WW 5.5 Q7

$$\frac{d}{dt}\int_{10}^{t} \sec(19x+10)\,dx = \sec(19t+10)$$

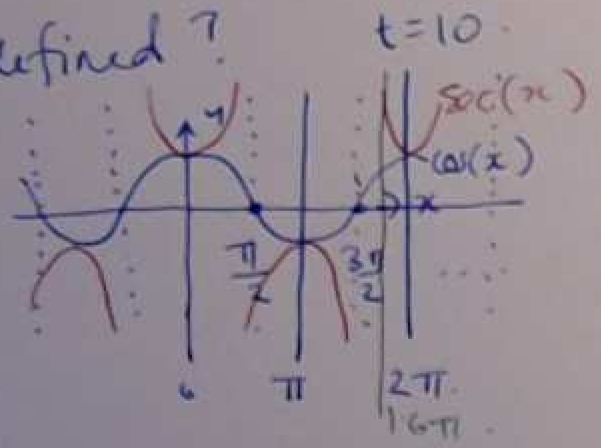
FTC ② $\quad \frac{d}{dt}\int_{a}^{t} \cancel{\sec} f(x)\,dx = f(t)$

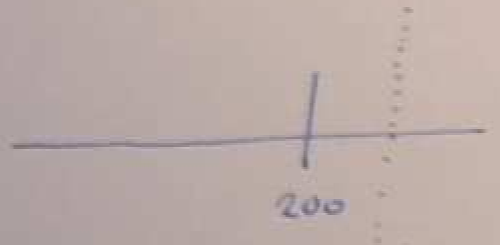
$t=10$

$\Leftrightarrow \sec(200)$

Q: where is this defined?

$\sec(x) = \frac{1}{\cos(x)}$





$\frac{200}{\pi} = 63.661\ldots$

$1.661\ldots$

$64.5\pi \leftarrow$ next vertical asymptote

$19t + 10 = 64.5\pi$

$t = \frac{64.5\pi - 10}{19}$

WW 5.5 Q7

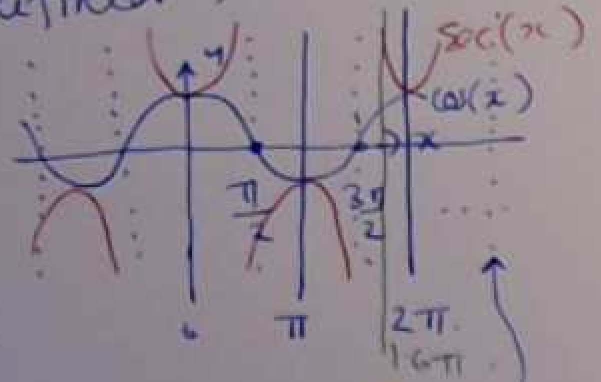
$$\frac{d}{dt}\int_{10}^{t} \sec(19x+10)\,dx = \sec(19t+10).$$

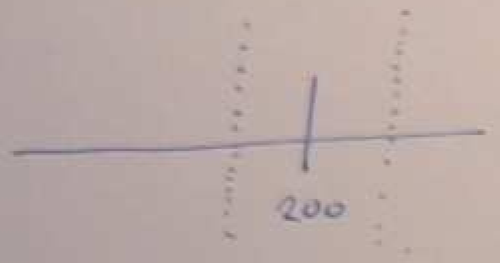
FTC ② $\quad \frac{d}{dt}\int_{a}^{t} \cancel{\sec} f(x)\,dx = f(t)$

