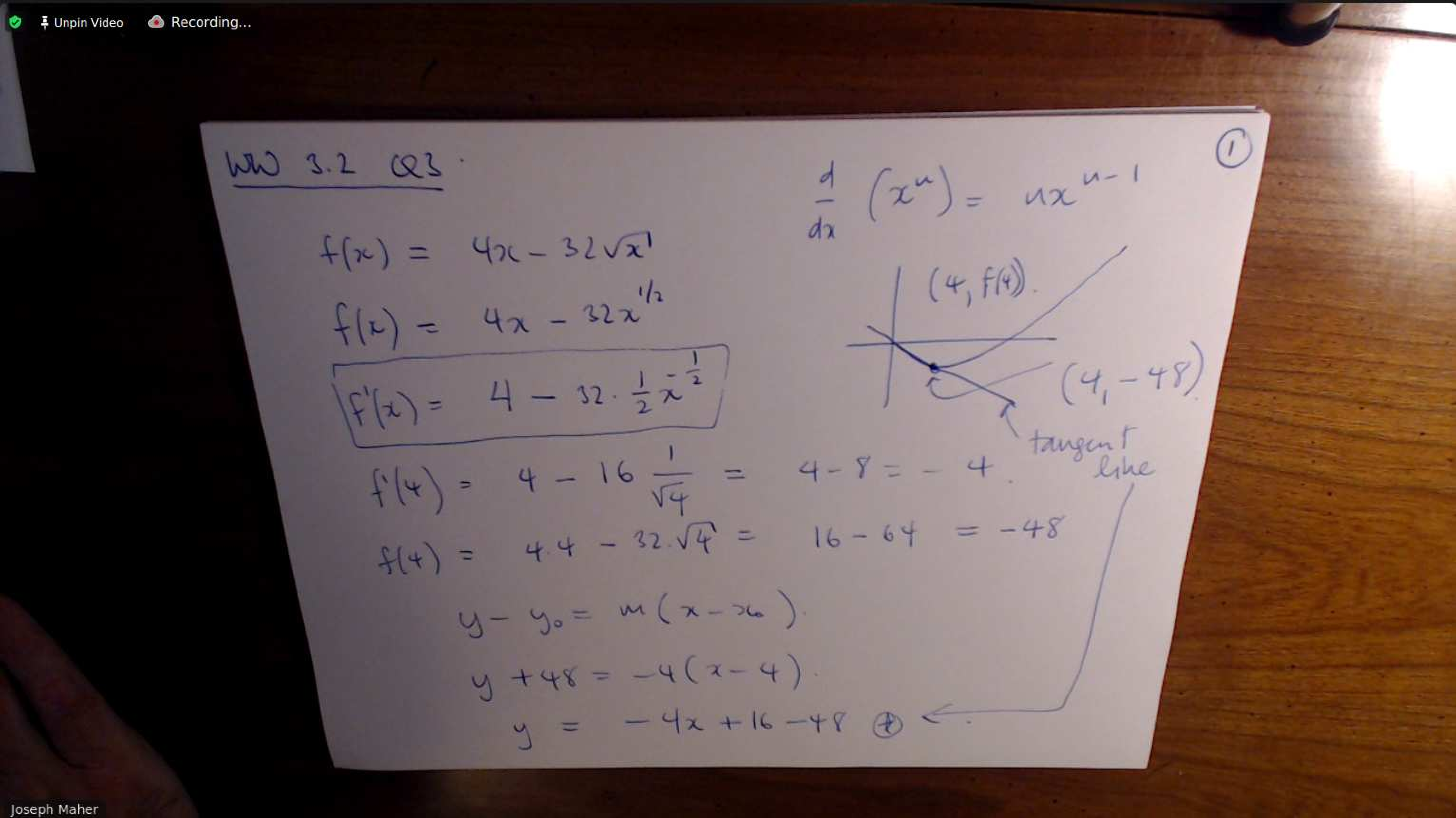


①

WW 3.2 Q3.

