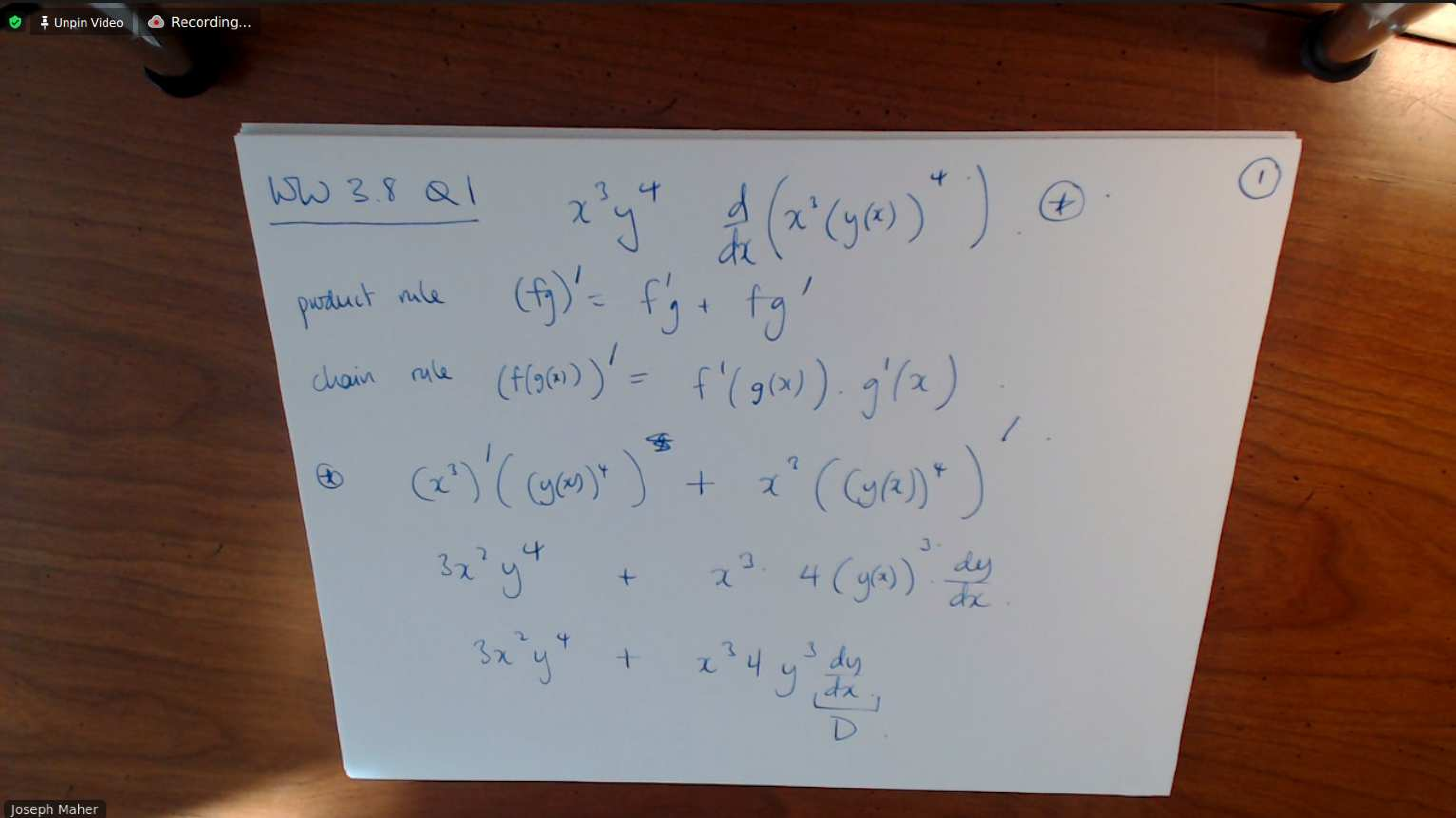

WW 3.8 Q1 x^3y^4 $\frac{d}{dx}\left(x^3(y(x))^4\right)$ $\circledast$

product rule $(fg)' = f'g + fg'$

chain rule $(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$\circledast$ $(x^3)'\left((y(x))^4\right) + x^3\left((y(x))^4\right)'$

$3x^2y^4 + x^3 \cdot 4(y(x))^3 \cdot \frac{dy}{dx}$

$3x^2y^4 + x^3 4 y^3 \underbrace{\frac{dy}{dx}}_{D}$

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②

ww 3.8 Q2

$(x^2+y^2)^7$ $\quad$ $(x^2+(y(x))^2)^7$ $\quad$ $\frac{d}{dx}(x^7)$

$=7x^6$

chain rule: $(f(g(x)))' = f'(g(x))\cdot g'(x)$

$$7(x^2+y(x)^2)^6\cdot(x^2+(y(x))^2)'$$

$$7(x^2+y^2)^6\left(2x+2(y(x))^1\frac{dy}{dx}\right)$$

$$7(x^2+y^2)^6\left(2x+2y\underbrace{\frac{dy}{dx}}_{D}\right)$$

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ww 3.8 Q10

③

$8x^6y = 6$ $\qquad$ $8(x(t))^6 y(t) = 6$.

diff wrt t

product rule: $(fg)' = f'g + fg'$

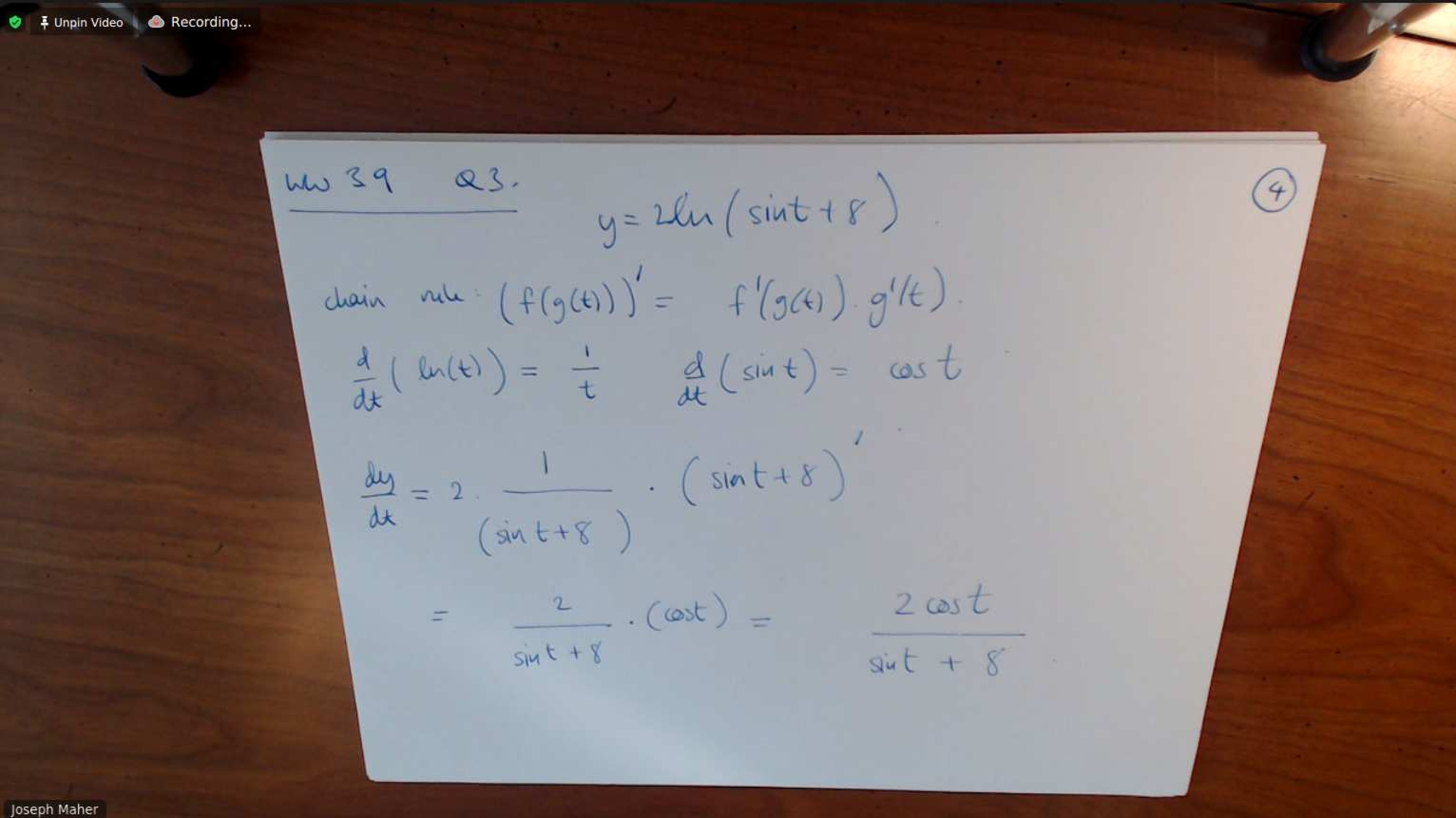
$$8\left[(x(t))^6\right]' y(t) + 8(x(t))^6 \left[y(t)'\right] = 0$$

