$y = 4x^{-3} + 5x^2$ find $\left.\frac{d^2y}{dx^2}\right|_{x=6}$ WW 3.5 Q4 1

$\frac{d}{dx}(x^n) = nx^{n-1}$

$$\frac{dy}{dx} = 4.-3x^{-4} + 5.2x^1$$

$$= -12x^{-4} + 10x.$$

$$\frac{d^2y}{dx^2} = -12.-4x^{-5} + 10.$$

$$= 48x^{-5} + 10.$$

$$\left.\frac{d^2y}{dx^2}\right|_{x=6} = \frac{48}{6^5} + 10.$$

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(2)

WW 3.5 Q5

$y = 4x^{-3/4}$ find $\left.\dfrac{d^2x}{dy^2}\right|_{x=2}$

$\dfrac{d}{dx}(x^n)$

$\shortparallel$

nx^{n-1}

$\dfrac{dy}{dx} = 4 \cdot -\dfrac{3}{4} x^{-\frac{3}{4}-1} = -3x^{-7/4}$

$-\dfrac{3}{4} - 1$

$\dfrac{d^2y}{dx^2} = -3 \cdot -\dfrac{7}{4} x^{-\frac{7}{4}-1}$

$-\dfrac{3}{4} - \dfrac{4}{4} = \dfrac{-7}{4}$

$= \dfrac{21}{4} x^{-11/4}$

$\dfrac{-7}{4} - 1 = \dfrac{-7-4}{4}$

$\left.\dfrac{d^2y}{dx^2}\right|_{x=2} = \dfrac{21}{4} 2^{-11/4}$

$= \dfrac{-11}{4}$

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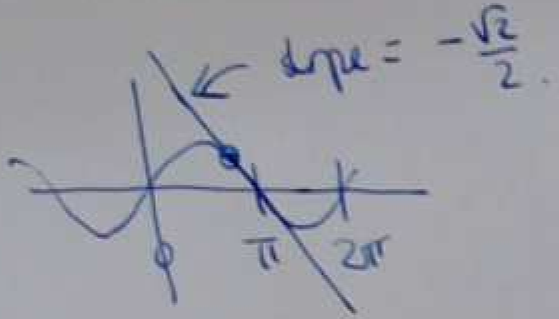
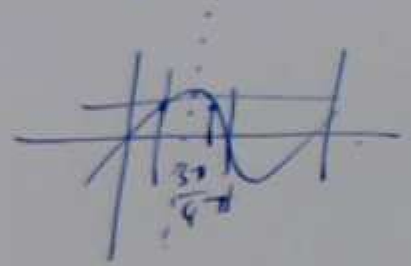
WW 3.6 Q1

$y=\sin(x)$ find tangent line at $x=\frac{3\pi}{4}$



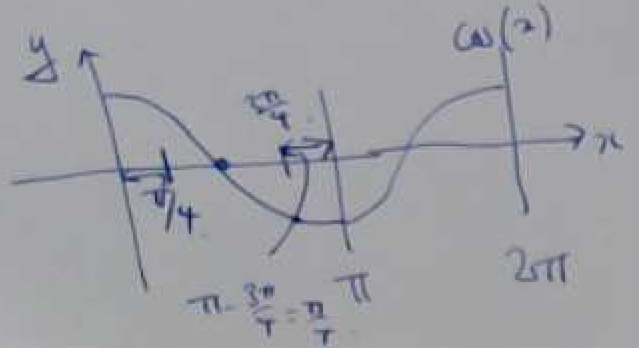
$$\frac{dy}{dx}=\cos(x)$$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

$$\frac{dy}{dx}\left(\frac{3\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)=-\cos\left(\frac{\pi}{4}\right)$$

$$=-\frac{\sqrt{2}}{2}=-\frac{1}{\sqrt{2}}$$



$$\left(\frac{3\pi}{4},\sin\left(\frac{3\pi}{4}\right)\right)=\left(\frac{3\pi}{4},\frac{\sqrt{2}}{2}\right)$$

