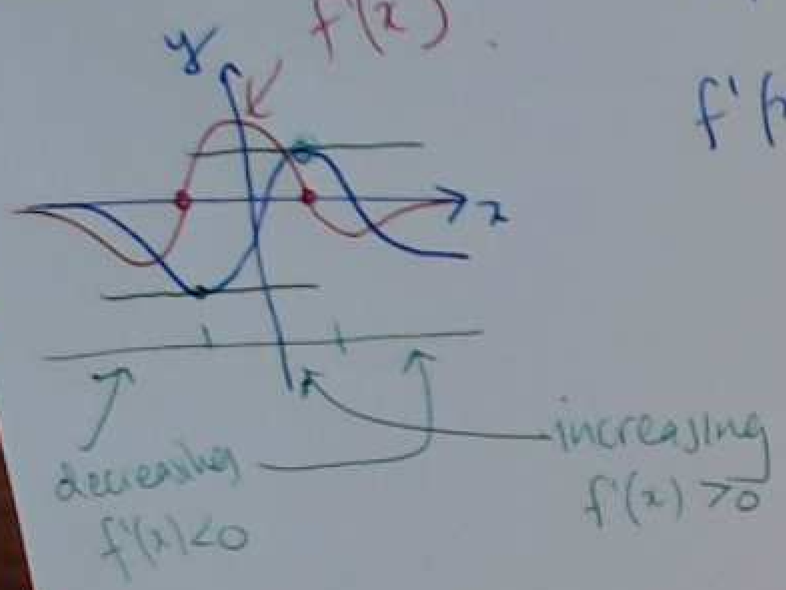


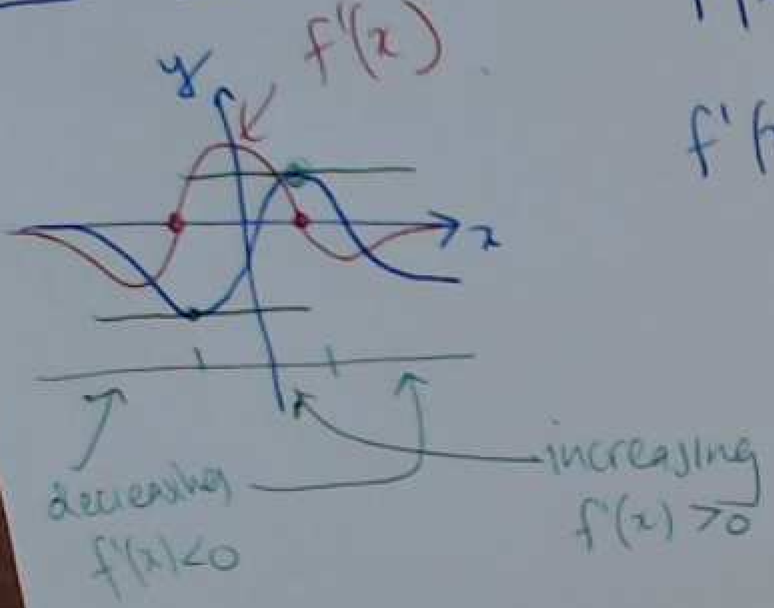
$f'(x) > 0 \leftrightarrow f$ increasing

$f'(x) < 0 \leftrightarrow f$ decreasing.

$$\lim_{x \to +\infty} f(x) = -2$$

$$\lim_{x \to -\infty} f(x) = 0.$$

SMT2 Q1



$f'(x) > 0 \iff f$ increasing

$f'(x) < 0 \iff f$ decreasing

$$\lim_{x \to +\infty} f(x) = -2$$

$$\lim_{x \to -\infty} f(x) = 0$$

Q4 a) $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$ ← product rule

at $x = 3$ $\quad h'(3) = \underbrace{f'(3)}_{-1}\underbrace{g(3)}_{1} + \underbrace{f(3)}_{1}\underbrace{g'(3)}_{-\frac{1}{2}} = -1 - \frac{1}{2} = -\frac{3}{2}$

b) $(f(g(x)))' = f'(g(x))\, g'(x)$

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$$(f(g(x)))' = f'(g(x))g'(x) \leftarrow \text{product rule}$$

$x=-1$: $h'(-1) = f'(\underbrace{g(-1)}_{2}) \cdot \underbrace{g'(-1)}_{1/2}$

$$\underbrace{f'(2)}_{-1} \cdot \frac{1}{2} = -\frac{1}{2}$$

Q7. Inflating spherical balloon, $\frac{dV}{dt} = 20 \text{ cm}^3/s$

$V = \frac{4}{3}\pi r^3$

$A = 4\pi r^2$

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$(f(g(x)))' = f'(g(x))\,g'(x)$. ← product rule.