↑ chain rule

$$\cancel{8} \cdot 6(x(t))^5 \cdot \frac{dx}{dt}\, y(t) + \cancel{8}(x(t))^6 \underbrace{\frac{dy}{dt}} = 0$$

$$x^6 \frac{dy}{dt} = -6x^5 \frac{dx}{dt}\, y$$

$$\frac{dy}{dt} = -\frac{6y}{x} \frac{dx}{dt}$$



hw 39 Q3. ④

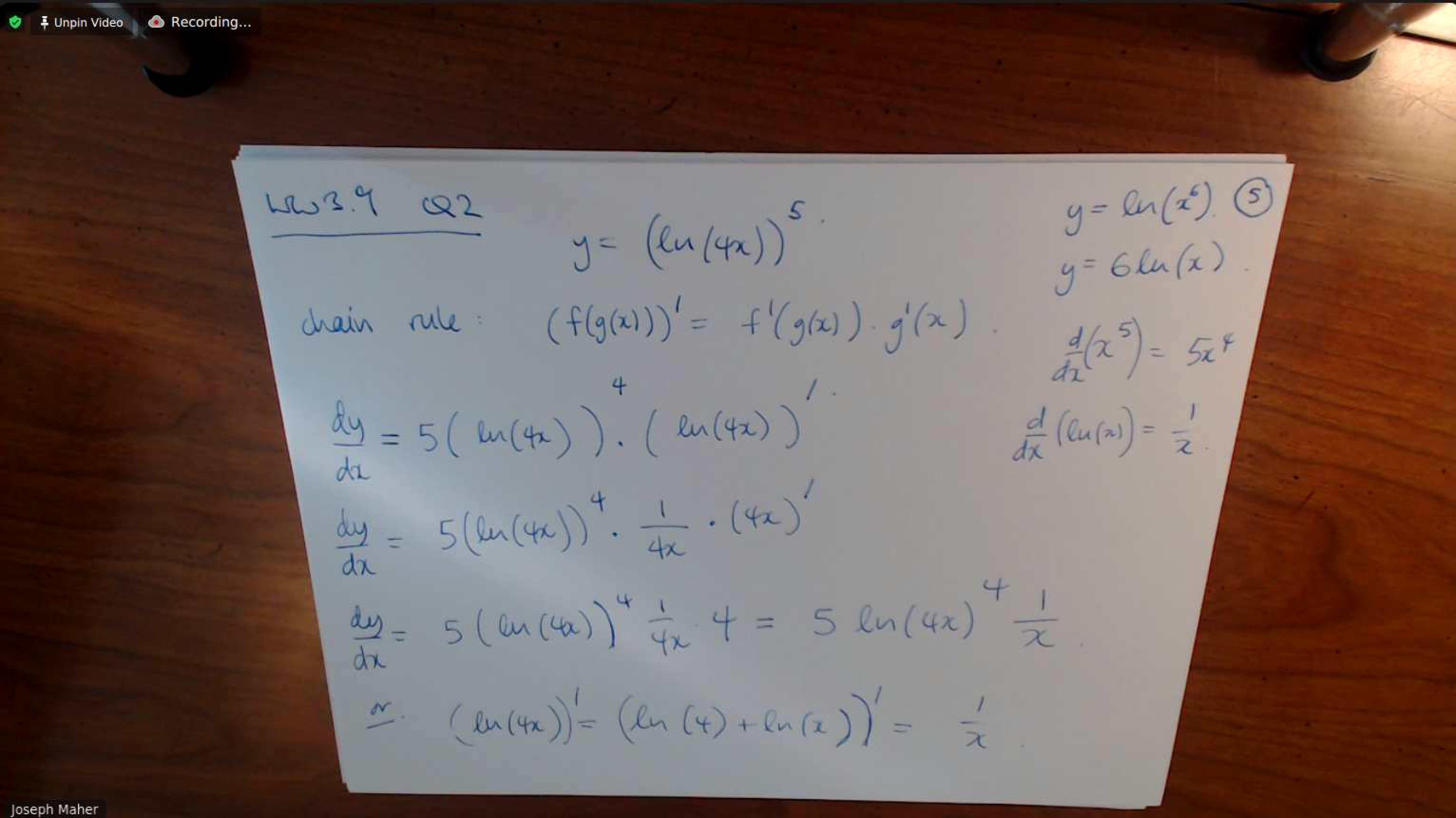
$$y = 2\ln(\sin t + 8)$$

chain rule: $(f(g(t)))' = f'(g(t)) \cdot g'(t)$

$\frac{d}{dt}(\ln(t)) = \frac{1}{t}$ $\frac{d}{dt}(\sin t) = \cos t$

$$\frac{dy}{dt} = 2 \cdot \frac{1}{(\sin t + 8)} \cdot (\sin t + 8)'$$

$$= \frac{2}{\sin t + 8} \cdot (\cos t) = \frac{2\cos t}{\sin t + 8}$$



HW3.9 Q2

$y = \ln(x^6)$ ⑤

$y = 6\ln(x)$.

$$y = (\ln(4x))^5.$$

chain rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

$\frac{d}{dx}(x^5) = 5x^4$

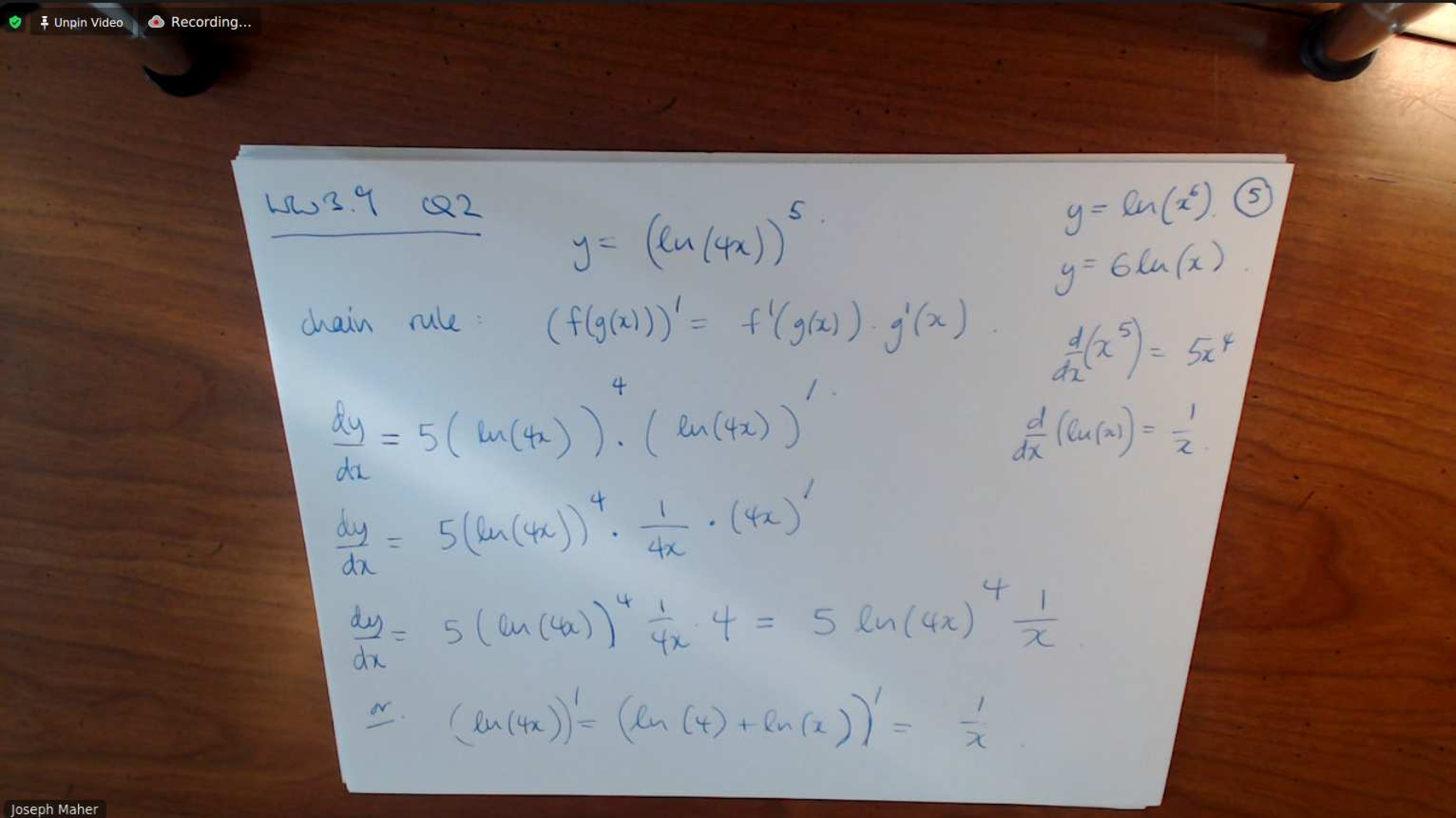
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$.