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$f(x) = 4x - 32\sqrt{x}$$

$$f(x) = 4x - 32x^{1/2}$$

$$\boxed{f'(x) = 4 - 32 \cdot \frac{1}{2} x^{-\frac{1}{2}}}$$

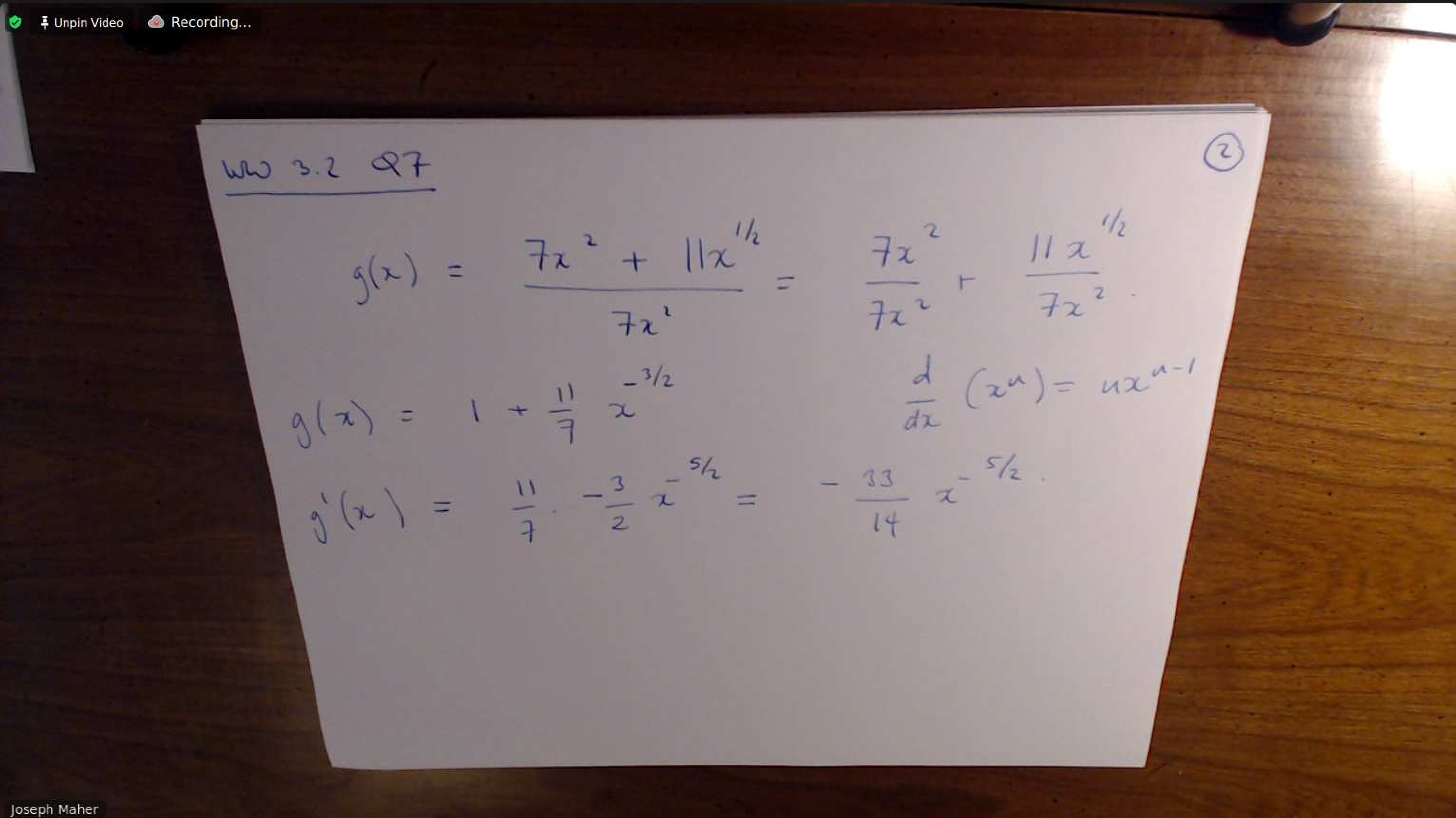
$$f'(4) = 4 - 16\frac{1}{\sqrt{4}} = 4 - 8 = -4.$$

$$f(4) = 4 \cdot 4 - 32 \cdot \sqrt{4} = 16 - 64 = -48$$

$$y - y_0 = m(x - x_0).$$

$$y + 48 = -4(x - 4).$$

$$y = -4x + 16 - 48 \quad (*) \leftarrow$$



Unpin Video Recording...

