HW 5.2 Q2

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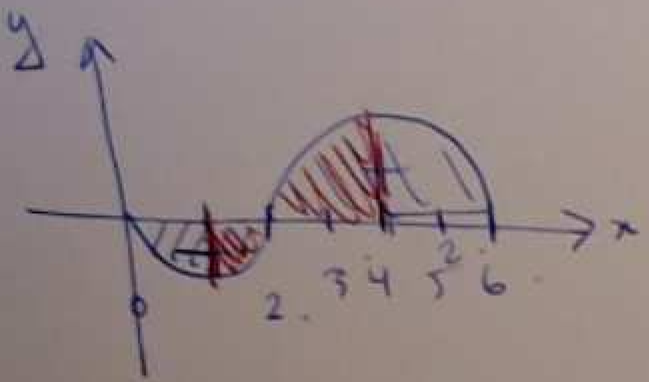
a) $\int_0^2 4f(x)\,dx$

b) $\int_0^6 5f(x)\,dx$

c) $\int_1^4 4f(x)\,dx$

d) $\underbrace{\int_1^6 |2f(x)|\,dx}$

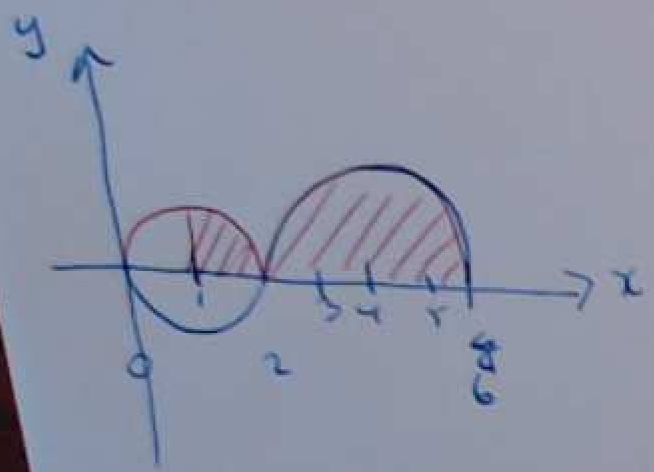


a) $\int_0^2 4f(x)\,dx = 4\underbrace{\int_0^2 f(x)\,dx}_{-\frac{\pi 1^2}{2}} = 4\cdot -\frac{\pi}{2} = -2\pi$

b) $\int_0^6 5f(x)\,dx = 5\int_0^6 f(x)\,dx = 5\left(-\frac{\pi}{2} + \frac{\pi 2^2}{2}\right) = 5\left(-\frac{\pi}{2} + 2\pi\right)$

$= 35\pi/8$

c) $4\int_1^4 f(x)\,dx = 4\left(-\frac{\pi 1^2}{4} + \frac{\pi 2^2}{4}\right) = 3\pi$



d) $\int_1^6 |2f(x)|\,dx$.

$= 2\left(\frac{\pi}{4} + \frac{\pi \cdot 2^2}{2}\right)$

$= \frac{\pi}{2} + \pi 4 = \frac{9\pi}{2}$.

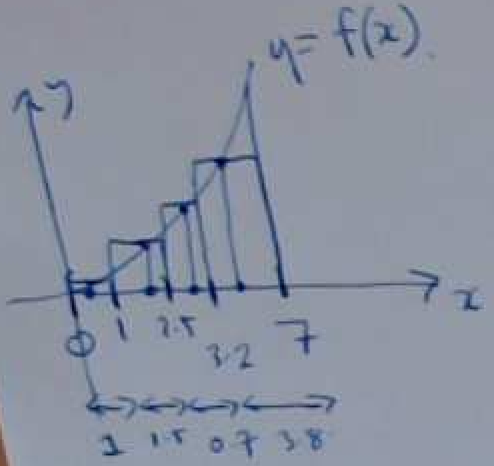
WW 5.2 Q4

$R(f, P, c)$

$f(x) = x^2+3x$

$P = \{0, 1, 2.5, 3.2, 7\}$

$C = \{0.5, 2, 3, 4.5\}$



widths	1	1.5	0.7	3.8
heights	$f(0.5)$	$f(2)$	$f(3)$	$f(4.5)$
	$\frac{1}{4}+\frac{3}{2}$	10	18	

area $\frac{7}{4} + 15 + 18\times 0.7 + 3.8\times f(4.5)$

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WW 5.3 Q6

$$\int x^{-15/6}\,dx$$

check

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\int 3x^2\,dx = \frac{\cancel{3}x^3}{\cancel{3}} = x^3 + c$$

recall $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\frac{d}{dx}(x^\pi) = \pi x^{\pi-1}$$

$$\int x^{-1}\,dx = \ln|x| + c$$

$x^{-1} = \frac{1}{x}$

$$-\frac{15}{6} + \frac{6}{6}$$

$$\int x^{-\frac{15}{6}}\,dx = \frac{x^{-\frac{15}{6}+1}}{-\frac{15}{6}+1} + c = \frac{x^{-3/2}}{-3/2} + c = -\frac{2}{3}x^{-3/2} + c$$