$x=-1$: $h'(-1) = f'(\underbrace{g(-1)}_{2})\cdot\underbrace{g'(-1)}_{1/2}$.

$\underbrace{f'(2)}_{-1}\cdot\frac{1}{2} = -\frac{1}{2}$.

find $\frac{dA}{dt}$ when $r=20$

Q7. Inflating spherical balloon, $\frac{dV}{dt} = 20\ \text{cm}^3/\text{s}$.

$$\left.\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ A &= 4\pi r^2 \end{aligned}\right\}$$

$\sqrt{\frac{A}{4\pi}} = r$

$V = \frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{3/2}$.

$\underbrace{\frac{dV}{dt}}_{20} = \frac{4}{3}\pi\,\frac{3}{2}\left(\frac{A}{4\pi}\right)^{1/2}\frac{dA}{dt}\,\frac{1}{4\pi}$

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②

find $\frac{dA}{dt}$ when $r=20$

$$\frac{dV}{dt} = 20 \text{ cm}^3/\text{s}$$

$$V = \frac{4}{3}\pi \left(\frac{A}{4\pi}\right)^{3/2}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{3}{2} \left(\frac{A}{4\pi}\right)^{1/2} \frac{dA}{dt} \frac{1}{4\pi}$$

③

$$A = 4\pi r^2$$

$$r = 20$$

$$\underbrace{\frac{dV}{dt}}_{20} = \cancel{\frac{4}{3}}\cancel{\pi} \frac{\cancel{3}}{2} \frac{1}{\cancel{4\pi}} \left(\frac{A}{4\pi}\right)^{1/2} \frac{dA}{dt}$$

$$20 = \frac{1}{2}\left(\frac{\cancel{4\pi} \cdot 20^2}{\cancel{4\pi}}\right)^{1/2} \frac{dA}{dt}$$

$$20 = \frac{1}{2} \cdot 20 \frac{dA}{dt}$$

$$\frac{dA}{dt} = 2 \text{ cm}^2/\text{sec}$$

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Q7 different method

$r = 20$

$$V = \frac{4}{3}\pi r^3 \quad \rightsquigarrow \quad \boxed{\frac{dV}{dt}}^{20} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

implicit diff wrt t

$$A = 4\pi r^2 \qquad \frac{dA}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

sub in $\rightsquigarrow$

$$20 = 4\pi \cdot 20^2 \frac{dr}{dt} \rightsquigarrow \frac{dr}{dt} = \frac{1}{4\pi \cdot 20}$$

$$\frac{dA}{dt} = 4\pi \cdot 2 \cdot 20 \cdot \frac{dr}{dt}$$

$$\frac{dA}{dt} = 4\pi \cdot 2 \cdot 20 \frac{1}{4\pi \cdot 20} = 2 \text{ cm}^2/\text{sec}$$

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Q7 method 3.

$$V = \frac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$

$$\frac{V}{A} = \frac{4\pi r^3}{3 \cdot 4\pi r^2} = \frac{r}{3}$$

$$V = \frac{1}{3} r(t) A(t)$$

$$\boxed{\underbrace{\frac{dV}{dt}}_{20} = \frac{1}{3}\underbrace{\frac{dr}{dt}}_{\frac{1}{4\pi 20}} \underbrace{A}_{4\pi \cdot 20^2} + \frac{1}{3}\underbrace{r}_{20}\frac{dA}{dt}}$$

$$\frac{dA}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

$$\underbrace{\frac{dV}{dt}}_{20} = 4\pi \underbrace{r^2}_{20} \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{20}{4\pi \cdot 20^2} = \frac{1}{4\pi \cdot 20}$$

