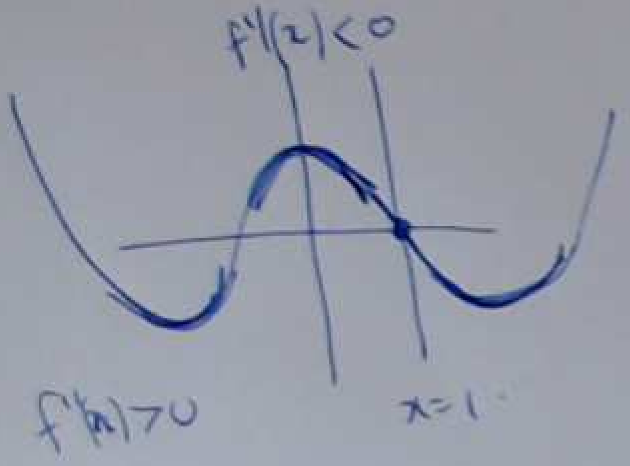
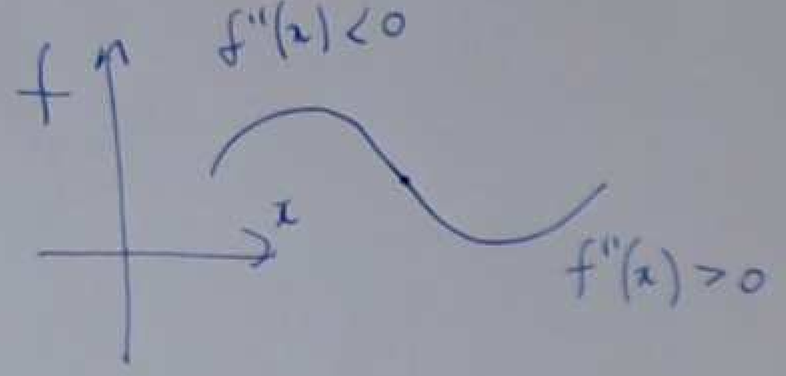
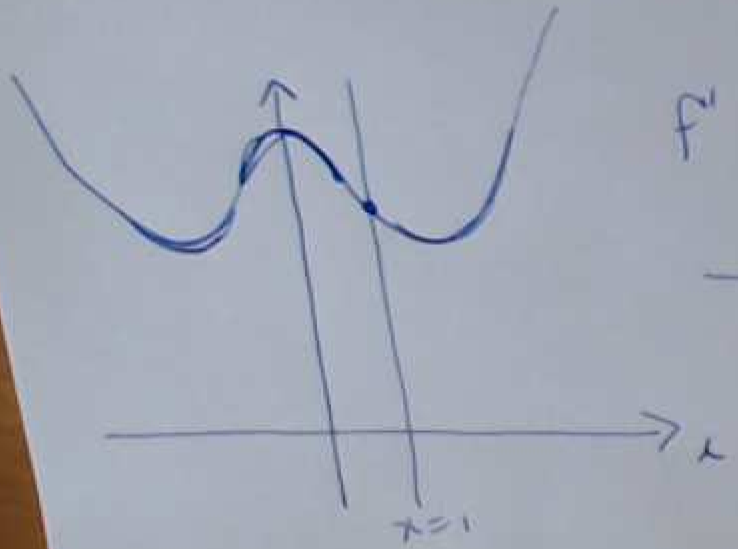
WW 4.6 Q1

