$$\frac{d}{dt}\int_{66}^{t} \sec(9x+57)\,dx = \sec(\underbrace{9t+57}_{u})$$

$t = 66.$

$u = 9 \times 66 + 57 = 651$

$\sec(u)$ ← vertical $2\pi n + \frac{\pi}{2}$

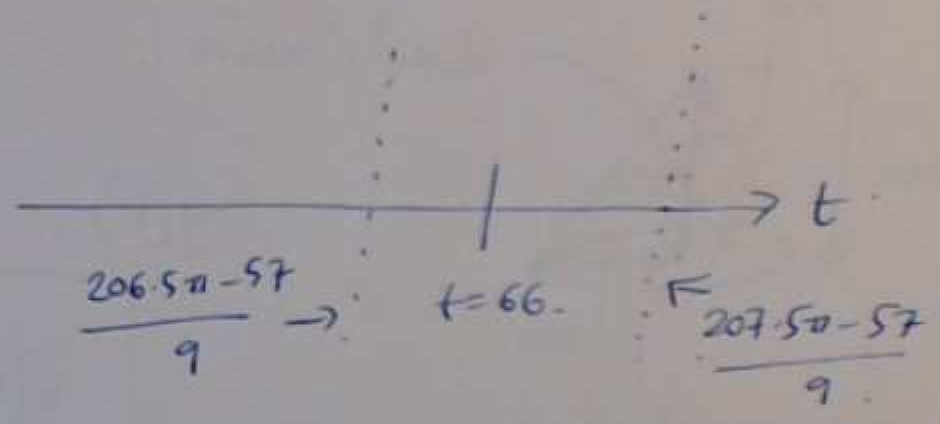
$2\pi n + \frac{3\pi}{2}.$

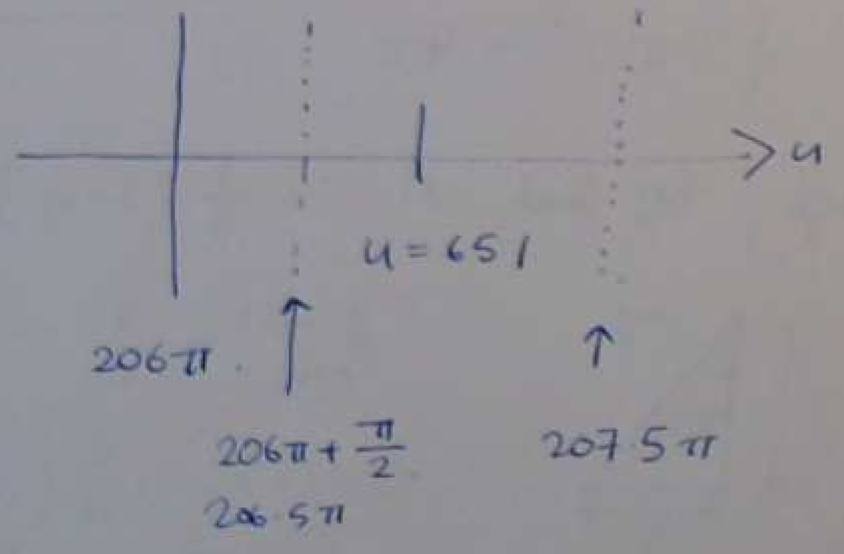
$= \frac{1}{\cos(u)}$

$\frac{651}{2\pi}$

$\frac{651}{\pi} = 207.22$

$u = 9t+57 \qquad \frac{u-57}{9} = t$





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5.5. Q10

$$\frac{d}{dx}\int_0^{28/x} \cos^6 t \, dt$$

$$u = 28/x = 28x^{-1} \quad ②$$

$$\frac{du}{dx} = -28x^{-2}.$$

$$\frac{d}{dx}\int_0^{u} \cos^6 t \, dt$$

$$\underbrace{\frac{d}{du}\int_0^{u} \cos^6 t \, dt}_{\text{FTC } ②} \cdot \frac{du}{dx}$$

$$\cos^6(u) \cdot \frac{-28}{x^2} = \cos^6\left(\frac{28}{x}\right) \cdot \frac{-28}{x^2}.$$