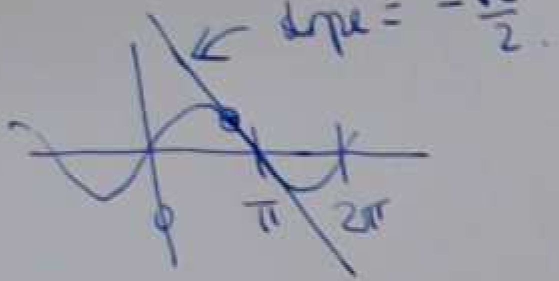
$$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$y-y_0=m(x-x_0)$$

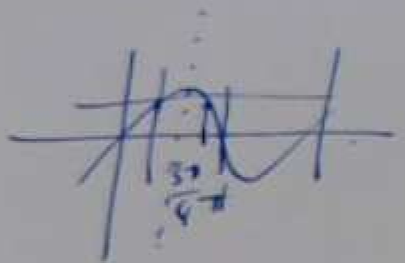
$$y-\frac{\sqrt{2}}{2}=-\frac{\sqrt{2}}{2}\left(x-\frac{3\pi}{4}\right)$$

WW 3.6 Q1

$y = \sin(x)$ find tangent line at $x = \frac{3\pi}{4}$



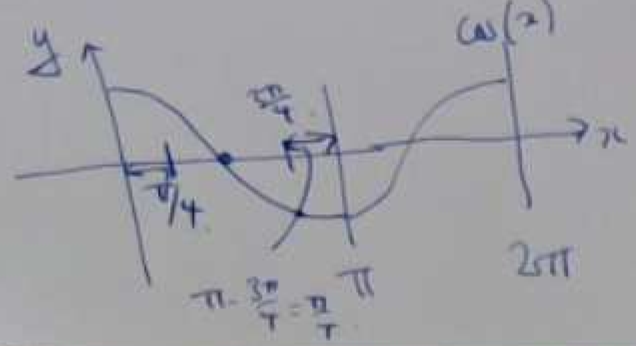
$$\frac{dy}{dx} = \cos(x)$$



θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

$$\frac{dy}{dx}\left(\frac{3\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right)$$

$$= -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$



$$\left(\frac{3\pi}{4}, \sin\left(\frac{3\pi}{4}\right)\right) = \left(\frac{3\pi}{4}, \frac{\sqrt{2}}{2}\right)$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$y - y_0 = m(x - x_0)$$

$$y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{3\pi}{4}\right)$$

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WW 3.5 Q 6

$$y = 3x^4 + ax^3 + bx^2 + cx + d$$

$$\frac{dy}{dx} \equiv y' \equiv y^{(1)}$$

$$y' = 12x^3 + 3ax^2 + 2bx + c$$

$$\frac{d^2y}{dx^2} \equiv y'' = y^{(2)}$$

$$y'' = 36x^2 + 6ax + 2b$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$y''' = y^{(3)} = 72x + 6a$$

$y^{(2)}(0)$

$$y^{(4)} = 72$$

$$y^{(5)} = 0.$$

$$y(0) = d \qquad y'(0) = c \qquad y''(0) = 2b.$$

$$y^{(3)}(0) = 6a \qquad y^{(4)} = 72 \qquad y^{(5)}(0) = 0$$

③

WW 3.6 Q6

$f(x) = -\sin(x)$

$f'(x) = -\cos(x)$

$f''(x) = +\sin(x)$

$f^{(3)}(x) = \cos(x)$

$f^{(4)}(x) = -\sin(x)$

$f^{(5)}(x) = -\cos(x)$

$\vdots$

$f^{(8)}(x) = f^{(4)}(x) = -\sin(x)$

$f^{(37)}(x) = f^{(1)}(x) = -\cos(x)$

$f^{(8)}$ $f^{(37)}$

5

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

1

chain rule.