$t = 10$
$\Leftrightarrow \sec(200)$

Q: where is this defined? $\quad t=10.$

$\sec(x) = \frac{1}{\cos(x)}.$





$\frac{200}{\pi} = 63.661\ldots$

$1.661\ldots$

1.5π

$64.5\pi \leftarrow$ next vertical asymptote

63.5π

↑ previous vertical asymptote

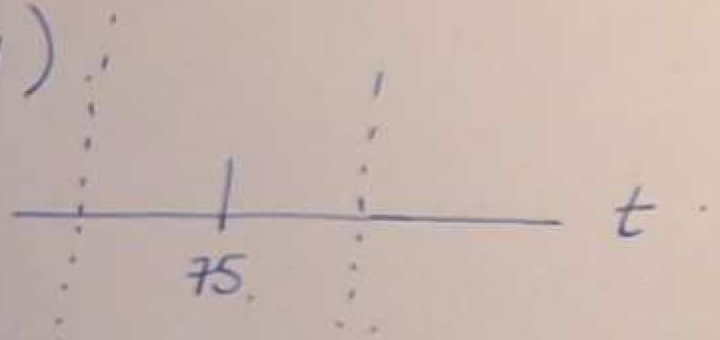
$19t + 10 = 64.5\pi$

$t = \frac{64.5\pi - 10}{19}$

$\left(\frac{63.5\pi - 10}{19}, \frac{64.5\pi - 10}{19} \right)$

$$\frac{d}{dt}\int_{75}^{t} \sec(13x - 91)\,dx \;=\; \sec(\overset{13}{\cancel{31}}\,t - 91)$$

1



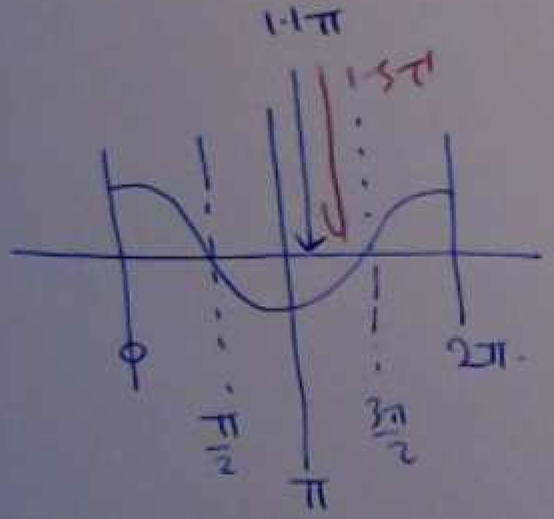
$\sec(\theta) \leftarrow$ vertical asymptotes

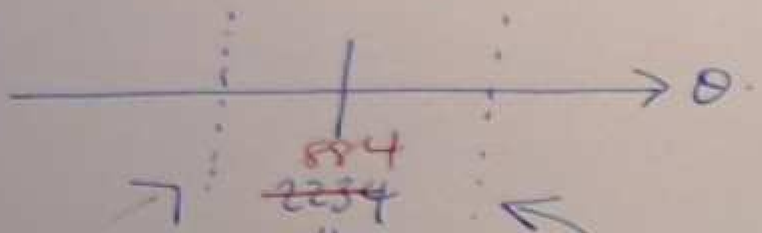
at $\quad 2\pi n + \frac{\pi}{2} \quad 2\pi n + \frac{3\pi}{2}$.

$\theta = \overset{13}{\cancel{31}}t - 91$

$= \cancel{2234}$ 884

$\frac{\theta + 91}{31}$





884

$\cancel{2234}$

$\parallel$

$281.3 \quad \cancel{711}\cdot\pi$

$\overset{}{\cancel{710}}_{280}\pi + \cancel{1.1}_{1.3}\pi$

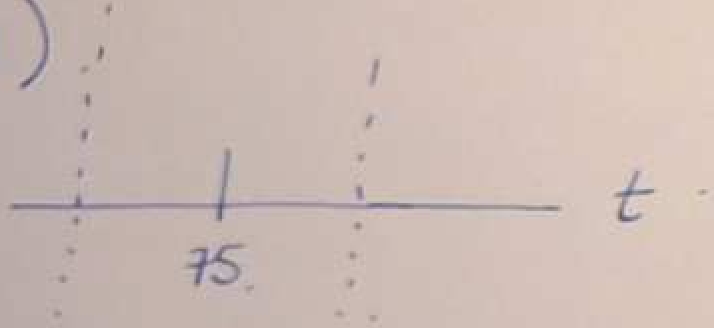
$$\frac{\left(\overset{280}{\cancel{710}}\pi + \frac{\pi}{2}\right) + 91}{\cancel{31}\,13} \qquad \theta$$

74.7859

$$\frac{\overset{\theta}{\left(\overset{280}{\cancel{710}}\pi + \frac{3\pi}{2}\right)} + \overset{91}{\cancel{31t}}}{\cancel{31}\,13}$$

75.02756

$$\frac{d}{dt}\int_{75}^{t} \sec(13x-91)\,dx = \sec(\overset{13}{\cancel{31}}\,t-91)$$