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②

5.5 Q10

$$\frac{d}{dx}\int_0^{28/x} \cos^6 t \, dt$$

$$F(u) = \int_0^u \cos^6 t \, dt$$

$$\cancel{\frac{d}{dx}\int_0^x \cos^6 t \, dt} \;\cancel{=}\; \cancel{\cos^6 x}$$

$$\frac{d}{dx}\left(F\left(\frac{28}{x}\right)\right) = \underbrace{\cancel{\frac{d}{dx}}}F\left(\frac{28}{x}\right) \cdot \frac{d}{dx}\left(\frac{28}{x}\right)$$

$$\frac{d}{dx}\int_0^x \cos^6 t \, dt \Big|_{\frac{28}{x}}$$

$$\cos^6\left(\frac{28}{x}\right) \cdot -\frac{28}{x^2}$$

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③

5.5 Q10

$$\frac{d}{dx}\int_0^{28/x} \cos^6 t\, dt$$

$$F(u) = \int_0^u \cos^6 t\, dt$$

$$\cancel{\frac{d}{dx}\int_0^x \cos^6 t\, dt} \cancel{=} \cancel{\cos^6 x}$$

$$\frac{d}{dx}\left(F\left(\frac{28}{x}\right)\right) = \frac{d}{dx}F\left(\frac{28}{x}\right) \cdot \frac{d}{dx}\left(\frac{28}{x}\right)$$

$$\frac{d}{dx}\int_0^x \cos^6 t\, dt \Big|_{\frac{28}{x}}$$

$$\cos^6\left(\frac{28}{x}\right) \cdot -\frac{28}{x^2}$$

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5.7. Q7

$$\int \sin 2x\,(\cos 2x+1)^{1/2}\,dx$$

ths

$u = \cos 2x + 1$

$\frac{du}{dx} = -\sin 2x \cdot 2$

$$\int \sin 2x\,(u)^{1/2}\,\frac{dx}{du}\,du$$

$$\int \cancel{\sin 2x}\; u^{1/2}\,\frac{1}{-2\cancel{\sin 2x}}\,du = -\frac{1}{2}\int u^{1/2}\,du$$

$$= -\frac{1}{2}\,\frac{2}{3}\,u^{3/2} + c = -\frac{1}{3}(\cos 2x+1)^{3/2} + c$$

④

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$$\frac{d}{dx}\int_{\sqrt{x}}^{x^2} \tan 6t \, dt$$ WW 5.5 Q11

Know: $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$. (5)

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{0} \tan 6t \, dt + \int_0^{x^2} \tan 6t \, dt\right)$$

$$\frac{d}{dx}\int_0^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$$

$$\int_a^b f(t)\,dt + \int_b^c f(t)\,dt = \int_a^c f(t)\,dt$$

$$\frac{d}{dx}\left(-\int_0^{\sqrt{x}} \tan 6t \, dt + \int_0^{x^2} \tan 6t \, dt\right).$$

$$\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$$

$$-\tan 6\sqrt{x}\,\cancel{dt}\cdot(\sqrt{x})' + \tan 6x^2 \cdot \frac{d}{dx}(x^2).$$

$$-\frac{1}{2}x^{-1/2}\tan(6\sqrt{x}) + 2x\tan 6x^2$$

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SF. Q3.

a) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$ $x = 3$: $\frac{0}{0}$ indeterminate form.

L'H: $\lim_{x \to 3} \frac{2x - 1}{1} = 5$.

b) $\lim_{x \to 0} \frac{1 - e^{3x}}{\cos(5x)}$ $x = 0$: $\frac{0}{1} = 0$.

c) $\lim_{x \to 0^+} x^{\sin(x)} = \lim_{x \to 0^+} e^{\ln(x)\sin(x)}$.

$\lim_{x \to 0^+} \ln(x)\sin(x) = \lim_{x \to 0^+} \frac{\ln(x)}{1/\sin(x)}$

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⑦

$$\lim_{x\to 0^+} \frac{\ln(x)}{1/\sin(x)} \overset{l'H}{=} \lim_{x\to 0^+} \frac{1/x}{-(\sin(x))^{-2}\cdot\cos(x)}$$

$$= \lim_{x\to 0+} -\frac{\sin^2(x)}{x\cos(x)}$$

method ① ← apply l'H twice.

$$= \underbrace{-\lim_{x\to 0+} \frac{\sin(x)}{x}}\cdot \underbrace{\lim_{x\to 0+} \frac{\sin(x)}{\cos(x)}}_{0} = -1\cdot 0 = 0.$$

$$\overset{l'H}{=} \underbrace{\lim_{x\to 0+} \frac{\cos(x)}{1}}_{1}$$

$$\lim_{x\to 0+} e^{\ln(x)\sin x}$$

$$= e^{\lim_{x\to 0+} \ln(x)\sin(x)}{}^{0} = e^0 = 1.$$

c)

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d) $\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{\sin^2 x} = \lim_{x\to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x}$. ← apply l'H 4 times.