WW 3.2 Q7

$$g(x) = \frac{7x^2 + 11x^{1/2}}{7x^2} = \frac{7x^2}{7x^2} + \frac{11x^{1/2}}{7x^2}.$$

$$g(x) = 1 + \frac{11}{7}x^{-3/2}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$g'(x) = \frac{11}{7} \cdot -\frac{3}{2}x^{-5/2} = -\frac{33}{14}x^{-5/2}.$$

Joseph Maher

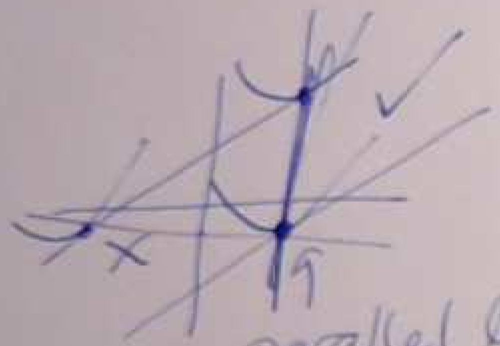
Unpin Video Recording...

WW 3.2 Q9

$\frac{d}{dx}(x^n) = nx^{n-1}$

$f(x) = x^6 \qquad f'(x) = 6x^5$

$g(x) = x^7 \qquad g'(x) = 7x^6.$



parallel lines have the same slope.