$$\frac{dy}{dx} = 5(\ln(4x))^4 \cdot (\ln(4x))'.$$

$$\frac{dy}{dx} = 5(\ln(4x))^4 \cdot \frac{1}{4x} \cdot (4x)'$$

$$\frac{dy}{dx} = 5(\ln(4x))^4 \frac{1}{4x} \cdot 4 = 5\ln(4x)^4 \frac{1}{x}.$$

or. $(\ln(4x))' = (\ln(4) + \ln(x))' = \frac{1}{x}$.



HW3.9 Q2

$$y = (\ln(4x))^5.$$

$y = \ln(x^6)$ ⑤

$y = 6\ln(x)$.

chain rule: $(f(g(x)))' = f'(g(x))\cdot g'(x)$.

$\frac{d}{dx}(x^5) = 5x^4$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$.

$$\frac{dy}{dx} = 5(\ln(4x))^4\cdot(\ln(4x))'.$$

$$\frac{dy}{dx} = 5(\ln(4x))^4\cdot\frac{1}{4x}\cdot(4x)'$$

$$\frac{dy}{dx} = 5(\ln(4x))^4\frac{1}{4x}\cdot 4 = 5\ln(4x)^4\frac{1}{x}.$$

or. $(\ln(4x))' = (\ln(4)+\ln(x))' = \frac{1}{x}$.

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WW 3.10 Q1.

$A = \pi r^2.$

$A(t) = \pi(r(t))^2$

diff wrt t: $\frac{dA}{dt} = \pi \; 2r(t) \cdot \frac{dr}{dt}.$

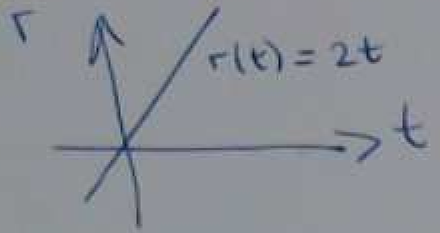
$\frac{dr}{dt} = 2m/min$

a) $\frac{dA}{dt}$ at $r = 24$

$\frac{dA}{dt} = \pi . 2 . 24 \cdot 2$

$= 96\pi.$

b) $r(0) = 0$ $t = 3?$



$r(3) = 2\times 3$

$= 6.$

$\frac{dA}{dt} = \pi . 2 . 6 . 2 .$

$= 24\pi.$

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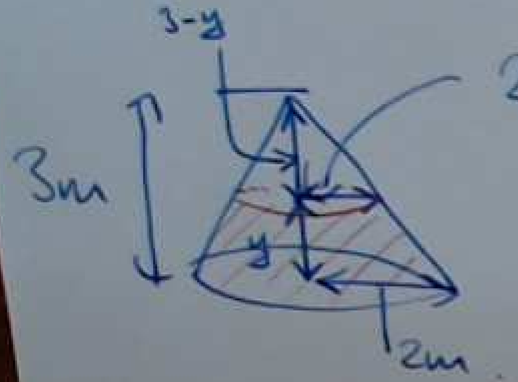
WW 3.10 Q 3.



$2m^3/min$.

volume of cone.

$$V = \frac{1}{3}h\,\pi r^2.$$

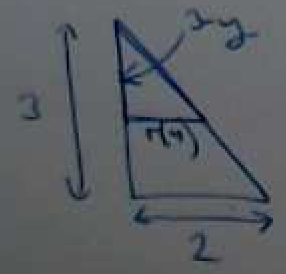
$y(t)$.

$$V(t) = 4\pi - \frac{1}{3}(3-y)\left(2-\frac{2}{3}y(t)\right)^2.$$

total vol of tank

$$\frac{1}{3}3\pi\cdot 2^2 = 4\pi$$

empty bit has vol $= \frac{1}{3}(3-y)\,\pi\, r(t)^2 = \frac{1}{3}(3-y)\left(2-\frac{2}{3}y(t)\right)^2.$



$$\frac{3-y(t)}{r(t)} = \frac{3}{2}$$

$$r(t) = \frac{2}{3}\left(3-y(t)\right)$$

$$= 2-\frac{2}{3}y(t).$$

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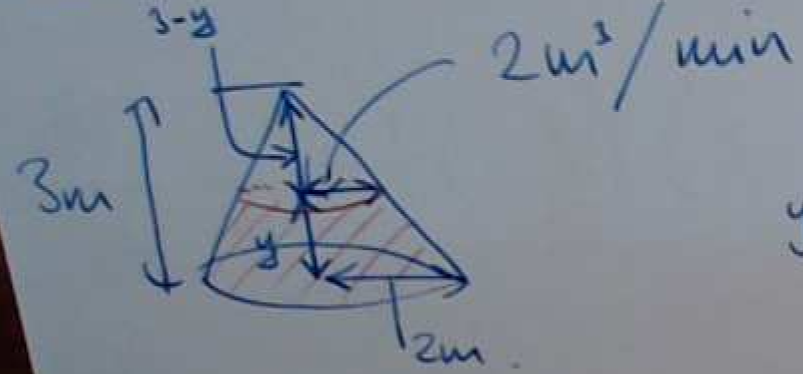
WW 3.10 Q3

volume of cone.

$$V = \frac{1}{3} h \pi r^2$$



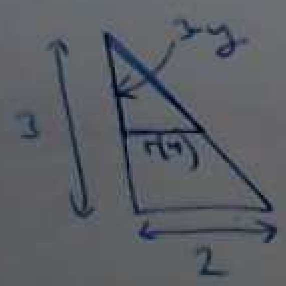
y(t)

$$\boxed{V(t) = 4\pi - \frac{1}{3}(3-y)\left(2-\frac{2}{3}y(t)\right)^2}$$

total vol of tank

$$\frac{1}{3} 3\pi \cdot 2^2 = 4\pi$$

empty bit has vol $= \frac{1}{3}(3-y)\pi r(t)^2 = \frac{1}{3}(3-y)\left(2-\frac{2}{3}y(t)\right)^2$.