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(\sin(x)) = \cos(x)$$

$$\frac{d}{dx}(2x) = 2.$$

$$f(x) = \sin(2x)$$

$$f'(x) = \cos(2x) \cdot 2$$

$$f''(x) = -\sin(2x) \cdot \underbrace{2 \cdot 2}_{4}.$$

$$f^{(3)}(x) = -\cos(2x) \cdot 8$$

$$f^{(4)}(x) = 16 \cdot \sin(2x)$$

$$f^{(4n)}(x) = 2^{4n} \sin(2x).$$

$$f^{(4n+1)}(x) = 2^{4n+1} \cos(2x).$$

$$\vdots$$

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$f(x) = \sin^2(x) = (\sin(x))^2$

chain rule

$(f(g(x)))' = f'(g(x))\, g'(x)$

$f'(x) = \boxed{2(\sin(x)).\cos x} = \sin 2x$ double angle

product rule

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$f''(x)$

$(fg)' = f'g + fg'$

$\frac{d}{dx}(x^2) = 2x$

$= 2\cos x . \cos x + 2\sin x . -\sin x$

$= 2\cos^2 x - 2\sin^2 x = 2\cos(x)$

$f(x) = x^2$

$g(x) = \sin(x)$

$f^{(n)}(x) = \cdots$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$\cos(2x) = \cos^2 x - \sin^2 x$

$1 - \sin^2 x$

$= 1 - 2\sin^2 x$

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7

$f(x) = \sin^2(x) = (\sin(x))^2$

chain rule

$(f(g(x)))' = f'(g(x))\, g'(x)$

double angle

$f'(x) = \boxed{2(\sin(x)).\cos x} = \sin 2x$

product rule

$(fg)' = f'g + fg'$

$\frac{d}{dx}(\sin(x)) = \cos(x)$

$\frac{d}{dx}(x^2) = 2x$

$f''(x)$

$= 2\cos x . \cos x + 2\sin x . -\sin x$

$= 2\cos^2 x - 2\sin^2 x = 2\cos(2x)$

$f(x) = x^2$

$g(x) = \sin(x)$

$f^{(3)}(x) = \dots$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$\cos(2x) = \cos^2 x - \sin^2 x$

$1 - \sin^2 x$

$= 1 - 2\sin^2 x$

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WW 3.7 Q 1

$$y = (x+\sin x)^5$$

chain rule

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$f(x) = x^5$

$g(x) = x + \sin x$

$f'(x) = 5x^4$

$g'(x) = 1 + \cos x$

$$5(x+\sin x)^4 \cdot (1+\cos x)$$

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WW 3.7 Q2

$$(\cos(u))' = -\sin(u(x)) \cdot u'(x)$$

9

$u(x) = 7 - x^2 \qquad u'(x) = -2x$

$u(x) = x^{-3} \qquad u'(x) = -3x^{-4}$

$u(x) = \tan x \qquad u'(x) = \sec^2 x$

$\frac{d}{dx}(\cos(x))$

$=$

$-\sin(x)$

$-\sin(7-x^2) \cdot (-2x)$

$-\sin(x^{-3}) \cdot (-3x^{-4}) = \frac{3\sin\left(\frac{1}{x^3}\right)}{x^4}$

$-\sin(\tan(x)) \cdot \sec^2 x$

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WW 3.7 Q5 $f(x) = \cos(x) \qquad g(x) = x^8 + 1$

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$(g(f(x)))' = g'(f(x)) \cdot f'(x)$

$f'(x) = -\sin(x)$

$g'(x) = 8x^7$

$\rightarrow -\sin(x^8+1) \cdot (8x^7)$

$\rightarrow 8(\cos(x))^7 \cdot (-\sin(x))$

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WW 3.7 Q6 $\left(\frac{x+81}{x-81}\right)^{20}$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x) \quad ⊛$$

$f(x) = x^{20}$ $\qquad f'(x) = 20x^{19}$

$g(x) = \frac{x+81}{x-81}$ $\qquad g'(x) = \frac{(x-81)\cdot 1 - (x+81)\cdot 1}{(x-81)^2}$