$$20 = \frac{1}{3} \cdot \frac{1}{4\pi 20} \cdot 4\pi \cdot 20^2 + \frac{1}{3} \cdot 20 \frac{dA}{dt}$$

$$20 = \frac{1}{3} \cdot 20 + \frac{1}{3} \cdot 20 \frac{dA}{dt}$$

$$\frac{2}{3} \cdot 20 = \frac{1}{3} \cdot 20 \frac{dA}{dt}$$

$$\frac{dA}{dt} = \frac{\frac{2}{3} \cdot 20}{\frac{1}{3} \cdot 20} = 2 \text{ cm}^2/\text{sec}.$$

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Q9 $f(x) = e^x(x^2 - x - 5)$. $(fg)' = f'g + fg'$.

find critical points. solve $f'(x) = 0$.

$$f'(x) = e^x(x^2 - x - 5) + e^x(2x - 1).$$

$$f'(x) = \underbrace{e^x}_{\neq 0}(x^2 + x - 6) = 0.$$

$$(x+3)(x-2).$$

$x = -3$
$x = 2$

		-3		2	
e^x	$+$		$+$		$+$
$x+3$	$-$		$+$		$+$
$x-2$	$-$		$-$		$+$
$f'(x)$	$+$		$-$		$+$

		-3		2	
$f'(x)$	$+$		$-$		$+$

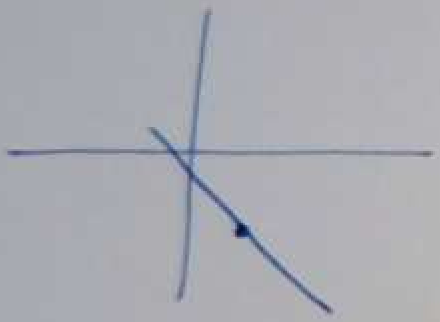
local max

local min.

first derivative test: -3 local max

2 local min.

Q5 $16x^2 - 3y^2 = 4$ $(1, -2)$



$16x^2 - 3(y(x))^2 = 4$ ← diff wrt x.

$$32\underset{1}{x} - 3 \cdot 2\left(\underset{-2}{y(x)}\right) \cdot \frac{dy}{dx} = 0$$

$$32 + 12\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{32}{12} = -\frac{8}{3}$$

$$y - \overset{-2}{y_0} = \underset{-8/3}{m}(x - \overset{1}{x_0})$$

$$y + 2 = -\frac{8}{3}(x - 1)$$

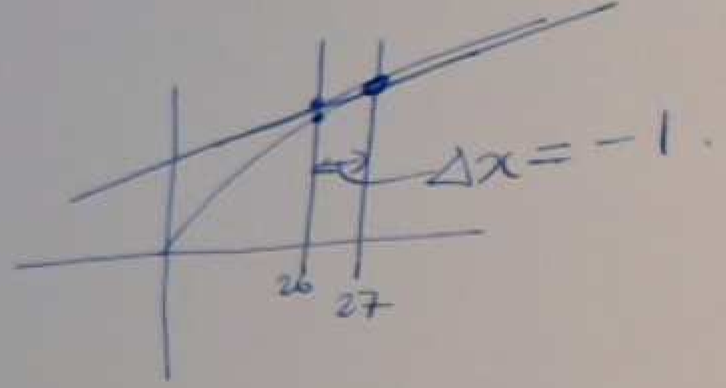
tangent line

Q8. $f(x) = \sqrt[3]{x} = x^{1/3}$. find approx for $\sqrt[3]{26}$.

use $3^3 = 27 = 26 + 1$.

$$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3 \cdot \sqrt[3]{x^2}}$$

$$f'(27) = \frac{1}{3 \cdot \sqrt[3]{27^2}} = \frac{1}{3 \cdot (3)^2} = \frac{1}{27}$$



linear approx:

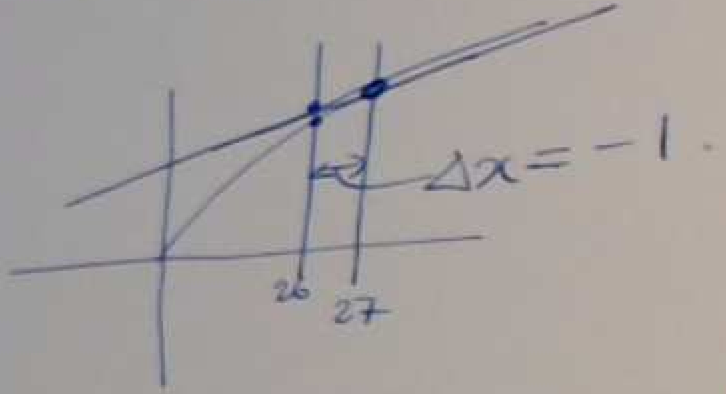
$$f(26) \approx f(27) + \underbrace{\Delta x}_{-1} \underbrace{f'(27)}_{\frac{1}{27}}$$

$$f(26) \approx 3 - \frac{1}{27} = 2\frac{26}{27}$$

Q8. $f(x) = \sqrt[3]{x} = x^{1/3}$. find approx for $\sqrt[3]{26}$.

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3 \cdot \sqrt[3]{x^2}}$.

use $3^3 = 27 = 26+1$.



$f'(27) = \frac{1}{3 \cdot \sqrt[3]{27^2}} = \frac{1}{3 \cdot (3)^2}$

$= \frac{1}{27}$.

linear approx:

$f(26) \approx f(27) + \underbrace{\Delta x}_{-1} \underbrace{f'(27)}_{\frac{1}{27}}$

$f(26) \approx 3 - \frac{1}{27} = 2\frac{26}{27}$

11

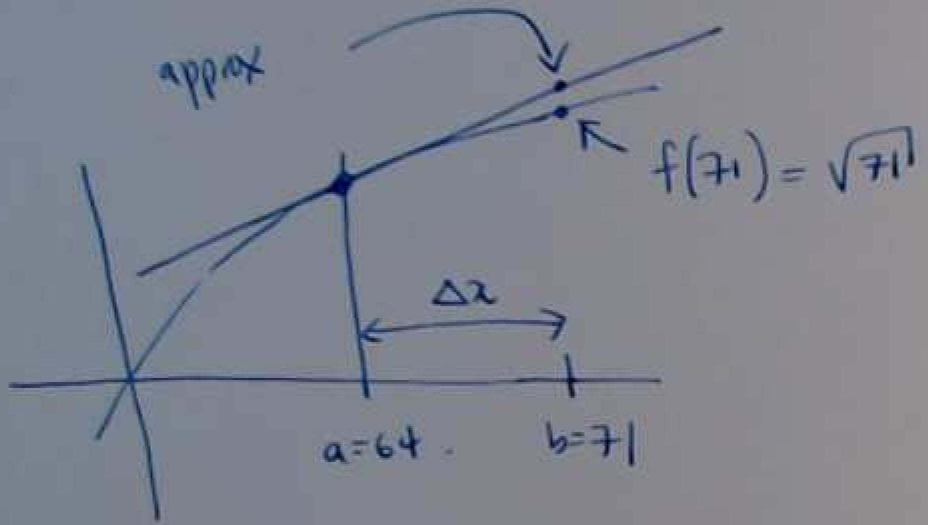
absolute error: $\left|\sqrt[3]{26} - 2\frac{26}{27}\right|$.

percentage error $\left|\frac{\left|\sqrt[3]{26} - 2\frac{26}{27}\right|}{\sqrt[3]{26}}\right| \times 100$.

$\approx 0.016\%$.

(12)

WW 4.1. Q4. $f(x) = \sqrt{x}$



$a = 64$
$b = 71$.

$\Delta x = b - a = 7$.

$f(x) = x^{1/2}$.

$f'(x) = \frac{1}{2} x^{-1/2}$.

$f'(64) = \frac{1}{2 \cdot \sqrt{64}} = \frac{1}{16}$.

approx : $f(64) + 7 \cdot f'(64)$

: $\sqrt{64} + 7 \cdot \frac{1}{16}$.