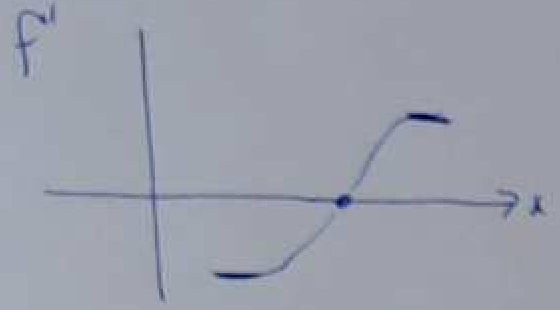
$f''(x) > 0 \leftrightarrow$ concave up

$f''(x) < 0 \leftrightarrow$ concave down

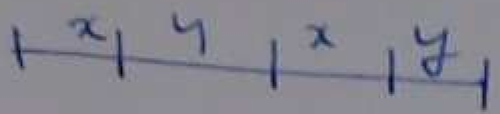




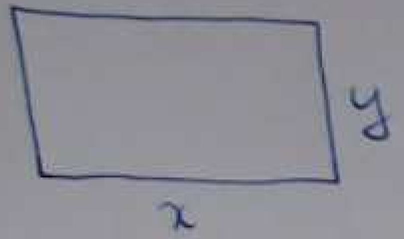




WW 4.7 Q1

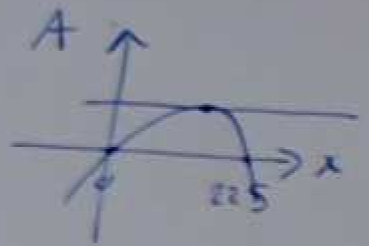


450 in



$$\left.\begin{aligned} 2x + 2y &= 450 \\ \text{area } A &= xy \end{aligned}\right\} \leftarrow \qquad \begin{aligned} 2y &= 450 - 2x \\ y &= 225 - x. \end{aligned}$$

$$A = x(225 - x) = 225x - x^2.$$



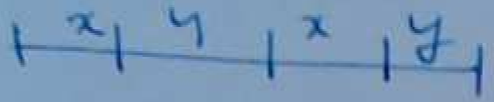
$$\frac{dA}{dx} = 225 - 2x = 0.$$

$x = 112.5$

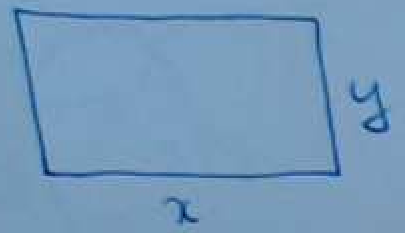
$y = 112.5$.

$A = (112.5)^2$

WW 4.7 Q1 ②



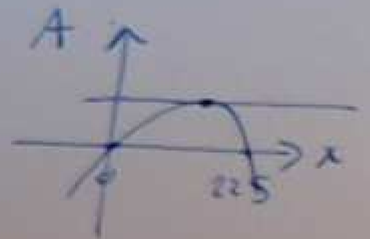
450 m



$$\left.\begin{aligned} 2x + 2y &= 450 \\ \text{area } A &= xy \end{aligned}\right\} \leftarrow$$

$2y = 450 - 2x$

$y = 225 - x$.



$A = x(225 - x) = 225x - x^2$.

$\frac{dA}{dx} = 225 - 2x = 0$.

$x = 112.5$

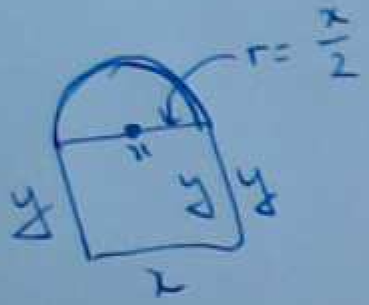
$y = 112.5$.

$A = (112.5)^2$

HW4.7 Q2

$$L = 491 \text{ ft} = x + 2y + \pi \frac{x}{2}$$

$$= \left(1+\frac{\pi}{2}\right)x + 2y = 491 \quad (*)$$



$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi x^2}{8} \quad (**)$$

$(*)$ $\left(1+\frac{\pi}{2}\right)x + 2y = 491$

$$2y = 491 - \left(1+\frac{\pi}{2}\right)x$$

$$y = \frac{491}{2} - \left(\frac{1}{2}+\frac{\pi}{4}\right)x$$

$(**)$ $A = x\left(\frac{491}{2} - \left(\frac{2+\pi}{4}\right)x\right) + \frac{\pi x^2}{8}$

```
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'citation()' on how to cite R or R packages in publications.

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'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[Previously saved workspace restored]

> 982 / (pi + 4)
[1] 137.5043
> 491 / 2 - (1/2 + pi/4) *  137.5043
[1] 68.75223
> 137.5043 *  68.75223 + 1/2 * pi * ( 137.5043/2 )^2
[1] 16878.66
> 
```

④

$$A = \frac{491}{2}x - \frac{2+\pi}{4}x^2 + \frac{\pi x^2}{8}$$

$$\frac{dA}{dx} = \frac{491}{2} - \frac{2+\pi}{2}x + \frac{\pi x}{4} = 0.$$

$$x\left(\frac{\pi}{4} - \frac{2+\pi}{2}\right) = -\frac{491}{2}$$

$$x = -\frac{491}{2} \cdot \frac{1}{\frac{\pi}{4} - \frac{2+\pi}{2}} \quad \Big\} \quad \frac{\pi - 4 - 2\pi}{4} = \frac{-\pi - 4}{4}$$

$$x = \frac{491 \cdot 4}{2(\pi+4)} = \frac{491 \times 2}{\pi + 4} = \frac{982}{\pi + 4}$$