quotient rule

$$\left(\frac{a}{b}\right)' = \frac{ba' - ab'}{b^2}$$

$a(x) = x+81 \qquad a'(x) = 1$

$b(x) = x-81 \qquad b'(x) = 1$

$$g'(x) = \frac{-162}{(x-81)^2}$$

⊛ $20\left(\frac{x+81}{x-81}\right)^{19} \cdot \frac{-162}{(x-81)^2}$

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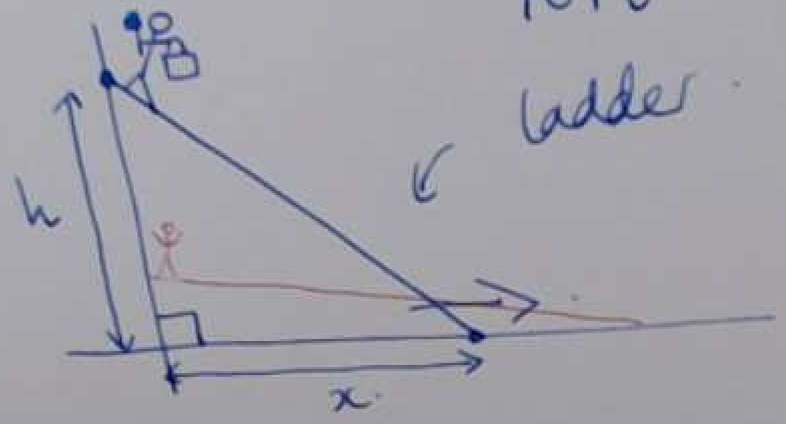
§ 3.10 Related rates

Example



16ft ladder.

$x(t)$ = distance from wall to foot of ladder

$h(t)$ = height of top of the ladder.

$$x^2+h^2 = 16^2.$$

Q: if $\frac{dx}{dt} = 3\text{ft/sec}$ what is $\frac{dh}{dt}$?

$$(x(t))^2 + (h(t))^2 = 16^2.$$

$$2(x(t))\underbrace{x'(t)}_{3} + 2(h(t))\cdot h'(t) = 0.$$

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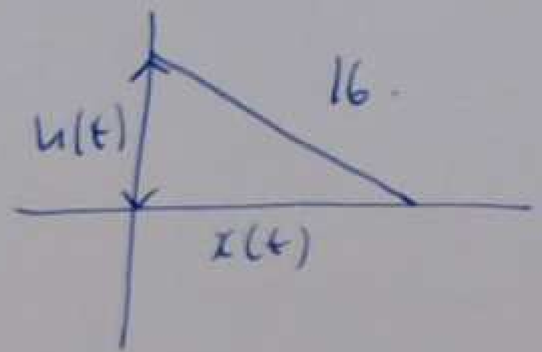
$x^2+h^2=16$ (13)

$$\cancel{2}x(t)\cdot 3 + \cancel{2}\cdot h(t)\frac{dh}{dt} = 0$$



$$\frac{dh}{dt} = \frac{-3x(t)}{h(t)}$$

assume $x(0) = 5$

$\frac{dx}{dt} = 3 \Rightarrow x(t) = 5 + 3t$

$h^2 + x^2 = 16$

$h^2 = 16 - x^2$

$h = \sqrt{16 - x^2}$

$$\frac{dh}{dt} = \frac{-3(5+3t)}{\sqrt{16-(5+3t)^2}}$$

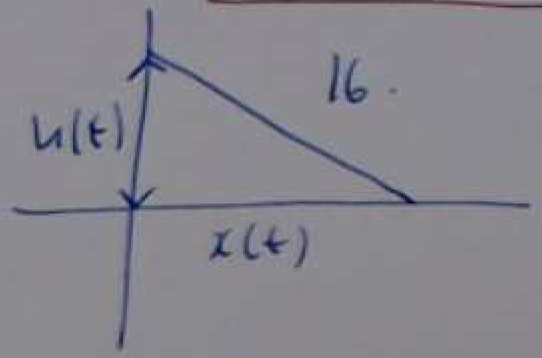
exercise:

$\frac{dh}{dt} \to \infty$ as $t \to$ gets large

$\boxed{x^2+h^2=16}$ 13

$$\cancel{2}x(t)\cdot 3+\cancel{2}\cdot h(t)\frac{dh}{dt}=0$$

$$\frac{dh}{dt}=\frac{-3x(t)}{h(t)}$$



assume $x(0)=5$

$\frac{dx}{dt}=3 \Rightarrow x(t)=5+3t$

$h^2+x^2=16$

$h^2=16-x^2$

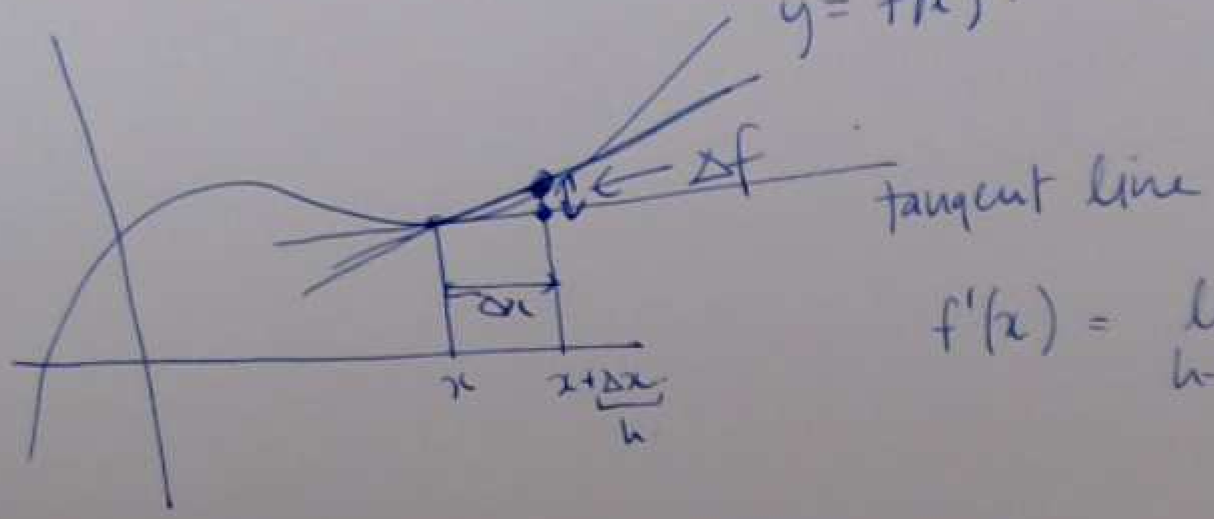
$h=\sqrt{16-x^2}$

$$\frac{dh}{dt}=\frac{-3(5+3t)}{\sqrt{16-(5+3t)^2}}$$

exercise:

$\frac{dh}{dt}\to\infty$ as $t\to$ gets large.

§4.1 Linear approximation



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If $f(x)$ is differentiable at x, and Δx is small, then $f(x+\Delta x) \simeq f(x) + \underbrace{\Delta x \cdot f'(x)}$

change of f is $\Delta f \simeq \Delta x \cdot f'(x)$.

Example

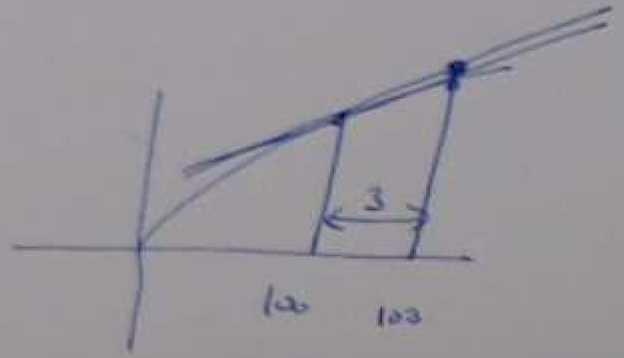
estimate $\sqrt{103}$

$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f(100) = \sqrt{100} = 10.$$

$$f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$$

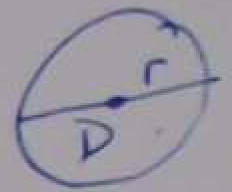


$$f(103) \approx f(100) + f'(100) \cdot 3.$$

$$\approx 10 + \frac{1}{20} \cdot 3.$$

$$\approx 10.15.$$

Example make an 18" pizza



$2r = D$

If your diameter is accurate to ± 0.4 in
how much pizza do you gain or lose?

$$A = \pi r^2 = \pi \left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$$

$$A'(D) = \frac{2\pi D}{4} = \frac{\pi D}{2} \qquad \Delta D = 0.4$$

$$\Delta A \approx A'(18) \cdot \Delta D = \frac{\pi \cdot 18}{2} \cdot 0.4 = 9\pi \, 0.4$$
$$\approx 11 \text{ in}^2$$