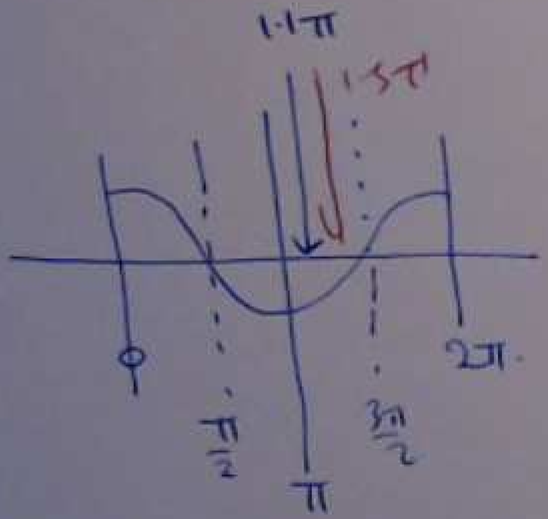


$\sec(\theta) \leftarrow$ vertical asymptotes

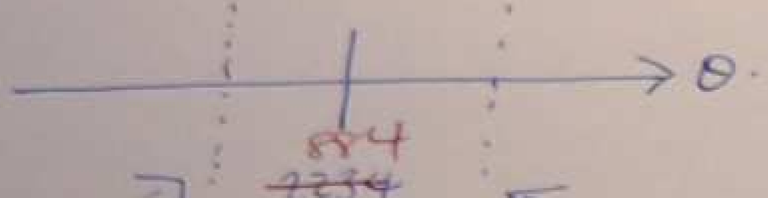
at $2\pi n+\frac{\pi}{2}$ $\quad 2\pi n+\frac{3\pi}{2}$

$$\theta = \overset{13}{\cancel{31}}\,t - 91$$

$$= \overset{884}{\cancel{2234}}$$



$$\frac{\theta+91}{31}$$



$\cancel{2234}$ $\overset{884}{}$

$=$

$281.2\ \ \cancel{711}\cdot\pi$

$\underset{280}{\cancel{710}}\pi + \underset{1.3}{\cancel{1.1}}\pi$

$$\frac{\left(\overset{280}{\cancel{710}}\pi+\frac{\pi}{2}\right)+91}{\cancel{31}\ 13} \quad (\theta)$$

