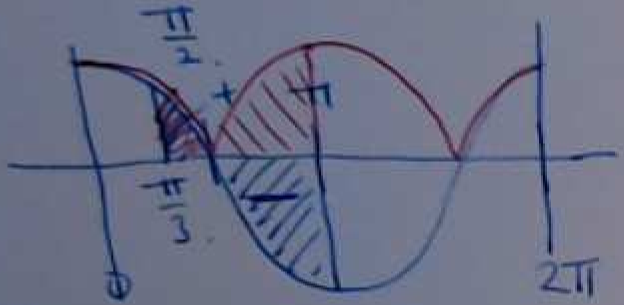
WW 5.4 Q6

$$\int_{\pi/3}^{\pi} |\cos(x)|\,dx$$

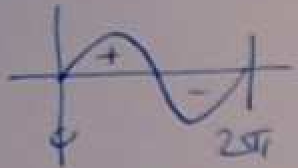


$$\boxed{\int_{\frac{\pi}{3}}^{\pi} \cos(x)\,dx}$$

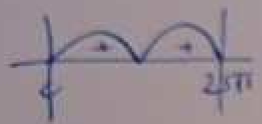
$$\boxed{\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos(x)\,dx + \int_{\frac{\pi}{2}}^{\pi} -\cos(x)\,dx.}$$

$$\int |f(x)|\,dx.$$

↑ manually check where $f(x)$ is positive and negative.



$$\int_0^{2\pi} \sin(x)\,dx = 0$$



$$\int_0^{2\pi} |\sin(x)|\,dx \neq 0.$$

$$= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx.$$

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$$\int_{\frac{\pi}{3}}^{\pi/2} \cos(x)\,dx + \int_{\frac{\pi}{2}}^{\pi} -\cos(x)\,dx.$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$

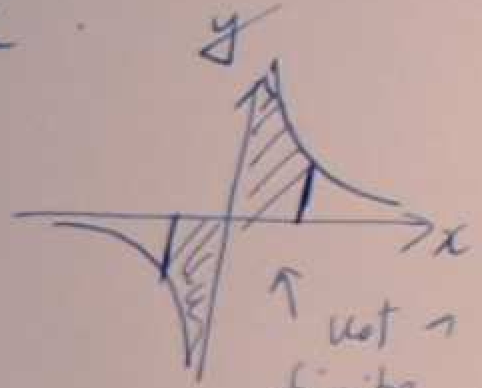
$$\Big[\sin(x)\Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \qquad \Big[-\sin(x)\Big]_{\pi/2}^{\pi}$$

$$\sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{3}\right) \qquad -\sin(\pi) + \sin\left(\frac{\pi}{2}\right)$$

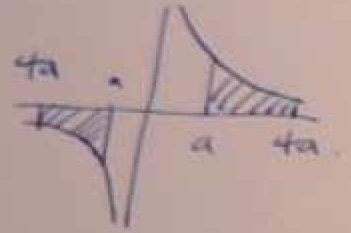
$$1 - \frac{\sqrt{3}}{2} \qquad 0 + 1 \qquad = 2 - \frac{\sqrt{3}}{2}$$

WW 5.4 Q7

$$\int_a^{4a} \frac{dx}{x} = \int_a^{4a} \frac{1}{x}\, dx.$$



not a finite integral.



Fact $\int \frac{1}{x} dx = \ln|x| + c.$

$$\Big[\ln|x| \Big]_a^{4a}$$

$$= \ln|4a| - \ln|a|$$

$$= \ln|4| + \cancel{\ln|a|} - \cancel{\ln|a|}$$

$$= \ln|4|$$

WW 5.5 Q7 $\quad \dfrac{d}{dt}\displaystyle\int_{10}^{t} \sec(19x+60)\,dx$.

FTC 2 $\quad \dfrac{d}{dt}\displaystyle\int_{a}^{t} f(x)\,dx = f(t)$.

$\sec(19t+60)$.