$$\frac{3-y(t)}{r(t)} = \frac{3}{2}$$

$$r(t) = \frac{2}{3}(3-y(t))$$

$$= 2-\frac{2}{3}y(t)$$

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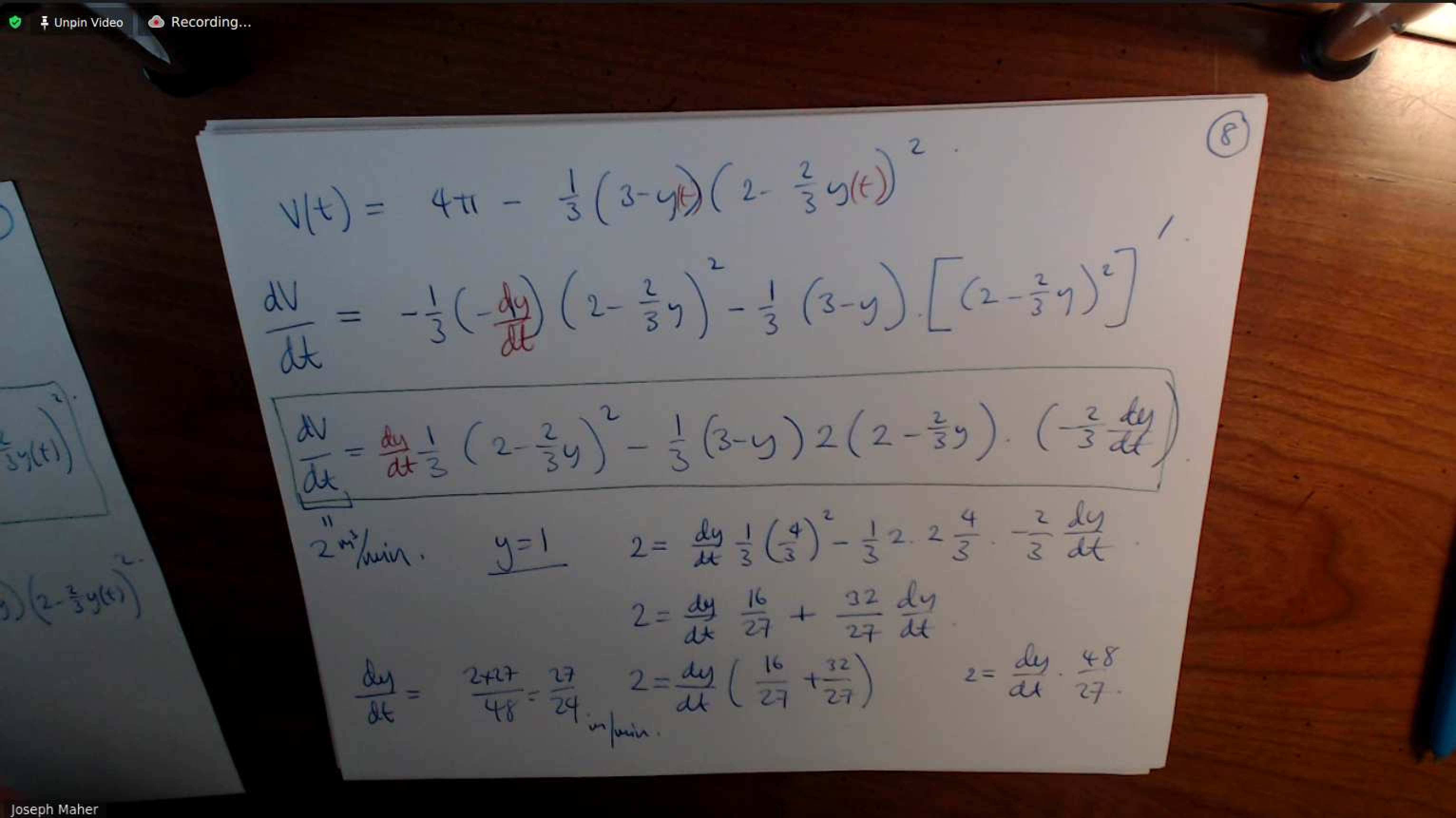

$$V(t) = 4\pi - \frac{1}{3}\left(3 - y(t)\right)\left(2 - \frac{2}{3}y(t)\right)^2 .$$

$$\frac{dV}{dt} = -\frac{1}{3}\left(-\frac{dy}{dt}\right)\left(2 - \frac{2}{3}y\right)^2 - \frac{1}{3}\left(3-y\right) . \left[\left(2 - \frac{2}{3}y\right)^2\right]' .$$

$$\boxed{\frac{dV}{dt} = \frac{dy}{dt}\frac{1}{3}\left(2 - \frac{2}{3}y\right)^2 - \frac{1}{3}\left(3-y\right)2\left(2 - \frac{2}{3}y\right) . \left(-\frac{2}{3}\frac{dy}{dt}\right)}$$

$$\overset{''}{2}\ m^3/min . \qquad \underline{y=1}$$

$$2 = \frac{dy}{dt}\frac{1}{3}\left(\frac{4}{3}\right)^2 - \frac{1}{3}2 . 2\frac{4}{3} . -\frac{2}{3}\frac{dy}{dt} .$$

$$2 = \frac{dy}{dt}\frac{16}{27} + \frac{32}{27}\frac{dy}{dt} .$$

$$2 = \frac{dy}{dt}\left(\frac{16}{27} + \frac{32}{27}\right) \qquad 2 = \frac{dy}{dt} . \frac{48}{27} .$$

$$\frac{dy}{dt} = \frac{2\times 27}{48} = \frac{27}{24}\ m/min .$$

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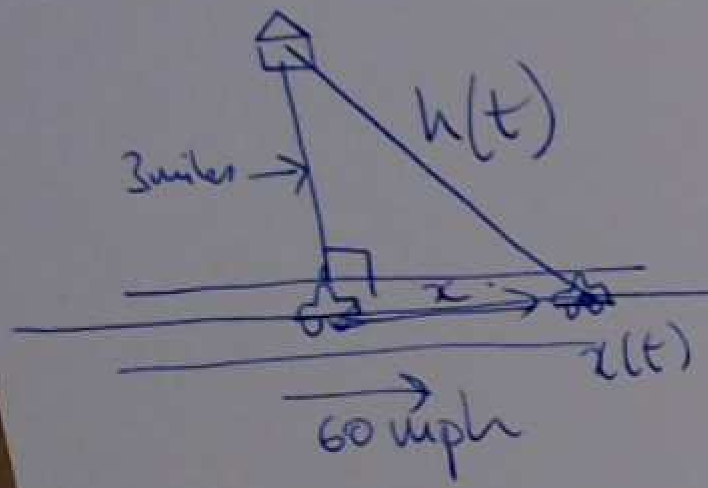
HW 3.10 Q5

$$\frac{dx}{dt} = 60 \text{ mph.}$$



x(t).

$$\boxed{h^2 = 3^2 + x^2}$$

diff. wrt t $\longrightarrow (h(t))^2 = 3^2 + (x(t))^2.$

$$\boxed{\cancel{2}\,\underbrace{h(t)}\,\frac{dh}{dt} = \cancel{2}\,\underbrace{x(t)}_{8}\,\underbrace{\frac{dx}{dt}}_{60}}$$

Q- what is $\frac{dh}{dt}$ when $x=8$

$h^2 = 3^2 + 8^2$

$h = \sqrt{9+64} = \sqrt{73}$

$$\frac{dh}{dt} = \frac{8 \times 60}{\sqrt{73}} \text{ mph}$$

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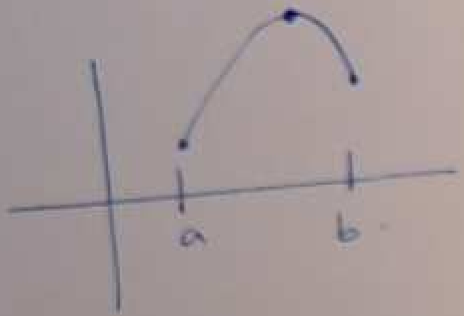
recall