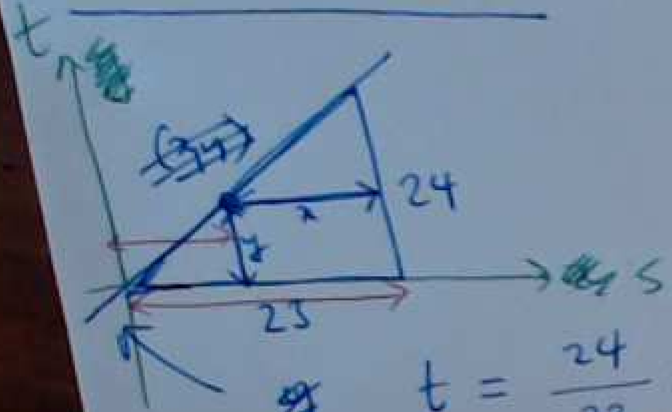
137.5043

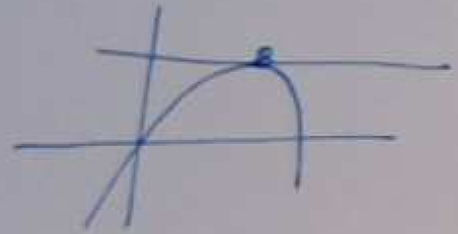
$$y = \frac{491}{2} - \left(\frac{1}{2} + \frac{\pi}{4}\right)x \quad 68.75223$$

$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 \quad 16873.66$$

WW 4.7 Q3

$$A = xy$$





$$(\overset{23}{\cancel{24}} - x,\ y)$$

$$\updownarrow \quad \updownarrow$$

$$(s,\ t)$$

$$t = \frac{24}{23} s$$

$$y = \frac{24}{23}(\overset{23}{\cancel{24}} - x)$$

$$A = x\,\frac{24}{23}(23 - x) = 24x - \frac{24}{23}x^2$$

$$\frac{dA}{dx} = 24 - \frac{48}{23}x = 0$$

$$\frac{48}{23}x = 24$$

$$x = \frac{24}{48}\cdot 23 = \frac{23}{2} = 11\tfrac{1}{2}$$

$$y = \frac{24}{23}\left(23 - \frac{23}{2}\right)$$

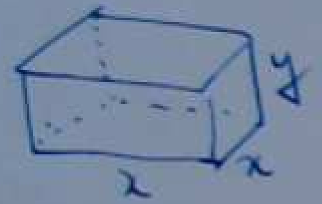
$$= \frac{24}{23}\,\frac{23}{2} = 12$$

$$A = 12 \times 11\tfrac{1}{2}$$

WW 4.7 Q5

height + girth = 118

〃 perimeter of base.

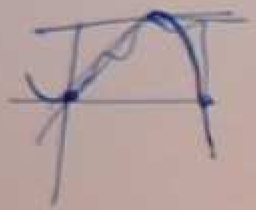


$$y + 4x = 118 \quad (*)$$

volume $V = x\,x\,y = x^2 y \quad (**)$

$(*)$ $y = 118 - 4x$

$(**)$ $V = x^2(118-4x) = 118x^2 - 4x^3$



find critical pts: solve $\frac{dV}{dx} = 0$

$$\frac{dV}{dx} = 236x - 12x^2 = 0$$

$$4x(59-3x) = 0$$

$x = \frac{59}{3}$

$y = 118 - \frac{4\times 59}{3}$

$118 \times \frac{1}{3}$

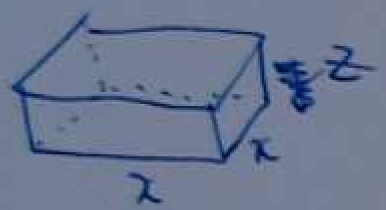
$V = \left(\frac{59}{3}\right)^2 118 \times \frac{1}{3}$