$f'(x) = g'(x)$ ← solve for x

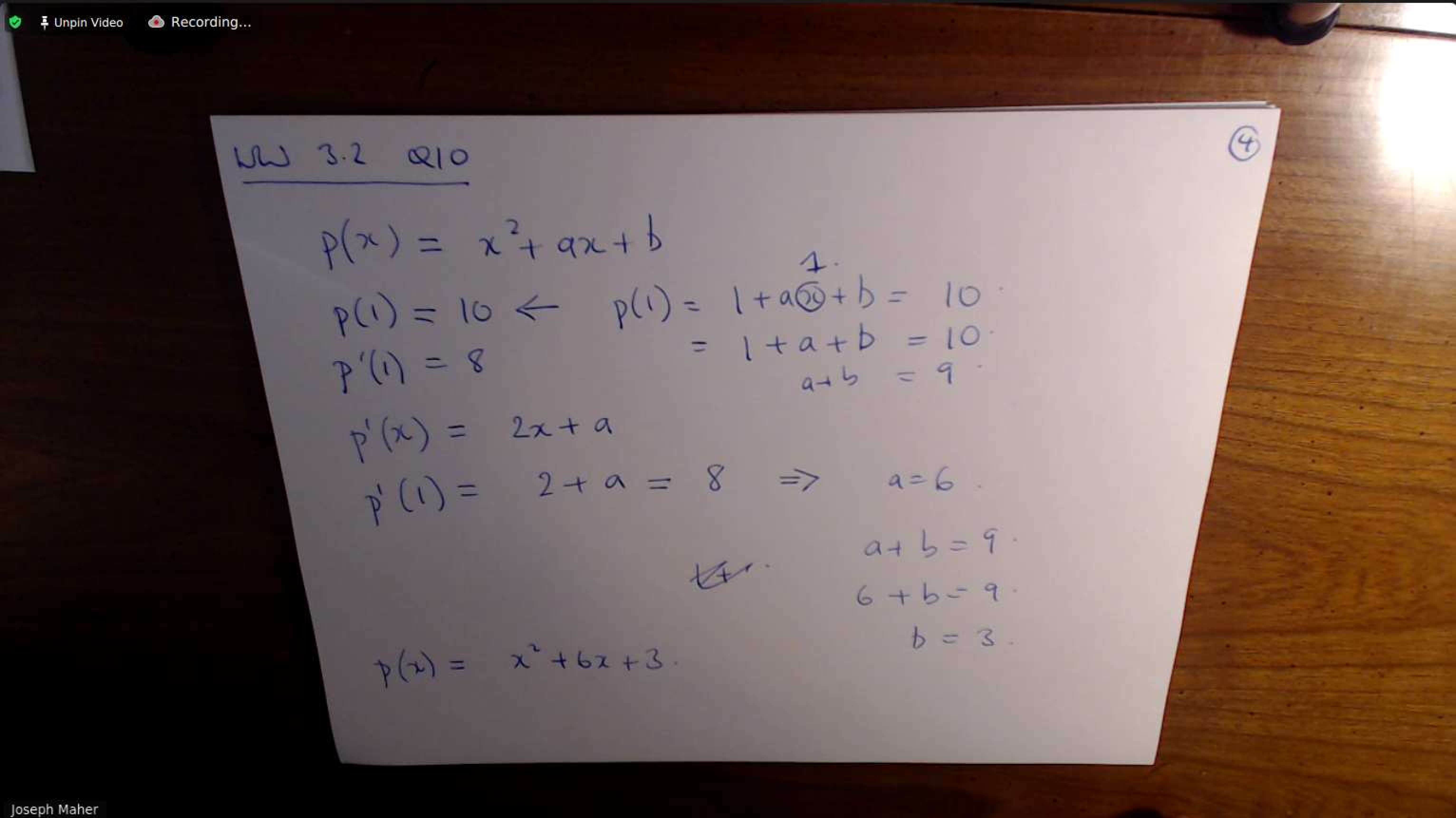
$6x^5 = 7x^6.$

$7x^6 - 6x^5 = 0$

$x^5(7x-6) = 0$

$x = 0 \qquad x = \frac{6}{7}$

Joseph Maher



HW 3.2 Q10

④

$$p(x) = x^2 + ax + b$$

$$p(1) = 10 \leftarrow \quad p(1) = 1 + a\overset{1.}{\boxed{x}} + b = 10.$$

$$= 1 + a + b = 10.$$

$$a + b = 9.$$

$$p'(1) = 8$$

$$p'(x) = 2x + a$$

$$p'(1) = 2 + a = 8 \Rightarrow a = 6.$$

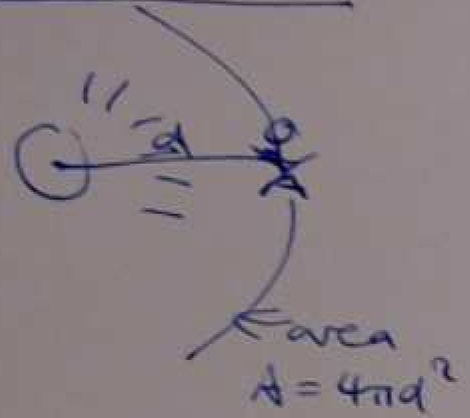
$$a + b = 9.$$

$$6 + b = 9.$$

$$b = 3.$$

$$p(x) = x^2 + 6x + 3.$$

HW 3.2 Q11

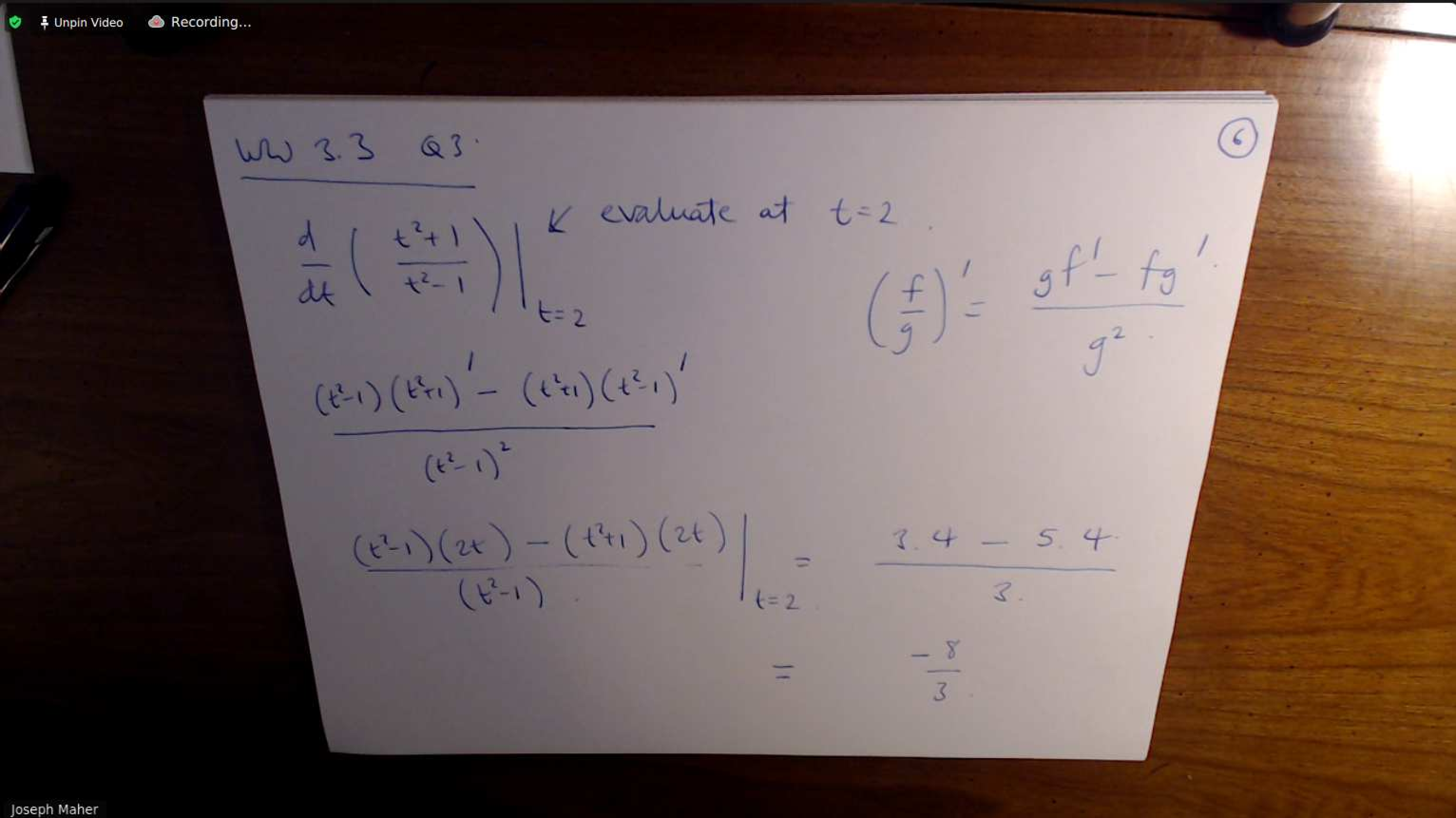


$$b = \frac{L}{4\pi d^2} \qquad L = 3.9\times10^{26}$$
$$d = 1.5\times10^{11}$$