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WW 5.3 Q12

$$\frac{dy}{dx} = -9x^3 \, , \quad y(0) = 3.$$

$$y = \int -9x^3 \, dx$$

recall $\int x^n \, dx = \frac{x^{n+1}}{n+1}$.

$$y(x) = -\frac{9x^4}{4} + C.$$

$$y(0) = C = 3$$

$$y(x) = -\frac{9x^4}{4} + 3$$

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WW 5.3 Q13

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$$\frac{dy}{dt} = -4\sqrt{t} \qquad y(1) = -5$$

$$= -4t^{1/2}$$

recall

$$\int t^n\,dt = \frac{t^{n+1}}{n+1} + c$$

$$y = \frac{-4t^{3/2}}{3/2} + c = -\frac{8t^{3/2}}{3} + c.$$

$y(1) = -5$:

$$-\frac{8}{3} + c = -5.$$

$$c = -5 + \frac{8}{3} = -\frac{7}{3}.$$

$$y(t) = -\frac{8t^{3/2}}{3} - \frac{7}{3}.$$

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WW 5.3 Q14

(7)

$$f''(x) = \cos(x)$$

$$\boxed{\begin{aligned} f'\left(\tfrac{\pi}{2}\right) &= 13 \\ f\left(\tfrac{\pi}{2}\right) &= 10 \end{aligned}}$$

$$f'(x) = \sin(x) + cx^0$$

$$f(x) = -\cos(x) + cx + d$$

$$f'\left(\tfrac{\pi}{2}\right) = \underbrace{\sin\left(\tfrac{\pi}{2}\right)}_{1} + c = 13$$

$$c = 12$$

$$f'(x) = \sin(x) + 12$$

$$f(x) = -\cos(x) + 12x + d$$

$$f(x) = -\cos(x) + 12x + 10 - 6\pi$$

recall

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\int \cos(x)\,dx = \sin(x) + c$$

$$\frac{d}{dx}(\cos(x)) = -\sin x$$

$$\int \sin x\,dx = -\cos x + c$$

$$f\left(\tfrac{\pi}{2}\right) = \overbrace{-\cos\left(\tfrac{\pi}{2}\right)}^{0} + 12 \cdot \tfrac{\pi}{2} + d = 10$$

$$6\pi + d = 10 \quad d = 10 - 6\pi$$

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<u>recall</u> integration by substitution / change of variable.

$$\int_{x=a}^{x=b} f(x)\,dx, \quad x(u) \qquad \int_{u=x^{-1}(a)}^{u=x^{-1}(b)} f(x(u))\,\frac{dx}{du}\,du$$

"reverse chain rule".

<u>recall</u>

$$\frac{d}{dx}(\tan x) = \sec^2 x.$$

<u>Example</u> $\displaystyle\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx.$

<u>fact</u>

$$\frac{dx}{du} = 1 \Big/ \frac{du}{dx}$$

try $u = \tan x$

$$\frac{du}{dx} = \sec^2 x.$$

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$$u = \tan(x)$$

$$\frac{du}{dx} = \sec^2(x)$$

$$\int_{x=0}^{x=\pi/4} \tan^3 x \sec^2 x \, dx$$

$$\int_{u=0}^{u=1} u^3 \cdot \sec^2 x \frac{dx}{du} \, du$$

$$\int_0^1 u^3 \cdot \cancel{\sec^2 x} \cdot \frac{1}{\cancel{\sec^2 x}} \, du$$

$$\int_0^1 u^3 \, du = \left[\frac{u^4}{4}\right]_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$$

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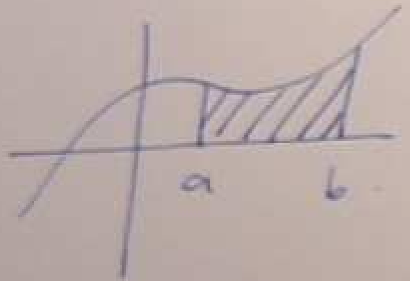
notation