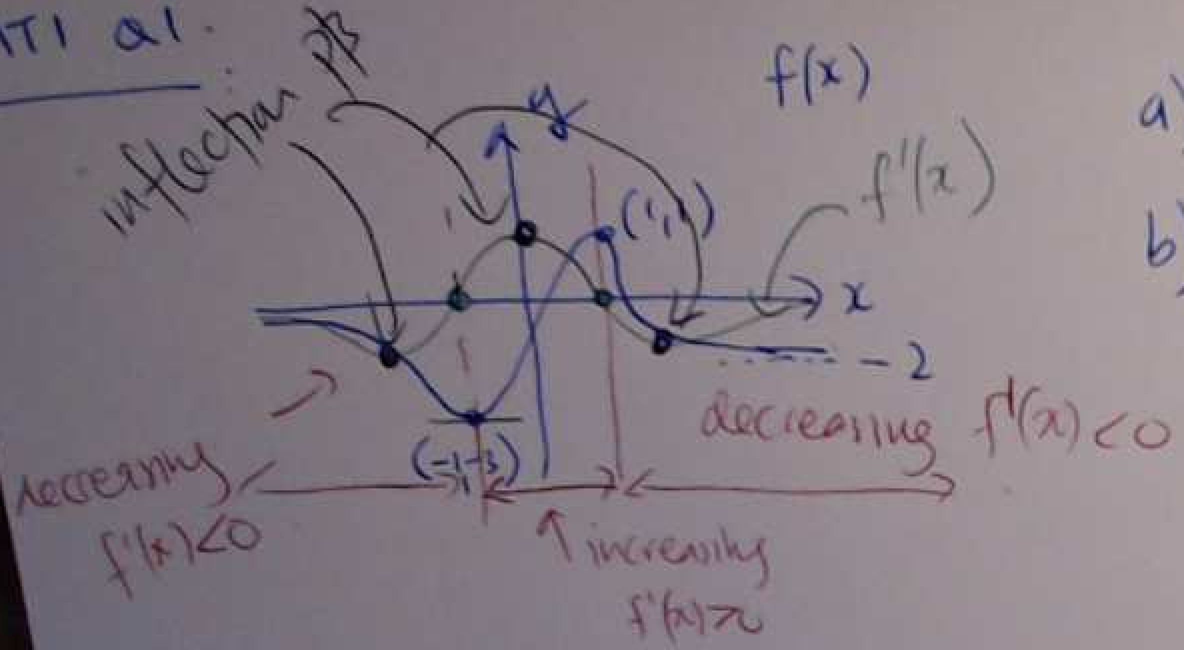
74.7859

$$\frac{\left(\overset{280}{\cancel{710}}\pi+\frac{3\pi}{2}\right)+91}{\cancel{31}\ 13} \quad (\theta)$$

75.02756

Joseph Maher

SMT1 Q1.



a) $f'(x) < 0 \leftrightarrow$ decreasing

b) $f'(x) > 0 \leftrightarrow$ increasing

c) $\lim_{x \to \infty} f'(x) = 0$.

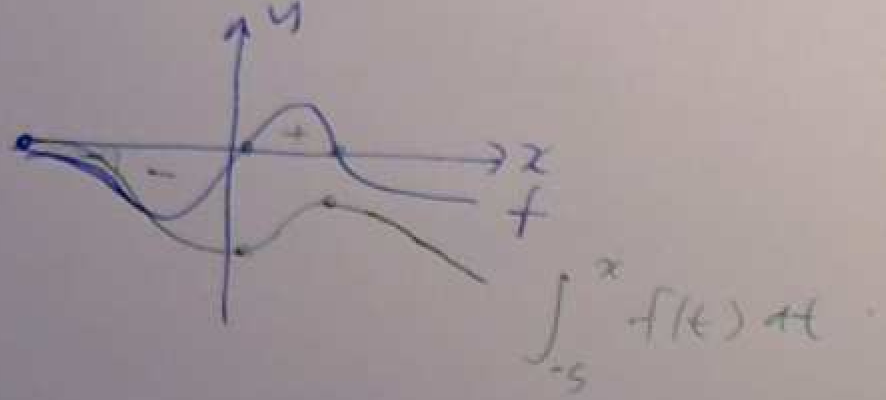
d) $\lim_{x \to -\infty} f'(x) = 0$.

e) sketch $f'(x)$

f) sketch $\int_{-5}^{x} f(t)\,dt$.