WW 5.5 Q10

$$\frac{d}{dx}\int_0^{\frac{71}{x}} \cos^3 t \, dt \qquad u = \frac{71}{x} = 71x^{-1}$$

FTC 1

$$\frac{d}{dx}\int_0^{x} f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_0^{u(x)} f(t)\,dt = f(u(x))\frac{du}{dx}$$

$$= \frac{d}{du}\int_0^{u(x)} f(t)\,dt \ \frac{du}{dx}$$

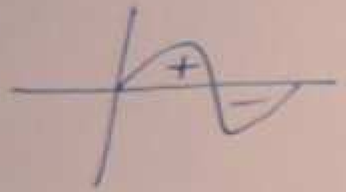
$$f(u(x)) \cdot \frac{du}{dx}$$

$$\cos^3\left(\frac{71}{x}\right) \cdot -71x^{-2}$$

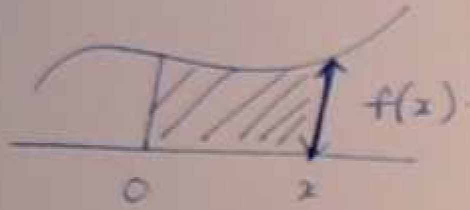
FTC Fundamental Theorem of Calculus I, II

FTC ① suppose $F(x)$ is an antiderivative of $f(x)$ ⑥

then $\int_a^b f(x)\,dx = F(b) - F(a)$



FTC ② $\frac{d}{dx}\int_0^x f(t)\,dt = f(x)$



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WW 5.7 Q 11

$$\int_{x=0}^{x=\pi/2} \cos^2 x \sin x \, dx$$

try

sub $u = \cos x$

$\frac{du}{dx} = -\sin x$

$\frac{dx}{du} = 1/\frac{du}{dx}$

$$\int_{u=1}^{u=0} u^2 \cdot \sin x \cdot \frac{dx}{du} \, du$$

$$\int_1^0 u^2 \cdot \cancel{\sin x} \, \frac{-1}{\cancel{\sin x}} \, du$$

$$\int_1^0 -u^2 \, du$$

$$\left[-\frac{u^3}{3} \right]_1^0 = 0 + \frac{1}{3} = \frac{1}{3}$$

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a) $\int x \cos(x^2)\, dx$

b) $\int x\sqrt{x^2-1}\, dx$

c) $\int \frac{1}{x \ln x}\, dx$

d) $\int \frac{e^{2x}+e^{4x}}{e^{x}}\, dx$

e) $\int 2^{x}\, dx$

f) $\int x e^{x^2}\, dx$

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a) $\int x\cos(x^2)\,dx$

$u = x^2 \qquad \frac{du}{dx} = 2x$

$\int x.\cos(u)\frac{dx}{du}\,du$

$\int \cancel{x}.\cos(u)\frac{1}{2\cancel{x}}\,du$

$\int \frac{1}{2}\cos(u)\,du$

consider

$\frac{d}{dx}\left(\cos(x^2)\right)$

$-\sin(x^2).2x$

$\frac{1}{2}\sin(u) + c$

$\frac{1}{2}\sin(x^2) + c$

check differentiate!

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(10)

$$\boxed{\int \frac{e^{2x}+e^{4x}}{e^x}\,dx}$$

<u>try</u> $u = e^{x}$

$$\frac{du}{dx} = e^x$$

$$\int \frac{e^{2x}+e^{4x}}{e^x}\frac{dx}{du}\,du$$

$$\int e^x + e^{3x}\,dx$$

$$\int \frac{e^{2x}+e^{4x}}{e^x}\cdot\frac{1}{e^x}\,du$$

$$e^x + \int e^{3x}\,dx$$

$u = 3x$

$$\int \frac{u^2+u^4}{u^2}\,du$$

$$\int 1+u^2\,du \qquad u+\tfrac{1}{3}u^3+c \qquad e^x+\tfrac{1}{3}e^{3x}+c$$

$$\int x\sqrt{x^2-1}\,dx \qquad x = \ln(u)$$

$$\frac{dx}{du} = \frac{1}{u}$$

$$\int \ln(u)\sqrt{\ln(u)^2-1}\,\frac{dx}{du}\,du$$

$$\int \ln(u)\sqrt{\ln(u)^2-1}\,\frac{1}{u}\,du \quad \leftarrow \text{this is harder}$$

$$\int e^{-x^2}\,dx \quad \leftarrow \text{no substitution makes this easier}$$

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b) $\int x\sqrt{x^2-1}\,dx$.

