Math 232 Calculus 2 Spring 20 Midterm 1a

Name: Solutions

- I will count your best 8 of the following 10 questions.
- You may use a calculator, and a 3×5 index card of notes.

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
	80	

Midterm 1	
Overall	

(1) (10 points) Find $\int \sin x\sqrt{\cos x}\, dx$.

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$$\int \sin x \cdot \sqrt{u}\, \frac{dx}{du}\, du$$

$$= \int \sin x\, u^{1/2} \cdot \frac{1}{-\sin x}\, du = \int -u^{1/2}\, du$$

$$= -\frac{2}{3} u^{+3/2} + c = -\frac{2}{3} (\cos x)^{3/2} + c$$

(2) (10 points) Find $\int x(x+1)^{11}\,dx$.

$u = x+1$

$\frac{du}{dx} = 1$

$$\int (u-1)u^{11} \frac{dx}{du}\,du = \int u^{12} - u^{11}\,du$$

$$= \frac{1}{13}u^{13} - \frac{1}{12}u^{12} + c = \frac{1}{13}(x+1)^{13} - \frac{1}{12}(x+1)^{12} + c$$

(3) (10 points) Find $\int x^2 e^{3-x^3}\, dx$.

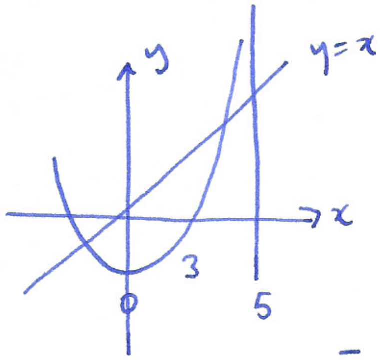
$u = 3 - x^3$

$\frac{du}{dx} = -3x^2$

$\int x^2 e^u \frac{dx}{du} du = \int x^2 \cdot e^u \cdot \frac{1}{-3x^2} du$

$= \int -\frac{1}{3} e^u du = -\frac{1}{3} e^u + c = -\frac{1}{3} e^{3-x^3} + c$

(4) (10 points) Find the area bounded between the curves $y = x$ and $y = x^2 - 6$ over the interval $0 \leq x \leq 5$.



intersections : $x = x^2 - 6$

$x^2 - x - 6 = (x-3)(x+2) = 0$

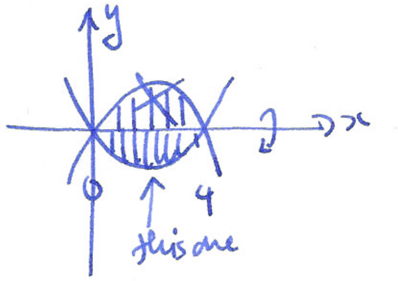
$$\int_0^3 x - x^2 + 6 \, dx + \int_3^5 x^2 - 6 - x \, dx$$

$$= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x\right]_0^3 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x\right]_3^5.$$

$$= -9 + \frac{9}{2} + 18 + \frac{125}{3} - \frac{25}{2} - 30 - \left(9 - \frac{9}{2} - 18\right)$$

$$= -18 + 9 + 36 - 30 + \frac{250}{6} - \frac{75}{6} = -3 + \frac{175}{6} = \frac{157}{6}$$

(5) (10 points) Draw a picture of the region bounded by the curve $y = x^2 - 4x$ and the x-axis. Find the volume of revolution of this region rotated about the x-axis.

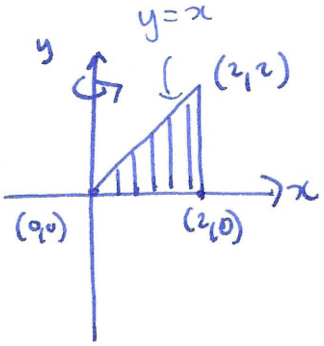


$$V = \int_0^4 \pi r^2\,dx = \int_0^4 \pi (x^2-4x)^2\,dx$$

$$= \pi \int_0^4 x^4 - 8x^3 + 16x^2\,dx$$

$$= \pi\left[\frac{1}{5}x^5 - \frac{8}{4}x^4 + \frac{16}{3}x^3\right]_0^4 = \pi\left(\frac{1024}{5} - 512 + \frac{1024}{3}\right) = \frac{512}{15}\pi$$

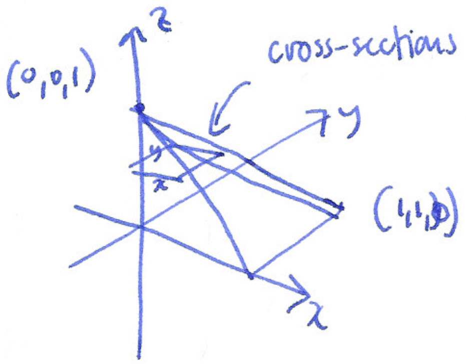
(6) (10 points) Use shells to find the volume of revolution of the triangle with vertices $(0,0)$, $(2,0)$ and $(2,2)$, rotated about the y-axis.