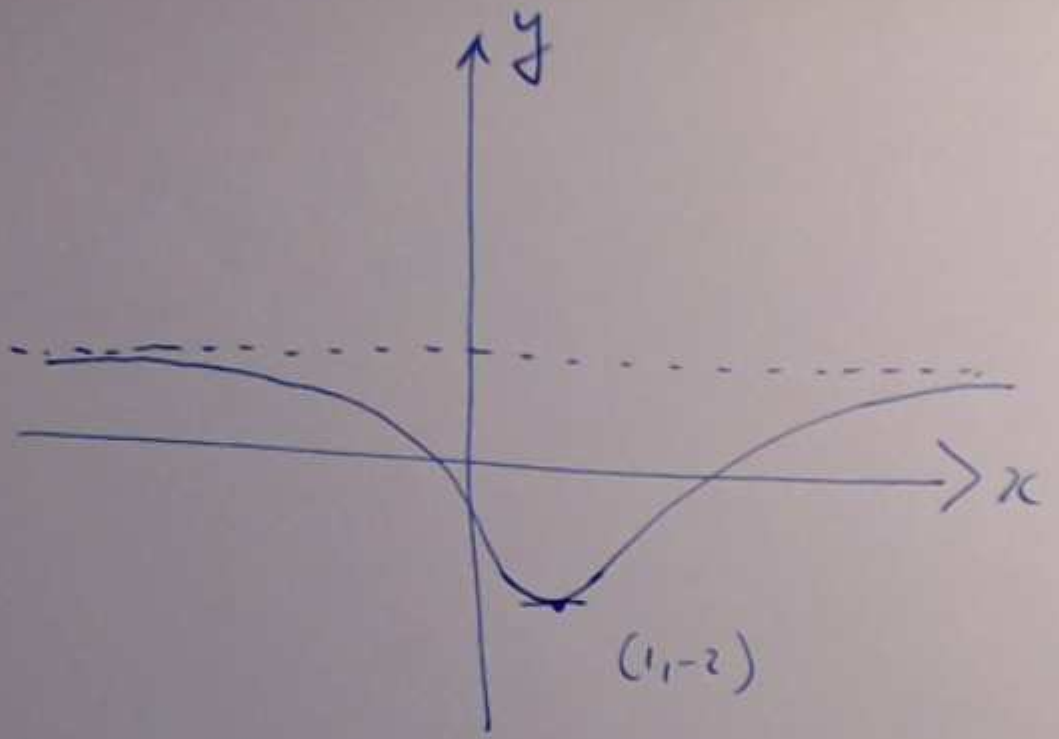
g) points of inflection

$f''(x) = 0$.



Q2

4



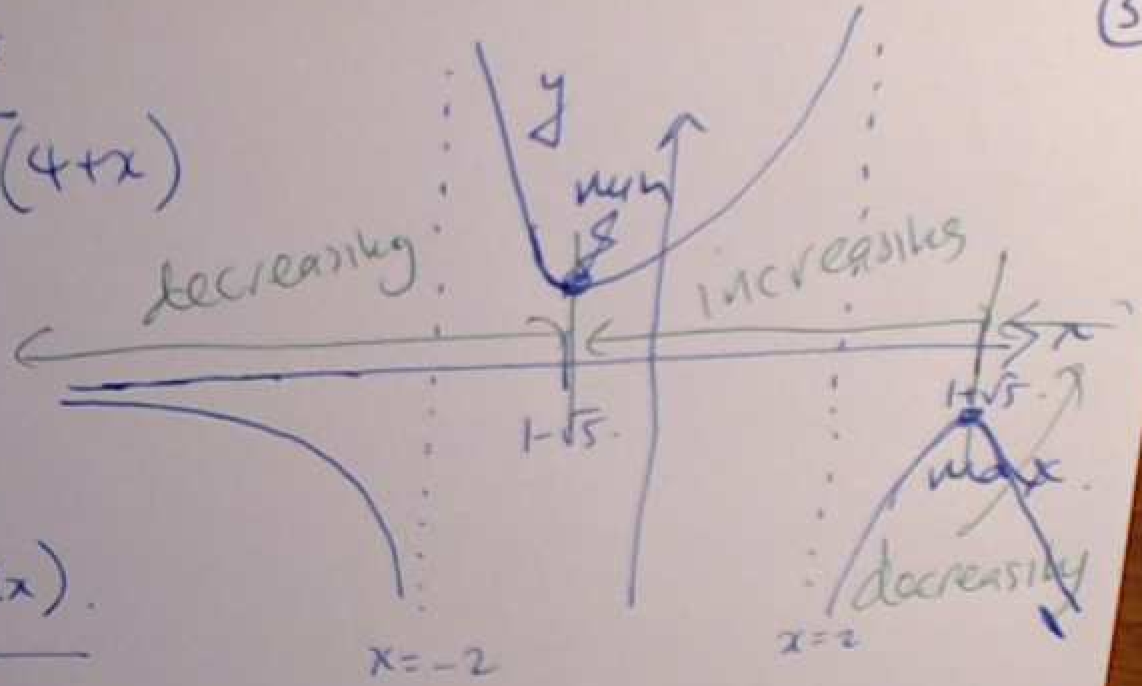
$$\lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 1.$$

Q3 $f(x) = \dfrac{e^x}{4-x^2} = \dfrac{e^x}{(4-x)(4+x)}$

$\lim_{x \to \infty} \dfrac{e^x}{4-x^2} = -\infty$

$\lim_{x \to -\infty} \dfrac{e^x}{4-x^2} = 0$

c)



b) $f'(x) = \dfrac{(4-x^2)e^x - e^x(-2x)}{(4-x^2)^2}$

$e^x(\underline{-x^2+2x+4}) = 0 \qquad x = \dfrac{-2 \pm \sqrt{4+16}}{-2} = 1 \pm \sqrt{5}$

$\sim \geq 0$

d) 2nd derivative test $\rightarrow$ don't use this, use 1st derivative test

e)