$u = x^2 - 1$

$\frac{du}{dx} = 2x$.

$u = x^2$

$\frac{du}{dx} = 2x$

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$\int x \sqrt{u} \frac{dx}{du} du$

$\int \cancel{x} \sqrt{u} \frac{1}{2\cancel{x}} du$

$\int \frac{1}{2} u^{1/2} du$.

$\frac{1}{2}\frac{2}{3} u^{3/2} + c$ $\qquad \frac{1}{3} u^{3/2} + c$.

$\frac{1}{3} (x^2-1)^{3/2} + c$.

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f) $\int x e^{(x^2)} dx$

$(e^x)^2 = e^{2x}$

$u = x^2$

$\frac{du}{dx} = 2x$

$\int x e^u \frac{dx}{du} du$

$\int \cancel{x} \cdot e^u \frac{1}{2\cancel{x}} du$

$\frac{1}{2} \int e^u du = \frac{1}{2} e^u + c = \frac{1}{2} e^{x^2} + c$

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e) $\int 2^x \, dx$.

$$e^{\ln(2)} = 2.$$

$$u = \ln(2)x$$

$$\int \left(e^{\ln(2)}\right)^x dx = \int e^{\ln(2)x} \, dx. \qquad \frac{du}{dx} = \ln(2).$$

$$\int e^u \frac{dx}{du} \, du = \int e^u \frac{1}{\ln(2)} \, du$$

$$= \frac{1}{\ln(2)} e^u + c = \frac{1}{\ln(2)} e^{\ln(2)x} + c$$

$$= \frac{1}{\ln(2)} 2^x + c.$$

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e) $\int 2^x \, dx$.

$e^{\ln(2)} = 2$.

$u = \ln(2)x$

$\frac{du}{dx} = \ln(2)$.

$\int \left(e^{\ln(2)}\right)^x dx = \int e^{\ln(2)x} \, dx$.

$\int e^u \frac{dx}{du} du = \int e^u \frac{1}{\ln(2)} du$

$= \frac{1}{\ln(2)} e^u + c = \frac{1}{\ln(2)} e^{\ln(2)x} + c$

Fact

$\int a^x dx = \frac{1}{\ln(a)} a^x + c$.

$= \frac{1}{\ln(2)} 2^x + c$.

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c) $\displaystyle\int \frac{1}{x \ln x}\, dx$ $\qquad u = \ln x$. (15)

$$\frac{du}{dx} = \frac{1}{x}.$$

$$\int \frac{1}{x \cdot u} \frac{dx}{du}\, du$$

$$\int \frac{1}{\cancel{x} u} \cancel{x}\, du = \int \frac{1}{u}\, du = \ln|u| + c$$

$$= \ln|\ln(x)| + c$$

check: $\displaystyle\frac{1}{\ln(x)} (\ln(x))' = \frac{1}{\ln(x)} \cdot \frac{1}{x}$.

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$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\tan^{-1}(\sqrt{x})\right) = \frac{1}{1+(\sqrt{x})^2} \cdot (\sqrt{x})'$$

$x^{1/2}$

↑ chain rule.

$$\int \frac{\sqrt{x}}{x+x^2}\,dx$$

$$= \frac{1}{1+x} \cdot \frac{1}{2} x^{-1/2}.$$

$$\parallel$$

$$\frac{1}{2}\tan^{-1}(\sqrt{x}) + c$$

$$= \frac{1}{2\sqrt{x} + 2x^{3/2}} \quad \frac{\times\sqrt{x}}{\times\sqrt{x}}.$$

$$= \frac{\sqrt{x}}{2x+2x^2}$$

16

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\tan^{-1}(\sqrt{x})\right) = \frac{1}{1+(\sqrt{x})^2} \cdot (\sqrt{x})'$$

$x^{1/2}$

chain rule.

$$\int \frac{\sqrt{x}}{x+x^2}\,dx$$

$$= \frac{1}{1+x} \cdot \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{2\sqrt{x} + 2x^{3/2}} \quad \frac{\times\sqrt{x}}{\times\sqrt{x}}$$

$$= \frac{\sqrt{x}}{2x+2x^2}$$

$\shortparallel$

$\frac{1}{2}\tan^{-1}(\sqrt{x}) + c$

sub

$u = \sqrt{x}$.