HW 4.7 Q7

top | bottom \$4/ ft^2

sides \$6/ ft

$V = 30 \text{ ft}^3$



$V = x^2 z$ *

$x^2 z = 30$

$z = \frac{30}{x^2}$

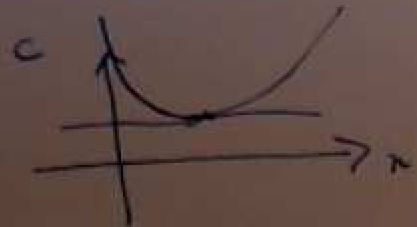
$C = 2x^2 \times 4 + 4xz \times 6$

$= 8x^2 + 24xz$

**

$C = 8x^2 + 24x \frac{30}{x^2}$

$C = 8x^2 + \frac{24 \times 30}{x}$



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8

$$C = 8x^2 + \frac{720}{x}$$

find critical pts
solve $\frac{dC}{dx} = 0$

$$\frac{dC}{dx} = 16x - \frac{720}{x^2} = 0$$

$\frac{1}{x} = x^{-1}$

$\frac{d}{dx}(x^{-1})$

$-x^{-2}$

$$16x = \frac{720}{x^2}$$

$$x^3 = \frac{720}{16} = \frac{360}{8} = 45$$

$x = \sqrt[3]{45}$

$z = \frac{30}{x^2} = \frac{30}{(45)^{2/3}}$

$$\cancel{V = x^2 z = (45)^{2/3} \frac{30}{(45)^{2/3}} = 30}$$

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9

$$x = (45)^{1/3} \qquad \cancel{y}\, z = \frac{30}{(45)^{2/3}}.$$

$$C = 8x^2 + 24xz.$$

$$= 8(45)^{2/3} + 24(45)^{1/3} \cdot \frac{30}{(45)^{2/3}}.$$

10

4.6 Q4

$$y = xe^{-71x^2}$$

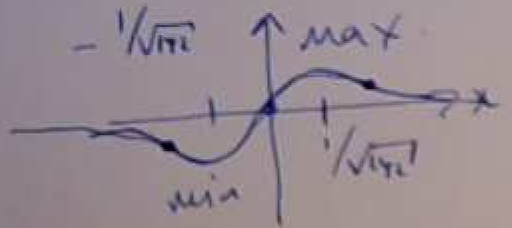
$$(ab)' = a'b + ab'$$

$$(a(b(x)))' = a'(b(x)) \cdot b'(x)$$

$$\frac{dy}{dx} = e^{-71x^2} + x \cdot e^{-71x^2} \cdot (-142x)$$

$$= e^{-71x^2}(1 - 142x^2)$$

$$= e^{-71x^2} - 142x^2 e^{-71x^2}$$



$$\frac{dy}{dx} = 0$$

$$142x^2 = 1$$

$$x^2 = \frac{1}{142}$$

$$x = \frac{\pm 1}{\sqrt{142}}$$

$$\frac{d^2y}{dx^2} = e^{-71x^2} \cdot -142x$$

$$+ -284x\, e^{-71x^2} + -142x^2 \cdot e^{-71x^2} \cdot -71 \cdot 2x$$

$$= e^{-71x^2}\left[-142x - 284x + 142 \times 71 \times 2x^3\right]$$

$$= 142x\, e^{-71x^2}\left[-1 - 2 + 142x^2\right]$$

$$142x^2 - 3$$

$$\frac{d^2y}{dx^2} = 0$$

$$x^2 = \pm\sqrt{\frac{3}{142}}$$

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4.6 Q4

$$y = xe^{-71x^2}$$

$$(ab)' = a'b + ab'$$

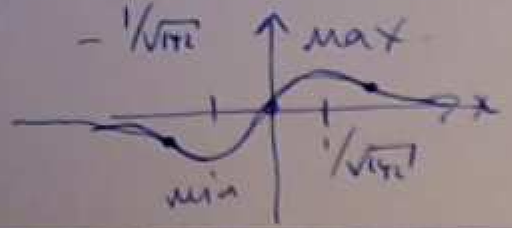
$$(a(b(x)))' = a'(b(x))\,b'(x)$$

(10)

$$\frac{dy}{dx} = e^{-71x^2} + x\cdot e^{-71x^2}\cdot(-142x)$$

$$= e^{-71x^2}(1 - 142x^2)$$

$$= e^{-71x^2} - 142x^2 e^{-71x^2}$$



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$$= 142x\,e^{-71x^2}\left[-1 - 2 + 142x^2\right]$$