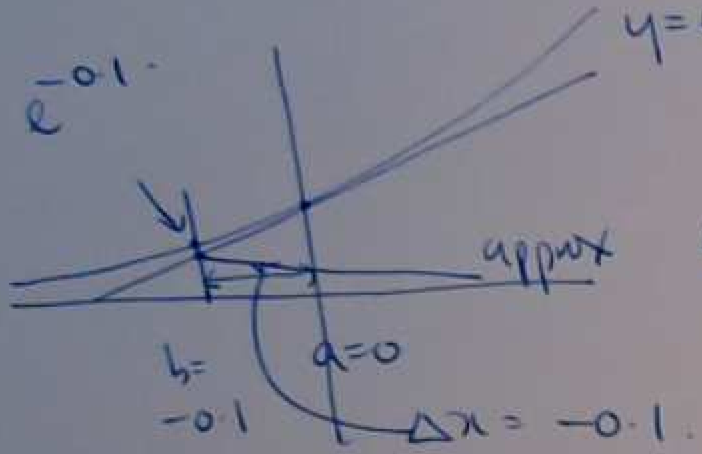
$8 + \frac{7}{16} = 8\frac{7}{16}$.

error : $\left| 8\frac{7}{16} - \sqrt{71} \right|$.

WW 4.1 Q7

$f(x) = e^x \qquad a = 0 \qquad b = -0.1$

$f'(x) = e^x$



$$e^{-0.1} \approx f(0) + \underbrace{\Delta x}_{-0.1} f'(0)$$

$$f(0) - 0.1\, f'(0)$$

$$\approx \underbrace{e^0}_{1} - 0.1 e^0$$

$$\approx 1 - 0.1 = 0.9$$

error: $\left| e^{-0.1} - 0.9 \right|$

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WW 4.2 Q1

$$f(x) = x^3 + 4x^2 - 7x - 3.$$

$$f'(x) = 3x^2 + 8x - 7$$

find critical points: solve $f'(x) = 0$.

$$3x^2 + 8x - 7.$$

3 7
1 1.

12
7.
70
14.

$$x = \frac{-8 \pm \sqrt{64 - 4 \cdot 3 \cdot (-7)}}{6} = \frac{-8 \pm \sqrt{64 + 84}}{6}$$

148.

$$= \frac{-4 \pm \sqrt{37}}{3}.$$

critical points: $\frac{-4 + \sqrt{37}}{3}$ $\frac{-4 - \sqrt{37}}{3}$.

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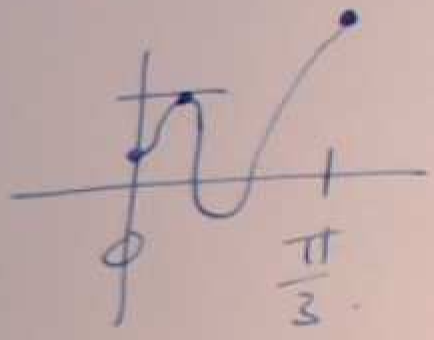
WW 4.2 Q6 $f(\theta) = \sqrt{12}\,\theta - \sqrt{6}\sec\theta \quad [0, \frac{\pi}{3}]$.



① find critical points.

$$f'(\theta) = \sqrt{12} - \sqrt{6}\sec\theta\tan\theta.$$

solve $f'(\theta) = 0$.

$$\sqrt{12} - \sqrt{6}\sec\theta\tan\theta = 0.$$

$$\sec\theta\tan\theta = \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2}.$$

$$\frac{1}{\cos\theta}\,\frac{\sin\theta}{\cos\theta} = \sqrt{2}.$$

$$\sec\theta = \frac{1}{\cos\theta} = (\cos\theta)^{-1}$$

$$\frac{d}{d\theta}\left(\frac{1}{\cos\theta}\right) = -(\cos\theta)^{-2}\cdot(-\sin\theta)$$