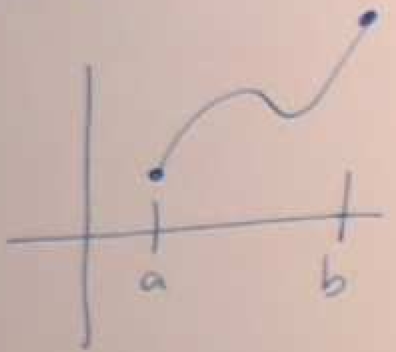
local max

local min

$\Rightarrow \quad f'(c) = 0$

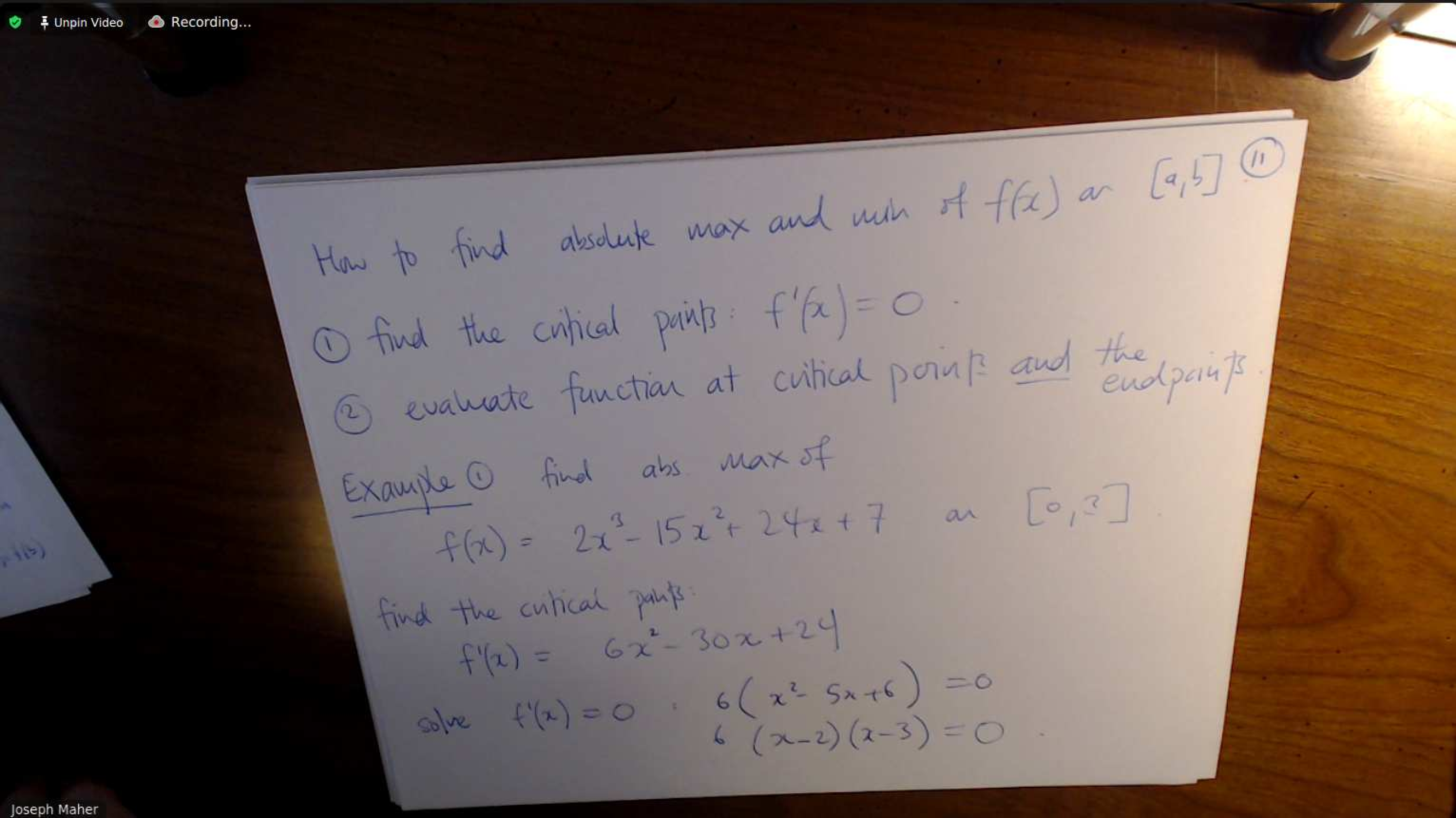
Q: find the absolute max or min of $f(x)$ on a closed interval $[a,b]$





max occurs: either at a
local max,
or an endpoint

min occurs: either at a local min
or an endpoint



How to find absolute max and min of $f(x)$ on $[a,b]$ ⑪

① find the critical points: $f'(x) = 0$.

② evaluate function at critical points <u>and</u> the endpoints.

Example ① find abs max of

$$f(x) = 2x^3 - 15x^2 + 24x + 7 \quad \text{on} \quad [0,3].$$

find the critical points:

$$f'(x) = 6x^2 - 30x + 24$$

solve $f'(x) = 0$: $6(x^2 - 5x + 6) = 0$

$$6(x-2)(x-3) = 0.$$

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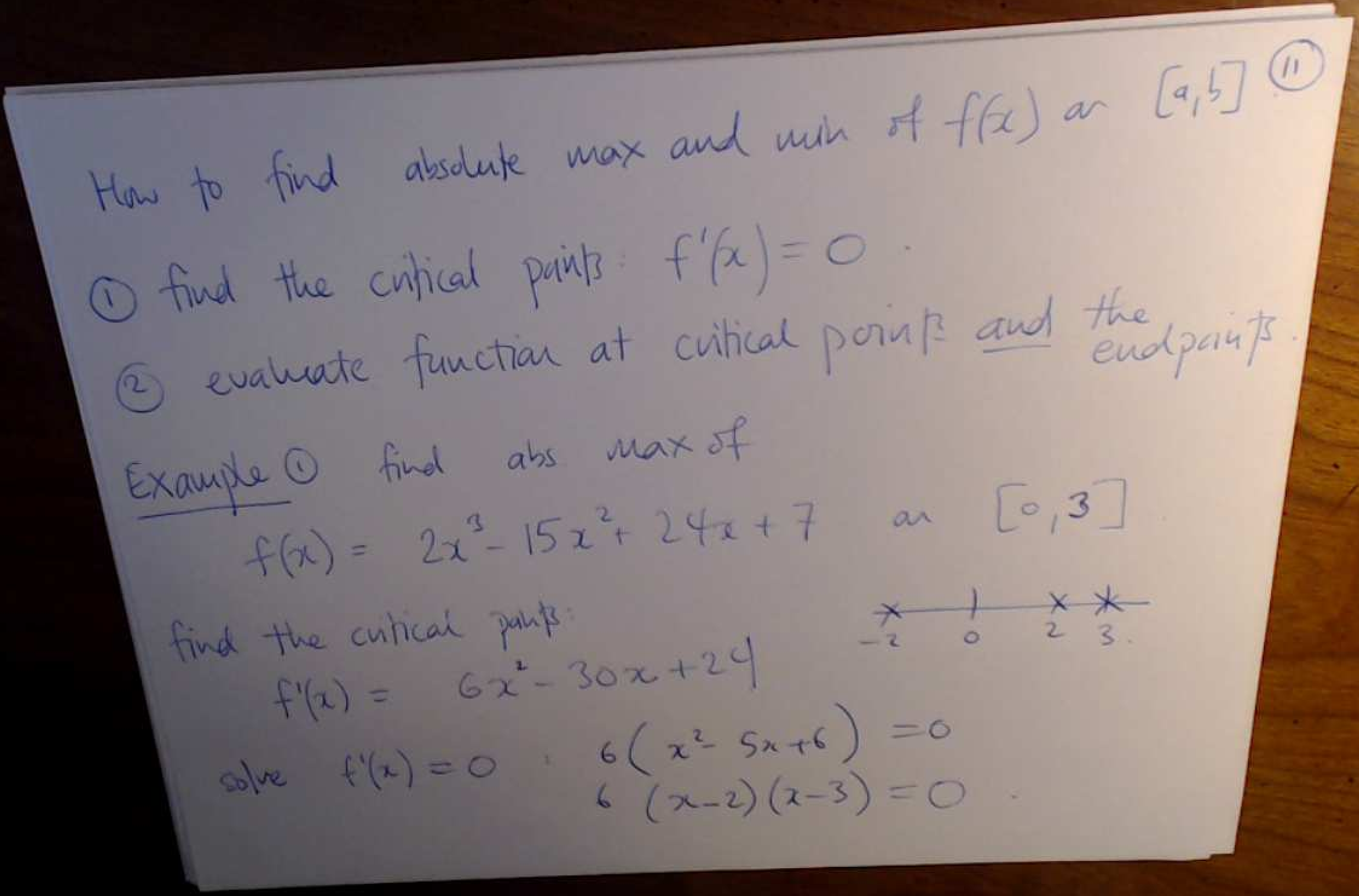

How to find absolute max and min of $f(x)$ on $[a,b]$ ⑪

① find the critical points: $f'(x) = 0$.

② evaluate function at critical points <u>and</u> the endpoints.