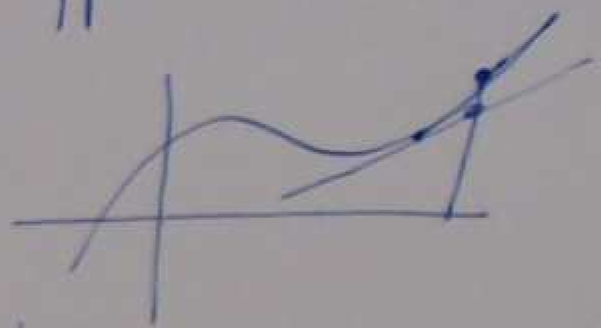
Q is this good or bad?

Absolute error 11 in^2.

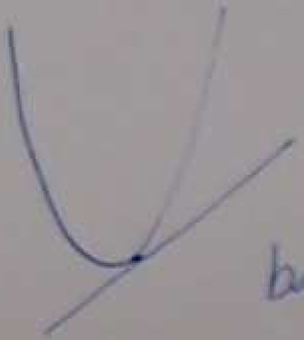
$$\text{percentage error} = \left| \frac{\text{absolute error}}{\text{actual value}} \right| \times 100.$$

$$\frac{11}{\underbrace{\pi \frac{18^2}{4}}_{\text{actual area of } D=18 \text{ pizza}}} \times 100 = 0.27\%.$$

<u>observation</u> when is the linear approximation a good approximation



good approx.



bad approx.

$|f''(x)|$ small

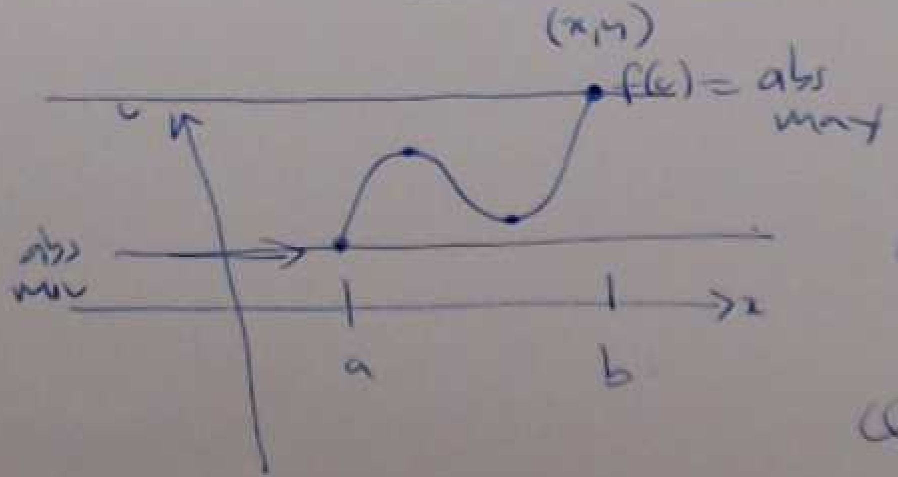
$|f''(x)|$ large

§4.2 Extreme values

suppose $f(x)$ is defined on a closed interval $[a,b]$.