8

$\lim_{x\to 0} \frac{\frac{1}{2} - \frac{1}{2}\cos 2x - x^2}{x^2\left(\frac{1}{2} - \frac{1}{2}\cos 2x\right)}$] $\frac{1}{2}x^2 - \frac{1}{2}x^2\cos 2x$

$\frac{d}{dx}(\sin^2 x)$

$= 2\sin x \cdot \cos x$

$= \sin 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

l'H

$= \lim_{x\to 0} \frac{\frac{1}{2}\sin 2x \cdot 2 - 2x}{x - x\cos 2x - \frac{1}{2}x^2 - \sin 2x}$

$= \lim_{x\to 0} \frac{\sin 2x - 2x}{x - x\cos 2x + x^2 \sin 2x}$.

$\stackrel{l'H}{=} \lim_{x\to 0} \frac{2\cos 2x - 2}{1 - \cos 2x - x - \sin 2x \cdot 2 + 2x\sin 2x + x^2\cos 2x \cdot 2}$

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$$\overset{l'H}{=} \lim_{x \to 0} \frac{-4\sin 2x}{\underbrace{2\sin 2x + 4\sin 2x}_{6\sin 2x} + \underbrace{4x\cos 2x \cdot 2 + 4x\cos 2x}_{12\,8x\cos 2x} - x^2 \sin 2x \cdot 4}$$

$$\overset{l'H}{=} \lim_{x \to 0} \frac{-\cancel{4}8\cos 2x}{12\cancel{4}\cos 2x + 12\cos 2x + 24x \cdot -\sin 2x - 2x\sin 2x \cdot 4 - x^2 \cos 2x \cdot 8}$$

$$= \frac{-8}{24} = -\frac{1}{3}.$$

10

Q8 $x^3y - xy^4 + 4x = 10$ $(-1,2)$

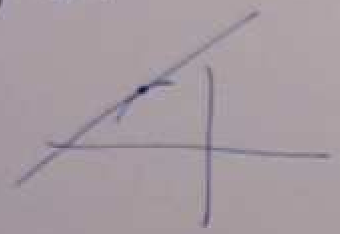
implicit diff. $y \rightsquigarrow y(x)$. $y^4 \rightsquigarrow (y(x))^4$.

$$3x^2y + x^3 \cdot y' - y^4 - x \cdot 4y^3 \cdot y' + 4 = 0.$$

$x=-1$ $y=2$

$$6 + -y' - 16 + 32y' + 4 = 0.$$

$31y' = 6.$ $y' = 6/31.$



$$y - 2 = \frac{6}{31}(x+1)$$

Q11

$V = \frac{4}{3}\pi r^3$ $\qquad A = 4\pi r^2$ $\qquad \frac{dV}{dt} = 8 \text{ in}^3/\text{s}.$

$V(t) \quad r(t) \quad A(t) \quad r(t) \qquad r = 6 \text{ in}$

$$\underbrace{\frac{dV}{dt}}_{8} = 4\pi \underset{\substack{\uparrow \\ r=6}}{r^2} \frac{dr}{dt} \qquad \frac{dA}{dt} = 8\pi r \frac{dr}{dt}.$$

$$1\cancel{8} = \cancel{4}\pi \cdot \overset{18}{\cancel{36}} \frac{dr}{dt}. \qquad \frac{dA}{dt} = 48\cancel{\pi}6 \cdot \frac{1}{4\cancel{18\pi}} = \frac{2}{3} \text{ in}^2/\text{sec}$$

$$\frac{dr}{dt} = \frac{1}{18\pi}.$$

```
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[Previously saved workspace restored]

> 70 * 0.8 + 0.2
[1] 56.2
> 70 * 0.8 + 20
[1] 76
>
Save workspace image? [y/n/c]: y
maher@ariadne:~$ R

R version 3.5.2 (2018-12-20) -- "Eggshell Igloo"
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Platform: x86_64-pc-linux-gnu (64-bit)

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Type 'q()' to quit R.

[Previously saved workspace restored]

> 9 * 66 + 57
[1] 651
> 651 / pi
[1] 207.2197
```

Q2 c) variant.

try $u = \sin(2x)$

$$\frac{du}{dx} = \cos(2x) \cdot 2.$$

$$\int_{x=0}^{x=\pi/4} \cos(2x)\sin^3(2x)\,dx.$$

$$\int_0^1 \cos(2x)\, u^3 \frac{dx}{du}\,du$$

$$\int_0^1 \cancel{\cos(2x)}\, u^3 \frac{1}{2\cancel{\cos(2x)}}\,du = \int_0^1 \frac{1}{2}u^3\,du$$

$$= \left[\frac{1}{8}u^4\right]_0^1 = \frac{1}{8}.$$