$$= \frac{\sin\theta}{\cos^2\theta} = \tan\theta\cdot\frac{1}{\cos\theta}$$

$$= \sec\theta\cdot\tan\theta.$$

$$\frac{\sin\theta}{\cos^2\theta} = \sqrt{2}$$

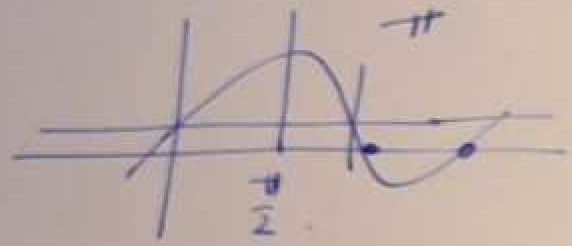
$$\sin\theta = \sqrt{2}\,\underbrace{\cos^2\theta}_{1-\sin^2\theta}$$

$[0, \frac{\pi}{3}]$

$$\sin\theta = \sqrt{2}\,(1-\sin^2\theta)$$

$$\sqrt{2}\sin^2\theta + \sin\theta - \sqrt{2} = 0$$

$$\sin^2\theta + \frac{1}{\sqrt{2}}\sin\theta - 1 = 0$$



$$\sin\theta = \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{1}{2} - 4\cdot 1\cdot(-1)}}{2}$$

$$= \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{1}{2}+4}}{2} = \frac{\frac{1}{\sqrt{2}} \pm \sqrt{\frac{9}{2}}}{2}$$

$$= \frac{\frac{1}{\sqrt{2}} \pm \frac{3}{\sqrt{2}}}{2} \qquad \frac{2}{\sqrt{2}} > 1 \qquad \frac{\cancel{2}\sqrt{2}}{\cancel{2}} \qquad \underbrace{-\frac{1}{\sqrt{2}}}_{-\frac{\sqrt{2}}{2}}$$

no solutions in $[0, \frac{\pi}{3}]$

$$f(\theta) = \sqrt{12}\,\theta - \sqrt{6}\sec\theta \qquad [0, \tfrac{\pi}{3}] \qquad (17)$$

$f'(\theta) = 0 \rightsquigarrow$ no solutions in $[0, \frac{\pi}{3}]$ $\Rightarrow$ no critical points in $[0, \frac{\pi}{3}]$

$$f(0) = -\sqrt{6}\sec(0) = -\sqrt{6} \quad \text{min} \qquad \frac{1}{\cos(\frac{\pi}{3})}$$

$$f(\tfrac{\pi}{3}) = \sqrt{12}\,\frac{\pi}{3} - \sqrt{6}\sec(\tfrac{\pi}{3})$$

$$= \frac{\sqrt{12}\,\pi}{3} - \sqrt{6}\,\frac{1}{1/\sqrt{2}}$$

$$= \sqrt{12}\,\frac{\pi}{3} - \sqrt{12}$$

$$= \sqrt{12}\left(\frac{\pi}{3} - 1\right) \quad \text{max}$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	0

$$f(\theta) = \sqrt{12}\,\theta - \sqrt{6}\sec\theta$$

$$f'(\theta) = \sqrt{12} - \sqrt{6}\sec\theta\tan\theta = 0$$

$$\sec\theta\tan\theta = \frac{\sqrt{12}}{\sqrt{6}} = \sqrt{2}$$

$$\frac{\sin\theta}{\cos^2\theta} = \sqrt{2}$$

$$\sin\theta = \sqrt{2}\cos^2\theta = \sqrt{2}(1-\sin^2\theta)$$

$$\sqrt{2}\sin^2\theta + \sin\theta - \sqrt{2} = 0$$

$$\frac{-1 \pm \sqrt{1 + 4\cdot 2}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}}$$

$\boxed{\sin\theta = \frac{1}{\sqrt{2}}}$, $-\frac{2}{\sqrt{2}}$ ← no solution

$$\boxed{\theta = \frac{\pi}{4}}$$

$$f\left(\frac{\pi}{4}\right)$$

$$\sqrt{12}\,\frac{\pi}{4} - \sqrt{6}\sec\left(\frac{\pi}{4}\right)$$

$$\sqrt{12}\left(\frac{\pi}{4} - \sqrt{2}\,\frac{1}{1/\sqrt{2}}\right.$$

$$\sqrt{12}\cdot\frac{\pi}{4} - 2$$

max