<u>Example</u> ① find abs max of

$$f(x) = 2x^3 - 15x^2 + 24x + 7 \quad \text{on} \quad [0,3]$$

find the critical points:

$$f'(x) = 6x^2 - 30x + 24$$

$-2 \quad 0 \quad 2 \quad 3$

solve $f'(x) = 0$: $6(x^2 - 5x + 6) = 0$

$$6(x-2)(x-3) = 0$$

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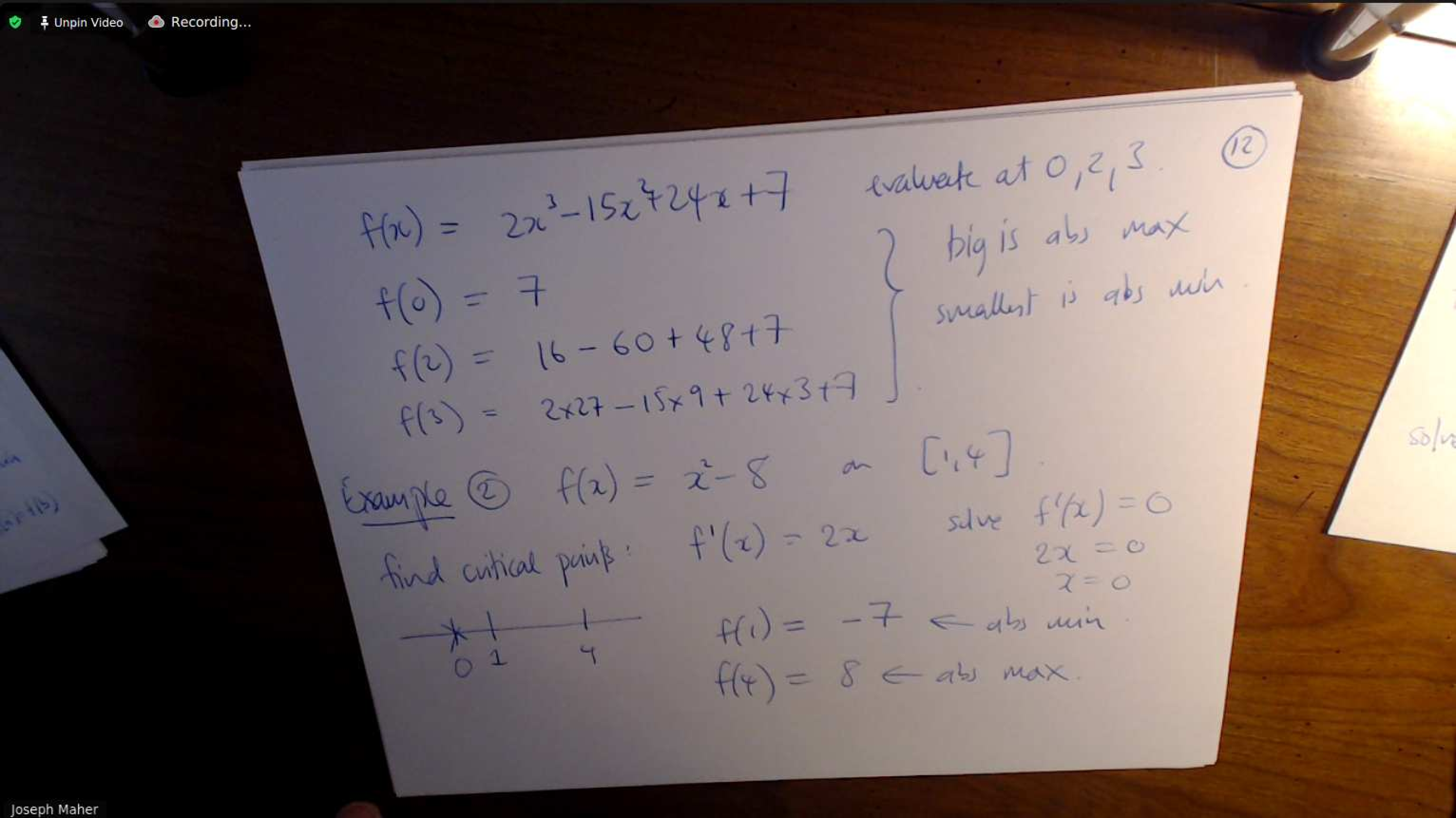


$f(x) = 2x^3 - 15x^2 + 24x + 7$ evaluate at 0, 2, 3.

$f(0) = 7$

$f(2) = 16 - 60 + 48 + 7$

$f(3) = 2 \times 27 - 15 \times 9 + 24 \times 3 + 7$

} big is abs max
smallest is abs min.

Example ② $f(x) = x^2 - 8$ on $[1,4]$.

find critical points: $f'(x) = 2x$ solve $f'(x) = 0$

$2x = 0$

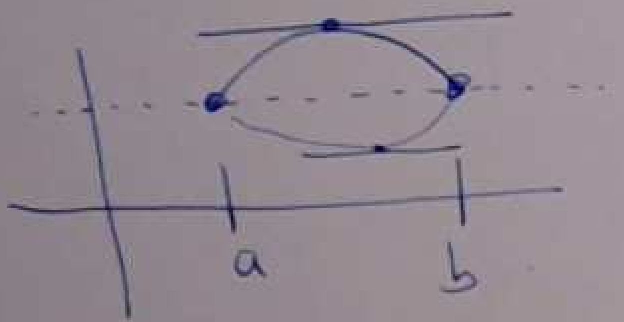
$x = 0$

$f(1) = -7$ ← abs min.

$f(4) = 8$ ← abs max.

Thm (Rolle's Thm) If $f(x)$ is cts on $[a,b]$ and differentiable on (a,b) if $f(a)=f(b)$, then there is a $c \in (a,b)$ s.t. $f'(c)=0$.

Proof



$f(a)=f(b)$

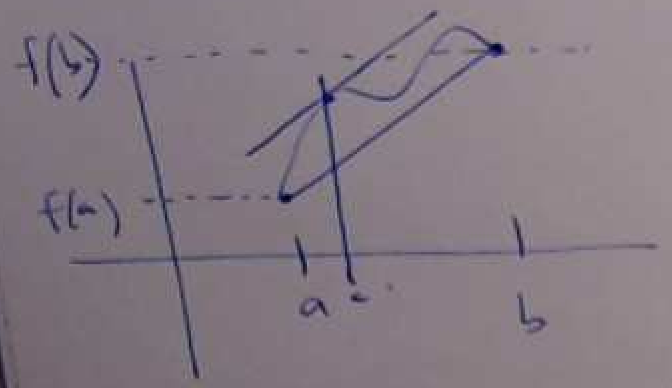
- If there is a $c \in (a,b)$ which is a local max or min then $f'(c)=0$ ✓

- If there is no local max or min, then $f(x) = \text{const} = f(a)=f(b)$, but $f'(c)=0$ for all $c \in (a,b)$ ✓ □

§4.3. First derivative test

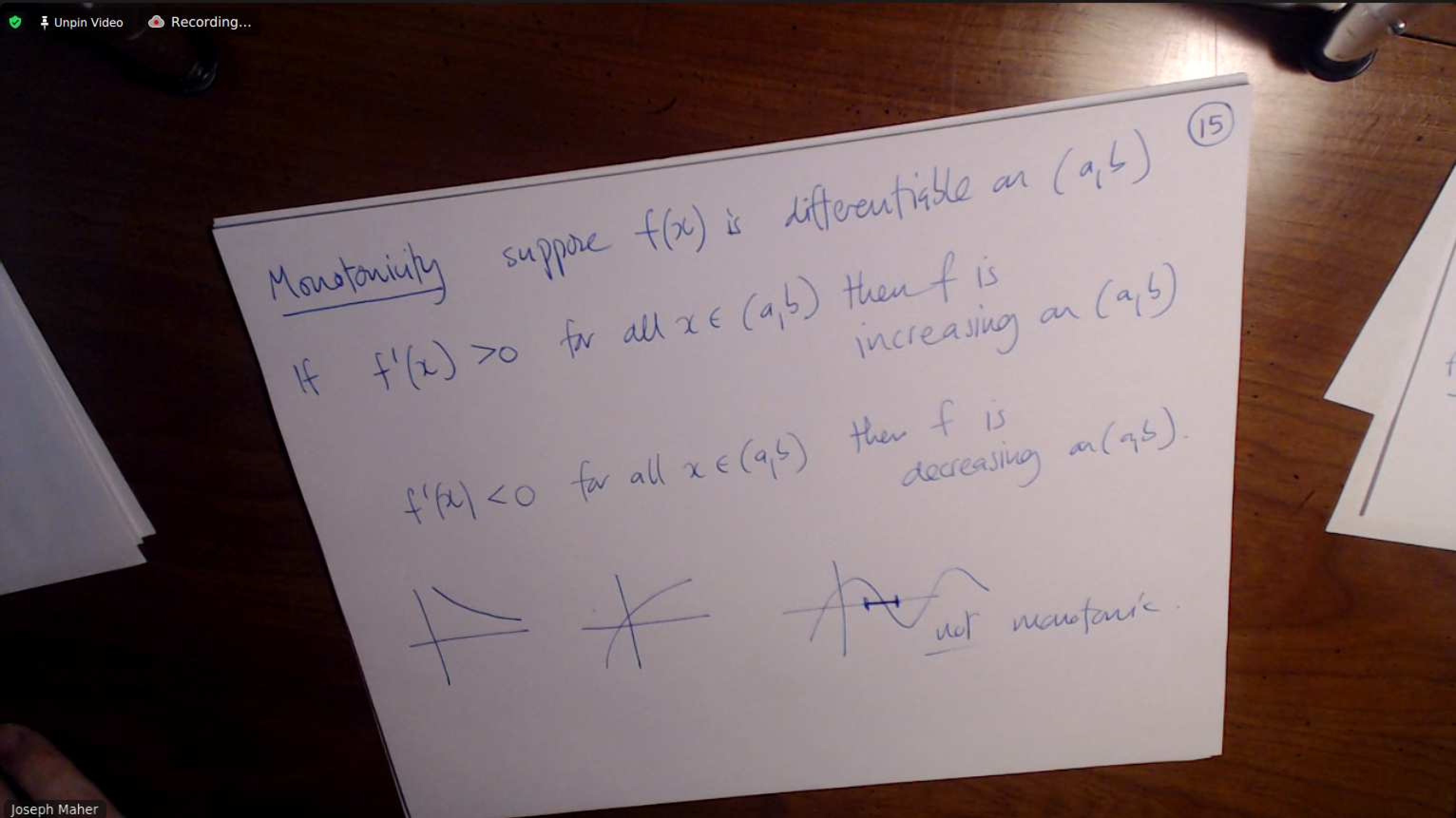
find critical pts
solve $f'(x)=0$.