Def^{n} $f(c)$ is the absolute max if $f(c) \geq f(x)$ for all $x \in [a,b]$

$f(c)$ is the absolute min if $f(c) \leq f(x)$ for all $x \in [a,b]$



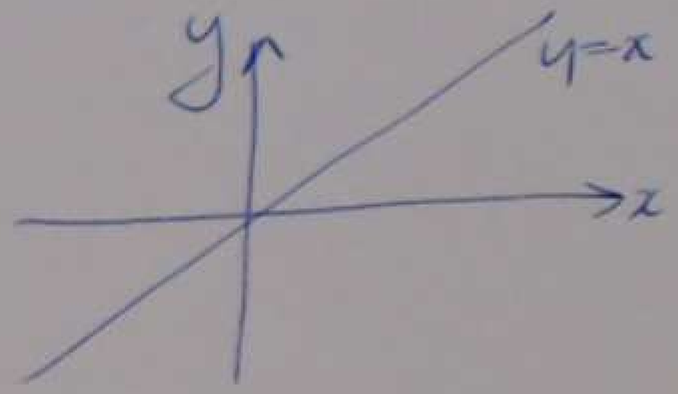
note

Q: where is the max (local/abs) ← x-value

Q: what is the max ← y-value

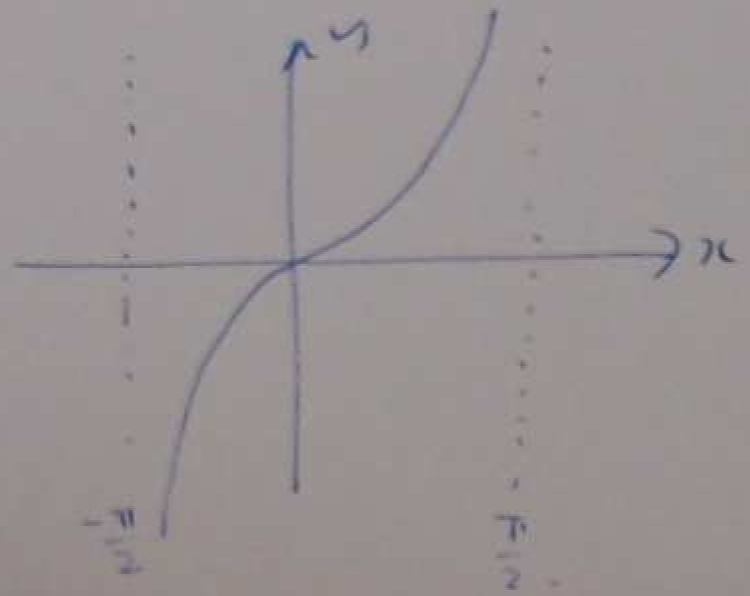
<u>warning</u> some functions do not have any max or min.

<u>Examples</u> $f: \mathbb{R} \to \mathbb{R}$
$x \mapsto x$



$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$

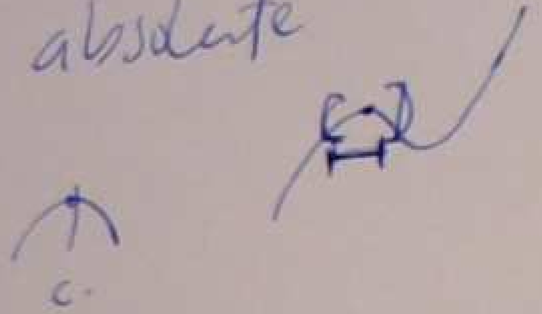
$x \longmapsto \tan(x)$



for max/min need <u>closed</u> interval

$[a, b]$.

Thm If $f(x)$ is continuous on a closed bounded interval $[a,b]$ then $f(x)$ has an absolute max and an absolute min.

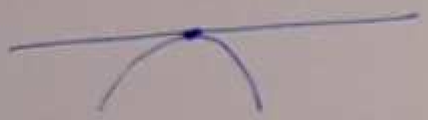


Defn $f(x)$ has a local max at $x=c$ if there is a small interval containing c s.t. $f(c)$ is a max on this interval

$f(x)$ has a local min at $x=c$ if there is a small interval containing c s.t. $f(c)$ is a min on this interval

Def^{n} We say that c is a critical point if $f'(c) = 0$ (or is undefined)

Thm If c is a local max or local min then $f'(c) = 0$

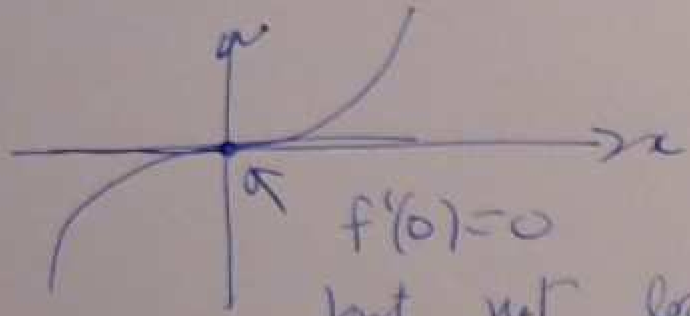


warning: $f'(c) = 0 \not\Rightarrow$ local max or min

Example. $y = x^3$.

$\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx}(0) = 0$



$f'(0) = 0$
but not local max or min.