$$142x^2 - 3$$

$$\frac{d^2y}{dx^2} = 0$$

$$x^2 = \pm\sqrt{3/142}$$

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4.6 Q4

10

$$y = xe^{-71x^2}$$

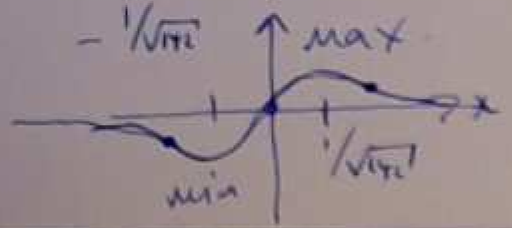
$$(ab)' = a'b + ab'$$

$$(a(b(x)))' = a'(b(x)) \cdot b'(x)$$

$$\frac{dy}{dx} = e^{-71x^2} + x \cdot e^{-71x^2} \cdot (-142x)$$

$$= e^{-71x^2}(1 - 142x^2)$$

$$= e^{-71x^2} - 142x^2 e^{-71x^2}$$



$$\frac{dy}{dx} = 0$$

$$142x^2 = 1$$

$$x^2 = \frac{1}{142}$$

$$x = \frac{\pm 1}{\sqrt{142}}$$

$$\frac{d^2y}{dx^2} = e^{-71x^2} \cdot -142x + -284x\, e^{-71x^2} + -142x^2 e^{-71x^2} \cdot -71 \cdot 2x$$

$$= e^{-71x^2}\left[-142x - 284x + 142 \times 71 \times 2x^3\right]$$

$$= 142x\, e^{-71x^2}\left[-1 - 2 + 142x^2\right]$$

$$142x^2 - 3$$

$$\frac{d^2y}{dx^2} = 0$$

$$x^2 = \pm\sqrt{3/142}$$

4.6 Q4

$$y = xe^{-71x^2}$$

$$(ab)' = a'b + ab'$$

$$(a(b(x)))' = a'(b(x)) \cdot b'(x)$$

$$\frac{dy}{dx} = e^{-71x^2} + x \cdot e^{-71x^2} \cdot (-142x)$$

$$= e^{-71x^2}(1 - 142x^2)$$

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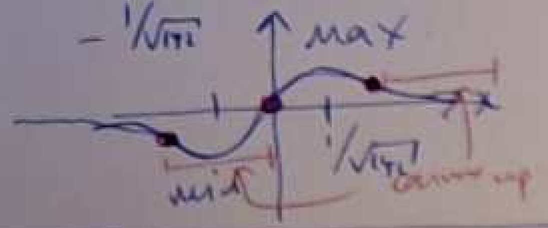

$$\frac{dy}{dx} = 0$$

$$142x^2 = 1$$

$$x^2 = \frac{1}{142}$$

$$x = \frac{\pm 1}{\sqrt{142}}$$

$$\frac{d^2y}{dx^2} = e^{-71x^2} \cdot -142x$$

$$+ -284x\, e^{-71x^2} + -142x^2 e^{-71x^2} \cdot -71 \cdot 2x$$

$$= e^{-71x^2}\left[-142x - 284x + 142 \times 71 \times 2x^3\right]$$

$$= 142x\, e^{-71x^2}\left[-1 - 2 + 142x^2\right]$$

$$142x^2 - 3$$

$$\frac{d^2y}{dx^2} = 0$$

$$x^2 = \pm\sqrt{3/142}$$

$$x = 0$$

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(11)

last time antiderivatives.

$f(x) = x + 1$ antiderivative $F(x)$

s.t. $F'(x) = f(x)$.

$F(x) = \frac{1}{2}x^2 + x + 7$

General antiderivative $F(x) = \frac{1}{2}x^2 + x + c$

Examples $f(x) = \sin(4x)$

guess $\frac{d}{dx}\left(-\frac{1}{4}\cos(4x)\right) = -\frac{1}{4} \cdot -\sin(4x) \cdot 4$

$F(x) = -\frac{1}{4}\cos(4x) + c$

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Notation indefinite integral

$$\int f(x)\,dx = F(x) + c$$

means $F(x)+c$ is the general antiderivative for $f(x)$.

Thm $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c \quad (n \neq -1)$.

Proof $\displaystyle\frac{d}{dx}\left(\frac{1}{n+1}x^{n+1} + c\right) = \frac{1}{n+1}(n+1)\,x^n = x^n$ $\square$

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13

<u>Thm</u> $\int \frac{1}{x}\,dx = \ln|x| + c$

<u>Proof</u> $(x > 0)$ $\frac{d}{dx}\left(\ln|x| + c\right) = \frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$ $\square$.