Thm (Mean value theorem MVT)
Suppose f is cts on $[a,b]$ and differentiable in (a,b), then there is a $c \in (a,b)$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$
= average rate of change.



Corollary If $f(x)$ is differentiable and $f'(x)=0$ then $f(x)=\underset{\text{const}}{c}$.

Proof (of corollary) suppose there is a, b with $a \neq b$. $f(a) \neq f(b)$)
MVT $\exists c$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a} \neq 0$ # $\square$



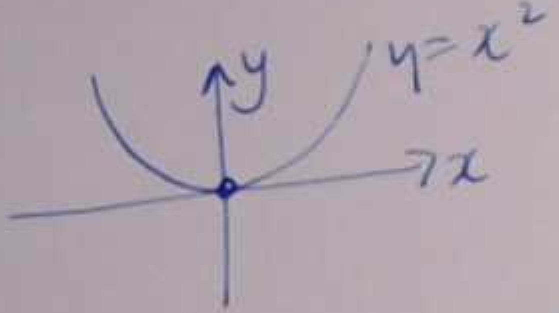
Monotonicity suppose $f(x)$ is differentiable on (a,b)

If $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)

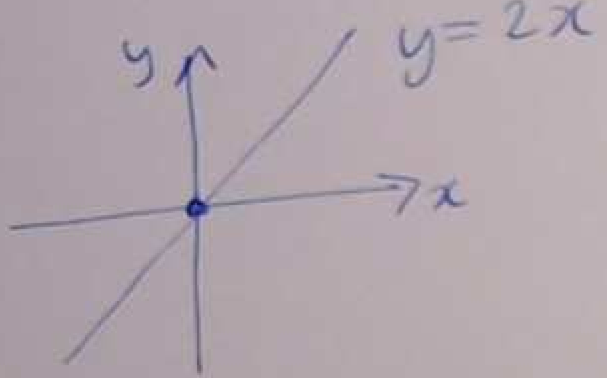
$f'(x) < 0$ for all $x \in (a,b)$ then f is decreasing on (a,b).

not monotonic.

Example $f(x) = x^2$



$f'(x) = 2x$



critical points:

$2x = 0 \Rightarrow x = 0$

$f'(x) < 0$ on $(-\infty, 0) \Rightarrow f$ is decreasing on $(-\infty, 0)$

$f'(x) > 0$ on $(0, \infty) \Rightarrow f$ is increasing on $(0, \infty)$

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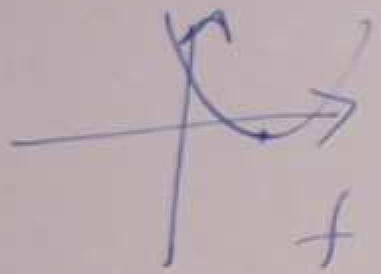
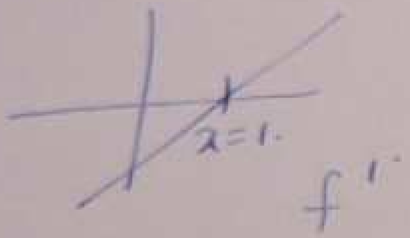
Example $f(x) = x^2-2x-3$.

$f'(x) = 2x-2$



find critical points: $f'(x) = 0$ $2x-2=0$
$x=1$.



Q: where is $f'(x) > 0$.

$2x-2 > 0$.
$+2 \quad +2$
$\frac{2x}{2} > \frac{2}{2}.$
$x > 1$.

inequality warning

$x > 1$.

$ax > a$.
only if $a > 0$!

if $a < 0$ you switch the inequality!

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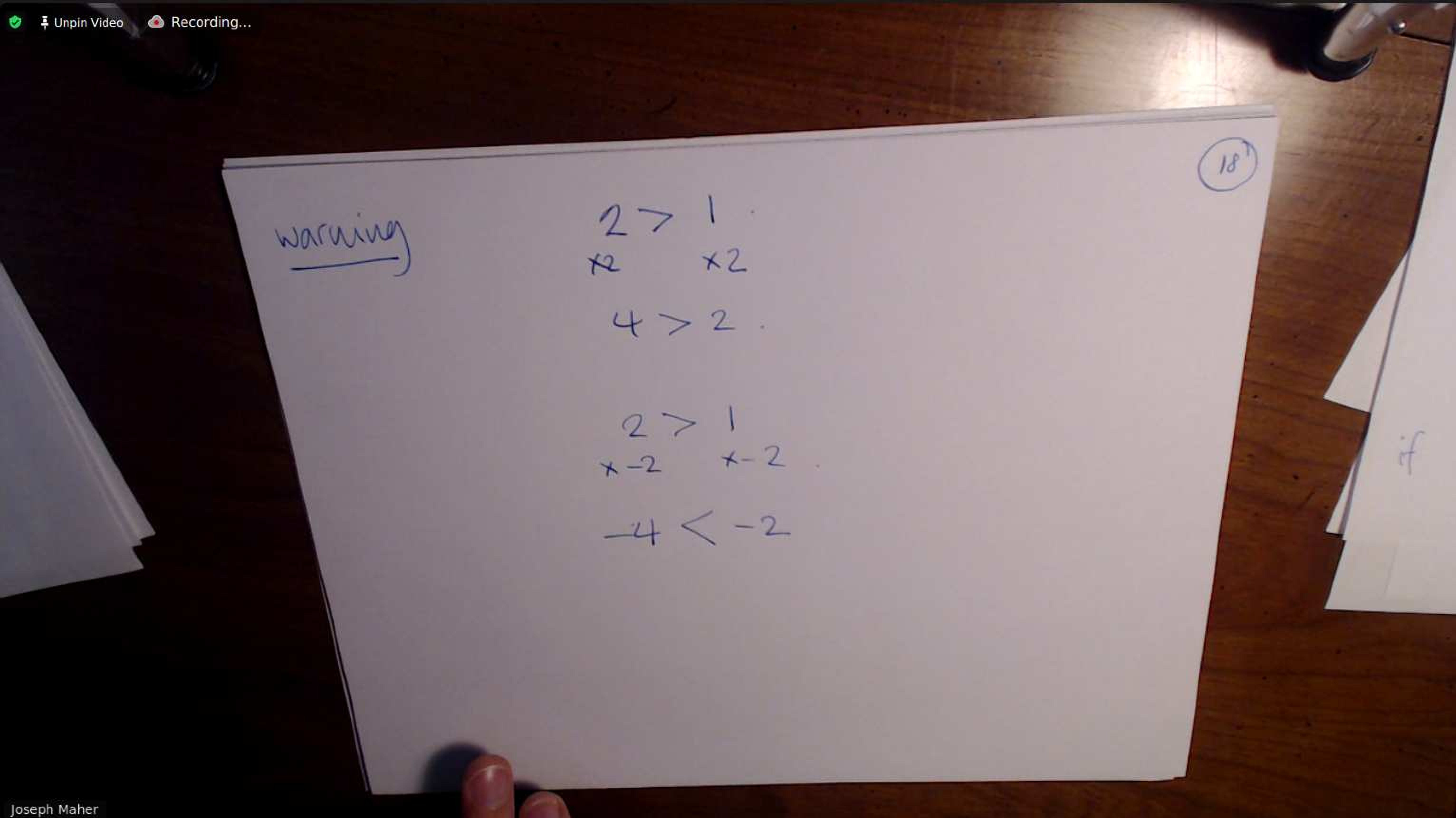

⑱

warning

$2 > 1$.