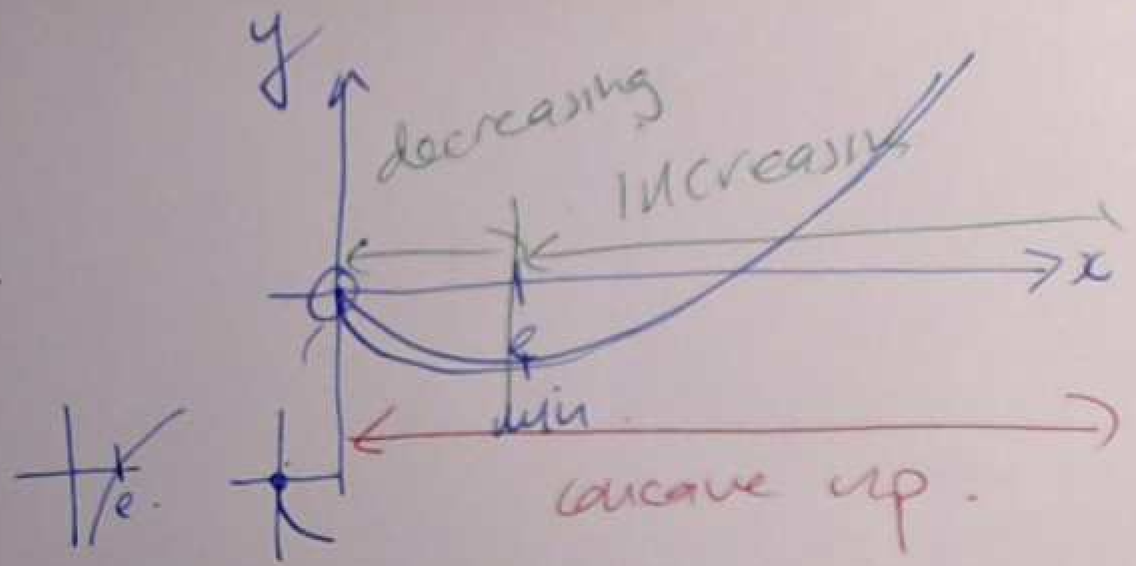
Q4 $f(x) = x\ln x - 2x$

$$f'(x) = \ln x + x\cdot\frac{1}{x} - 2$$

$$= \ln x + 1 - 2$$

$$= \ln x - 1$$

$f'(x)$



$f'(x) = 0$: $\ln x - 1 = 0$ $\ln x = 1$ $x = e$.

$f''(x) = \frac{1}{x} > 0$ for $x > 0$. $\leftrightarrow$ concave up. min.

Q6

$2\pi R$

$$V = \frac{1}{3}\pi r^2 h$$

$$R^2 = h^2 + r^2$$

$$(2\pi-\theta)R = 2\pi r \longleftrightarrow r = R\left(1-\frac{\theta}{2\pi}\right)$$

$$h = \sqrt{R^2 - r^2}$$

$$h = \sqrt{R^2 - R^2\left(1-\frac{\theta}{2\pi}\right)^2}$$

$$V = \frac{1}{3}\pi R^2\left(1-\frac{\theta}{2\pi}\right)^2\sqrt{R^2 - R^2\left(1-\frac{\theta}{2\pi}\right)^2}$$

$$V = \frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)^2\sqrt{\underbrace{1-\left(1-\frac{\theta}{2\pi}\right)^2}_{*}}$$

$$\frac{dV}{d\theta} = \frac{1}{3}\pi R^3\, 2\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)\sqrt{*} + \frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)^2\frac{1}{2}(*)^{-1/2}(-2)\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)$$

$$= \frac{\frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)\left[2\left(1-\left(1-\frac{\theta}{2\pi}\right)^2\right) - \left(1-\frac{\theta}{2\pi}\right)^2\right]}{\sqrt{*}} = 0$$

Q6

$2\pi R$

$$V = \frac{1}{3}\pi r^2 h$$

$$R^2 = h^2 + r^2$$

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$$h = \sqrt{R^2 - r^2}$$

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$$V = \frac{1}{3}\pi R^2\left(1-\frac{\theta}{2\pi}\right)^2\sqrt{R^2 - R^2\left(1-\frac{\theta}{2\pi}\right)^2}$$

$$V = \frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)^2\sqrt{\underbrace{1-\left(1-\frac{\theta}{2\pi}\right)^2}_{*}}$$

$$\frac{dV}{d\theta} = \frac{1}{3}\pi R^3 \cdot 2\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)\sqrt{*} + \frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)^2 \frac{1}{2}(*)^{-1/2}(-2)\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)$$

$$= \frac{\frac{1}{3}\pi R^3\left(1-\frac{\theta}{2\pi}\right)\left(-\frac{1}{2\pi}\right)\left[2\left(1-\left(1-\frac{\theta}{2\pi}\right)^2\right) - \left(1-\frac{\theta}{2\pi}\right)^2\right]}{\sqrt{*}} = 0$$