<u>Thm</u> (sums and constant multiples).

$$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int c f(x)\,dx = c\int f(x)\,dx$$

<u>warning</u>: no product / quotient / chain rule

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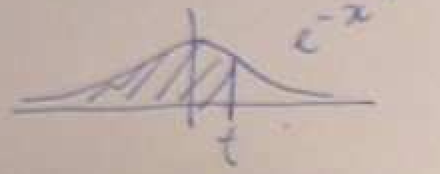
differentiation: we have complete rules to differentiation any elementary function

$\sin(x)$, x^n, $\ln(x)$, e^x ...

integration: some expressions do not have integrals which are elementary functions

examples: $\int_{-\infty}^{t} e^{-x^2}\,dx$



↑ no simple form for this integral

also $\int \frac{\sin x}{x}\,dx$

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15

useful integrals

$$\int \sin(x)\,dx = -\cos(x) + c$$

$$\int \cos(x)\,dx = \sin(x) + c$$

$$\int e^x\,dx = e^x + c$$

Examples

$$\int x^2 + \frac{1}{x} + \sin(x)\,dx$$

$$\int x^2\,dx + \int \frac{1}{x}\,dx + \int \sin(x)\,dx$$

$$\frac{1}{3}x^3 + \ln|x| + -\cos(x) + c$$

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15

useful integrals

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

$$\int e^x\,dx = e^x + C$$

Examples

$$\int x^2 + \frac{1}{x} + \sin(x)\,dx$$

$$\int x^2\,dx + \int \frac{1}{x}\,dx + \int \sin(x)\,dx$$

$$\frac{1}{3}x^3 + \ln|x| + -\cos(x) + C$$

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Alternative view

we can think of finding the indefinite integral as finding a function given its derivative / slope function.

This is an example of solving a differential equation

$\frac{dy}{dx} = f(x)$ In general there is a family of solutions $F(x)+c$

but if we know the value of a solution at $x=0$ (sometimes called an initial condition) then this gives (unique) particular solution.

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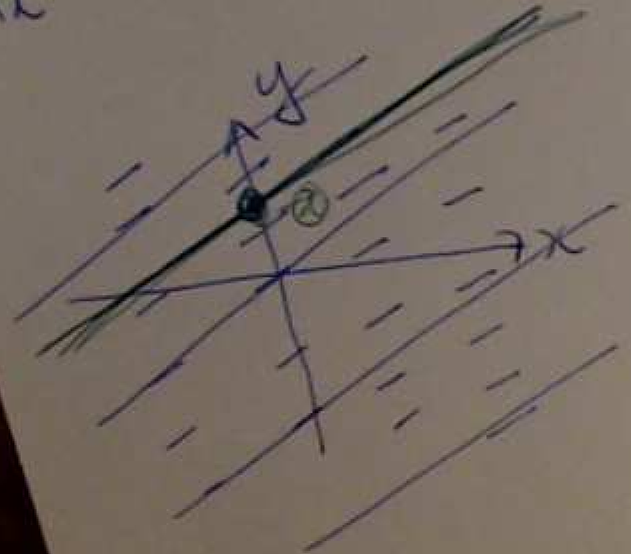
$\frac{dy}{dx} = 1$ general solution $F(x) = x + C$.

initial condition: $F(0) = 1$ (*)



then $\boxed{F(x) = x + 1}$

<u>Example</u> objects falling under gravity

acceleration $a(t) = -g$ constant $\qquad$ ($\cancel{a}$ $v'(t) = a(t)$).

$v(t) = \int a(t)\,dt = -gt + c$ general solution.

if speed at $t=0$ is v_0 $\quad v(0) = -g \cdot 0 + c \quad c = v_0$.

$v(t) = -gt + v_0$.

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(18)

$$x'(t) = v(t).$$

$$v(t) = -gt + v_0.$$

$$x(t) = \int v(t)\,dt = -\frac{1}{2}gt^2 + v_0 t + c.$$

if we know location at $t=0$ $\quad x(0) = x_0$

$$x(0) = x_0 = \underbrace{\cancel{-\tfrac{1}{2}g(0)^2}}_{0} + \underbrace{\cancel{v_0(0)}}_{0} + c \qquad c = x_0$$

$$x(t) = -\frac{1}{2}gt^2 + v_0 t + x_0.$$ ← equation for motion under gravity

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§5.4 Fundamental theorem of calculus

Thm (FTC ①) suppose $f(x)$ is continuous on $[a,b]$ and $F(x)$ is an antiderivative for $f(x)$.

Then $$\int_a^b f(x)\,dx = F(b) - F(a).$$