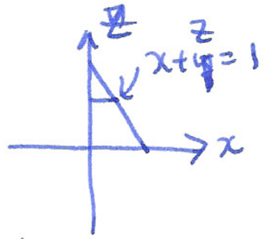
$$A = \int_0^2 2\pi r h \, dx = \int_0^2 2\pi x y \, dx$$

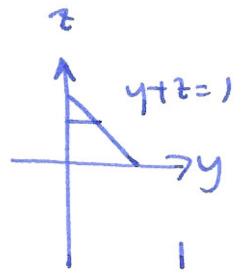
$$= \int_0^2 2\pi x^2 \, dx = 2\pi \left[\frac{1}{3} x^3 \right]_0^2 = \frac{16}{3}\pi$$

(7) (10 points) Find the volume of the pyramid whose base is the unit square in the first quadrant with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(1,1,0)$, and whose top vertex is $(0,0,1)$.



cross-sections squares.





$$V = \int_0^1 A(z)\,dz = \int_0^1 xy\,dz = \int_0^1 (1-z)^2\,dz$$

$$= \int_0^1 1-2z+z^2\,dz = \left[z - z^2 + \frac{1}{3}z^3\right]_0^1 = 1-1+\frac{1}{3} = \frac{1}{3}$$

$$\int uv'\,dx = uv - \int u'v\,dx$$

(8) (10 points) Find $\int_0^{\pi} e^{-3x}\sin(x)\,dx$.

$$\int_0^{\pi} \underbrace{e^{-3x}}_{u}\,\underbrace{\sin x}_{v'}\,dx = \left[-e^{-3x}\cos x\right]_0^{\pi} - \int_0^{\pi} -3e^{-3x}\cdot -\cos x\,dx$$

$$\left[-e^{-3x}\cos x\right]_0^{\pi} = e^{-3\pi} + 1$$

$u = e^{-3x}$ $\quad v' = \sin x$

$u' = -3e^{-3x}$ $\quad v = -\cos x$

$$\int_0^{\pi} e^{-3x}\sin x\,dx = e^{-3\pi} + 1 - \int_0^{\pi} \underbrace{3e^{-3x}}_{u}\,\underbrace{\cos x}_{v'}\,dx$$

$u = 3e^{-3x}$ $\quad v' = \cos x$

$u' = -9e^{-3x}$ $\quad v = \sin x$

$$\int_0^{\pi} e^{-3x}\sin x\,dx = e^{-3\pi} + 1 - \underbrace{\left[3e^{-3x}\sin x\right]_0^{\pi}}_{0} + \int_0^{\pi} -9e^{-3x}\sin x\,dx$$

$$\int_0^{\pi} e^{-3x}\sin x\,dx = \frac{1}{10}\left(e^{-3\pi} + 1\right)$$

(9) Find $\int \cos x \sin^2 x \, dx$.

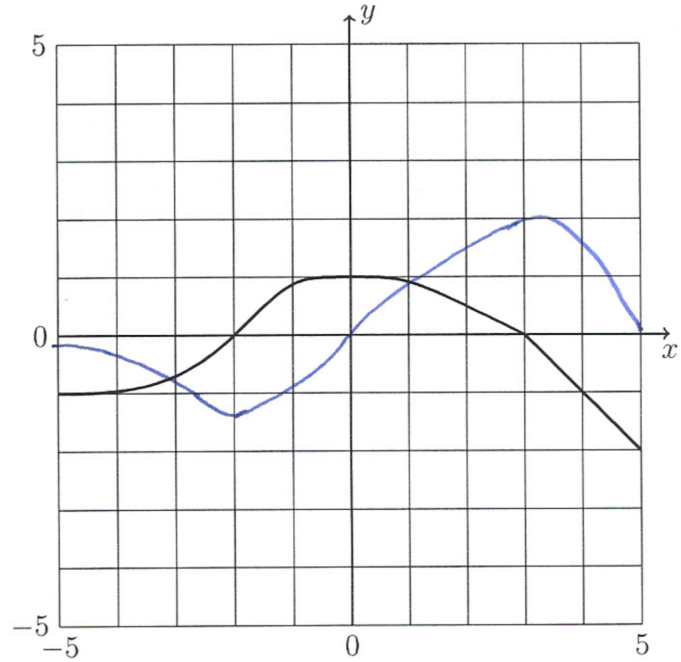
$u = \sin x$

$\frac{du}{dx} = \cos x$

$\int \cos x \cdot u^2 \cdot \frac{dx}{du} \cdot du = \int \cos x \cdot u^2 \cdot \frac{1}{\cos x} \, dx$

$= \int u^2 \, dx = \frac{1}{3} u^3 + c = \frac{1}{3} \sin^3 x + c$

(10) Consider the graph of the function f drawn below.



Let $g(x) = \int_0^x f(t)\,dt$.

Sketch $g(x)$, and find $g(0)$, $g'(0)$ and $g'(3)$.

$g(0) = 0$

$g'(0) = 1$

$g'(3) = 0$