$\times 2 \quad \times 2$

$4 > 2$.

$2 > 1$

$\times -2 \quad \times -2$.

$-4 < -2$

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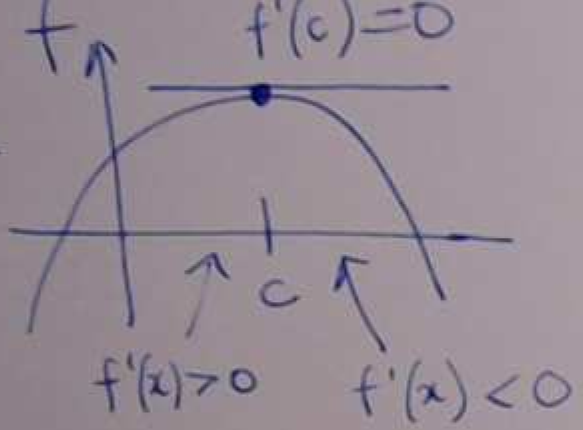
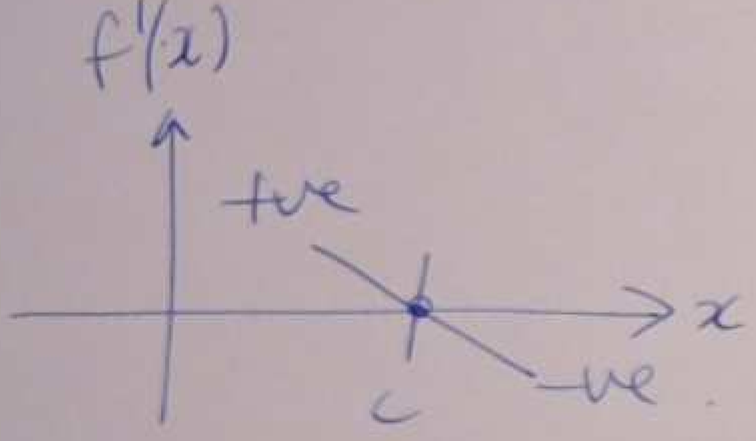
First derivative test

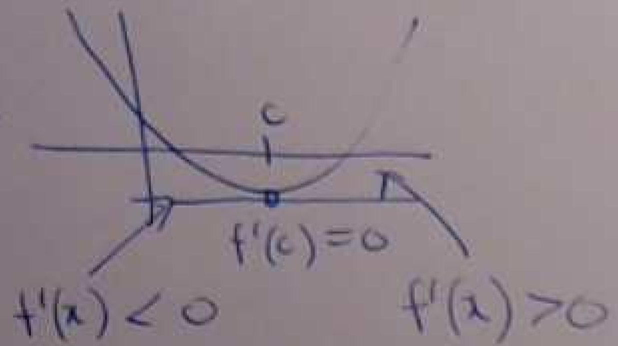
local max

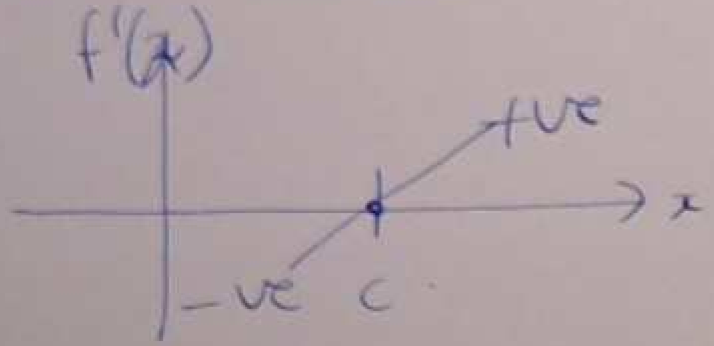




if $f'(x)$ goes from +ve to -ve at c ⇒ local max.

local min





if $f'(x)$ goes from -ve to +ve at c then c is a local min

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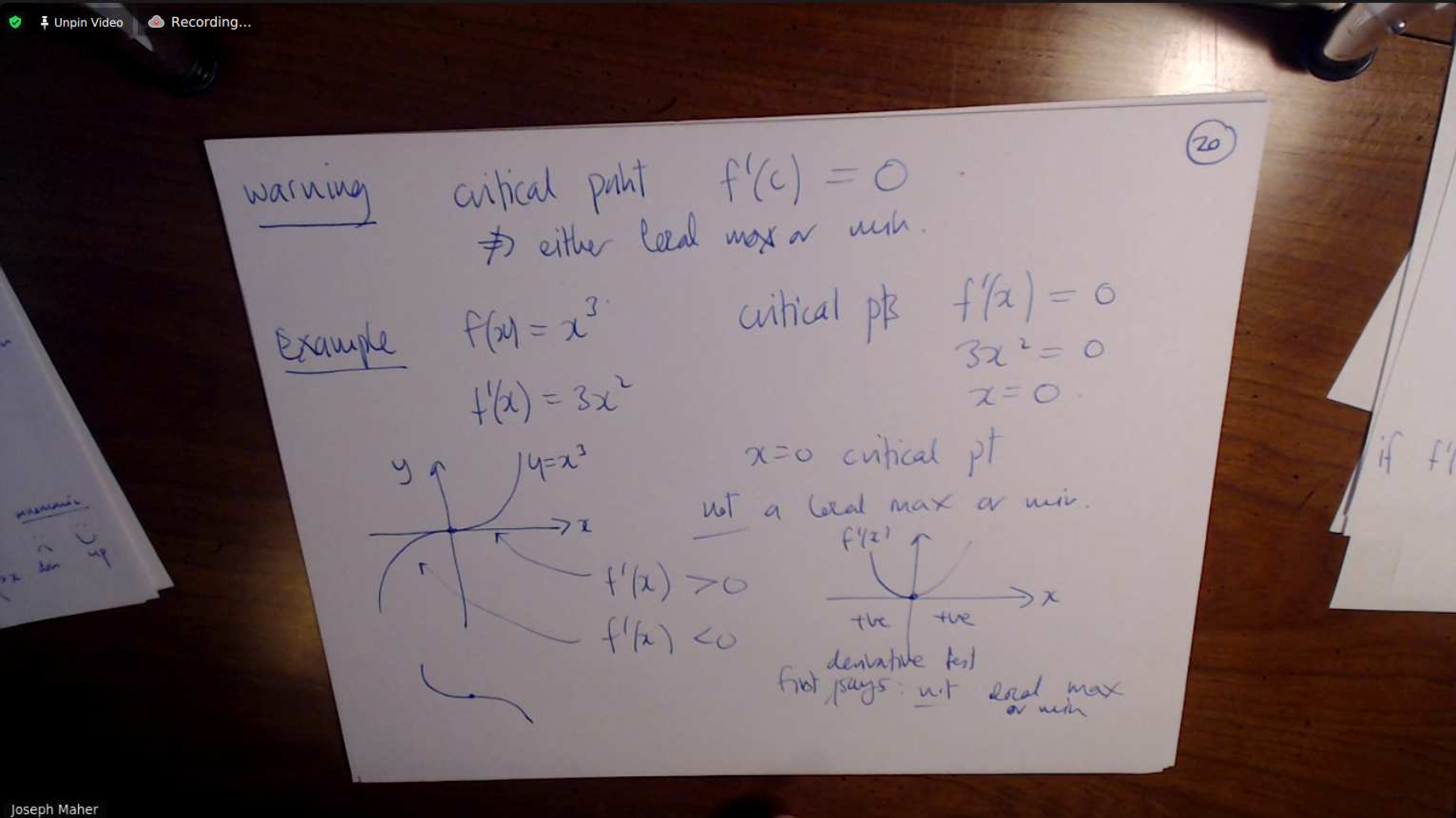

⑳

warning critical point $f'(c) = 0$

$\not\Rightarrow$ either local max or min.

Example $f(x) = x^3$ critical pts $f'(x) = 0$

$f'(x) = 3x^2$

$3x^2 = 0$

$x = 0$.

$x=0$ critical pt

not a local max or min.

$f'(x) > 0$

$f'(x) < 0$

+ve +ve

first derivative test says: not local max or min