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8

$$2\left(1-\left(1-\frac{\theta}{2\pi}\right)^2\right)-\left(1-\frac{\theta}{2\pi}\right)^2=0.$$

$$2-3\left(1-\frac{\theta}{2\pi}\right)^2=0.$$

$$\left(1-\frac{\theta}{2\pi}\right)^2=\frac{2}{3}.$$

$$1-\frac{\theta}{2\pi}=\sqrt{2/3}.$$

$$\theta=2\pi\left(1-\sqrt{2/3}\right).$$

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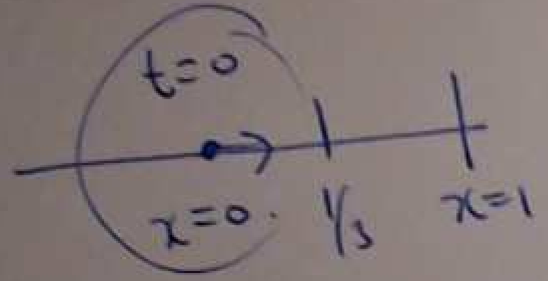
Q10



$$v(t) = (t+1)^{-4}$$
$$x'(t) = (t+1)^{-4}$$

$$x(t) = \int (t+1)^{-4}\,dt \qquad u = t+1 \qquad \frac{du}{dt} = 1$$

know

$x(0) = 0$

$$\int u^{-4} \frac{dt}{du}\,du$$

$$\int u^{-4}\,du$$

$$x(t) = -\frac{1}{3}u^{-3} + c$$

$$x(t) = -\frac{1}{3}(t+1)^{-3} + c$$

$$x(0) = -\frac{1}{3} + c = 0$$
$$c = \frac{1}{3}$$

$$x(t) = \frac{1}{3} - \frac{1}{3}(t+1)^{-3}$$

as $t \to \infty$

$x(t) \to \frac{1}{3}$

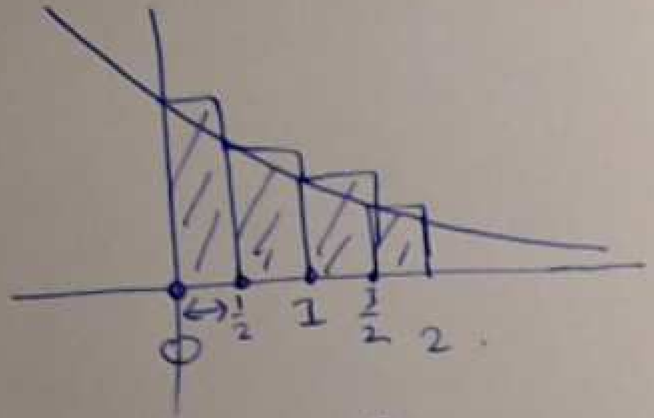
never get to 1

(10)

Q8 $y = e^{-2x}$

overestimate.

decreasing



area of rectangles: $\frac{1}{2}\left(f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right)\right)$

$\frac{1}{2}\left(1 + e^{-1} + e^{-2} + e^{-3}\right) \approx 0.77\ldots$

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(11)

Q9 c) $\int_1^8 \frac{2}{x}\,dx$ d) $\int_0^x \frac{1}{t+3}\,dt$

c) $2\int_1^8 \frac{1}{x}\,dx = 2\left[\ln|x|\right]_1^8 = 2\ln 8 - \underbrace{2\ln 1}_{0}$

$= 2\ln 8$

d) $\int_{t=0}^{t=x} \frac{1}{t+3}\,dt$ $u = t+3$, $\frac{du}{dt} = 1$

$\int_3^{x+3} \frac{1}{u}\frac{dt}{du}\,du = \int_3^{x+3}\frac{1}{u}\,du = \left[\ln|u|\right]_3^{3+x}$

$= \ln(3+x) - \ln(3)$