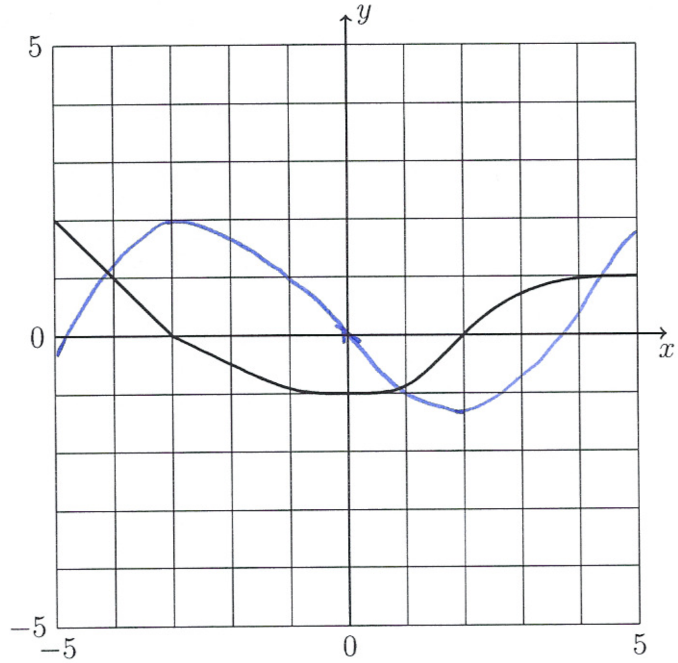
$u = \cos x$

$\dfrac{du}{dx} = -\sin x$

$$\int u^2 \sin x \frac{dx}{du} \, du = \int u^2 \sin x \frac{1}{-\sin x} \, du$$

$$= \int -u^2 \, du = -\frac{1}{3} u^3 + c = -\frac{1}{3} \cos^3 x + c$$

(10) Consider the graph of the function f drawn below.



Let $g(x) = \int_0^x f(t)\,dt$.
Sketch $g(x)$, and find $g(0)$, $g'(0)$ and $g'(2)$.

$g(0) = 0$

$g'(0) = -1$

$g'(2) = 0$