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\Big[F(x) \Big]_a^b \equiv F(b) - F(a)$$

pattern

$$\frac{d}{dx}\Big(\big(f(x)\big)^n \Big) = n\big(f(x)\big)^{n-1} \cdot f'(x)$$

$$\int n\big(f(x)\big)^{n-1} \cdot f'(x)\,dx$$

try

$u = f(x)$

$$\int_{x=0}^{x=1} \frac{x}{x^2+1}\,dx$$

$$u = x^2+1$$

$$\frac{du}{dx} = 2x$$

$$\int_{u=1}^{u=2} \frac{x}{u}\,\frac{dx}{du}\,du$$

$$\int_1^2 \frac{\cancel{x}}{u}\,\frac{1}{2\cancel{x}}\,du$$

$$\frac{1}{2}\int_1^2 \frac{1}{u}\,du$$

$$= \frac{1}{2}\Big[\ln|u|\Big]_1^2$$

$$= \frac{1}{2}\ln(2) - \frac{1}{2}\underbrace{\ln(1)}_{0}$$

$$= \frac{1}{2}\ln(2)$$

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pattern

$$\frac{d}{dx}\left(\ln(f(x))\right) = \frac{1}{f(x)} \cdot f'(x)$$

$$= \frac{f'(x)}{f(x)}$$

$$\int \frac{f'(x)}{f(x)}\,dx$$

try $u = f(x)$

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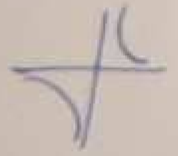
§ 5.8 More integrals



(check)

<u>recall</u> $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$. $\int \frac{1}{x}\,dx = \ln|x| + C$.

$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$ $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$.

$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$.

$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2-1}}$ $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C$.

$\frac{d}{dx}\left(e^x\right) = e^x$. $\int e^x\,dx = e^x + C$.

$$\frac{d}{dx}\left(e^x\right) = e^x . \qquad \int e^x \, dx = e^x + c .$$

$\int b^x \, dx$ <u>recall</u> $b^x = \left(e^{\ln(b)}\right)^x$

$= e^{x\ln(b)}$

$\int e^{x\ln(b)} \, dx$ <u>sub</u> $u = x\ln(b)$

$\frac{du}{dx} = \ln(b)$. $= \frac{1}{\ln(b)} b^x + c$

$\int e^u \frac{dx}{du} \, du$ $= \frac{1}{\ln(b)} e^{x\ln(b)} + c$

$$\int e^u \frac{1}{\ln(b)} \, du = \frac{1}{\ln(b)} \int e^u \, du = \frac{1}{\ln(b)} e^u + c$$

Examples

$$\int \frac{1}{9+x^2}\,dx.$$

$$\frac{1}{9}\int \frac{1}{1+x^2/9}\,dx.$$

$$\frac{1}{9}\int \frac{1}{1+(x/3)^2}\,dx.$$

try

$$u = \frac{x}{3}.$$

$$\frac{du}{dx} = \frac{1}{3}.$$

$$\frac{1}{9}\int \frac{1}{1+u^2}\frac{dx}{du}\,du$$

$$\frac{1}{9}\int \frac{1}{1+u^2}\cdot 3\,du = \frac{1}{3}\int \frac{1}{1+u^2}\,du$$

$$= \frac{1}{3}\tan^{-1}(u) + C.$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C.$$

check: differentiate!

know

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}.$$

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C.$$

$$\cancel{\int \frac{1}{\sqrt{1+c}}} \qquad \int \frac{1}{\sqrt{1-4x^2}}\,dx.$$

recall
$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C.$$

$$\int \frac{1}{\sqrt{1-(2x)^2}}\,dx. \qquad u = 2x \qquad \frac{du}{dx} = 2.$$

$$\int \frac{1}{\sqrt{1-u^2}}\,\frac{dx}{du}\,du = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{2}\sin^{-1}(u) + c = \frac{1}{2}\sin^{-1}(2x) + c.$$

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Example

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$$\int x(x+1)^{100}\,dx.$$

$u = x+1 \longleftrightarrow u-1 = x.$

$\frac{du}{dx} = 1.$

$$\int (u-1)u^{100}\frac{dx}{du}\,du = \int (u-1)u^{100}\,du.$$

$$\int u^{101} - u^{100}\,du = \frac{u^{102}}{102} - \frac{u^{101}}{101} + c.$$

$$= \frac{(x+1)^{102}}{102} - \frac{(x+1)^{101}}{101} + c.$$

$\sin 2\theta = 2\sin\theta\cos\theta$ 20

$$\int \sin x \cos x \, dx$$

1

$u = \sin x$

$\frac{du}{dx} = \cos x$

$$\int u \cdot \cos x \frac{dx}{du} \, du$$

$$\int u \cdot \cos x \frac{1}{\cos x} \, du$$

$$\int u \, du = \frac{1}{2} u^2 + c$$

$$= \frac{1}{2} \sin^2 x + c$$

2

$$\int \frac{1}{2} \sin 2x \, dx$$

$u = 2x$

$\frac{du}{dx} = 2$

$$\int \frac{1}{2} \sin u \frac{dx}{du} \, du$$

$$\int \frac{1}{4} \sin u \, du$$

$$-\frac{1}{4} \cos u + c$$

$$-\frac{1}{4} \cos 2x + c$$

Q: how can this be?