$$b(d) = \frac{L}{4\pi}\, d^{-2}$$

$$b'(d) = \frac{L}{4\pi} \cdot -2\, d^{-3} = -\frac{L}{2\pi} \cdot \frac{1}{d^3}$$

$$= -\frac{3.9\times10^{26}}{2\pi} \times \frac{1}{(1.5\times10^{11})^3}$$



WW 3.3 Q3.

$$\frac{d}{dt}\left(\frac{t^2+1}{t^2-1}\right)\Bigg|_{t=2}$$

↙ evaluate at $t=2$.

$$\left(\frac{f}{g}\right)' = \frac{gf'-fg'}{g^2}.$$

$$\frac{(t^2-1)(t^2+1)' - (t^2+1)(t^2-1)'}{(t^2-1)^2}$$

$$\left.\frac{(t^2-1)(2t)-(t^2+1)(2t)}{(t^2-1)}\right|_{t=2} = \frac{3\cdot 4 - 5\cdot 4}{3}.$$

$$= -\frac{8}{3}$$

WW 3.3 Q5

$$f(x) = (\sqrt{x}+62)(\sqrt{x}-61)$$

$$(fg)' = f'g + fg'.$$

$$f'(x) = (\sqrt{x}+62)'(\sqrt{x}-61) + \cancel{(\sqrt{x}+62)(\sqrt{x}-61)'}$$

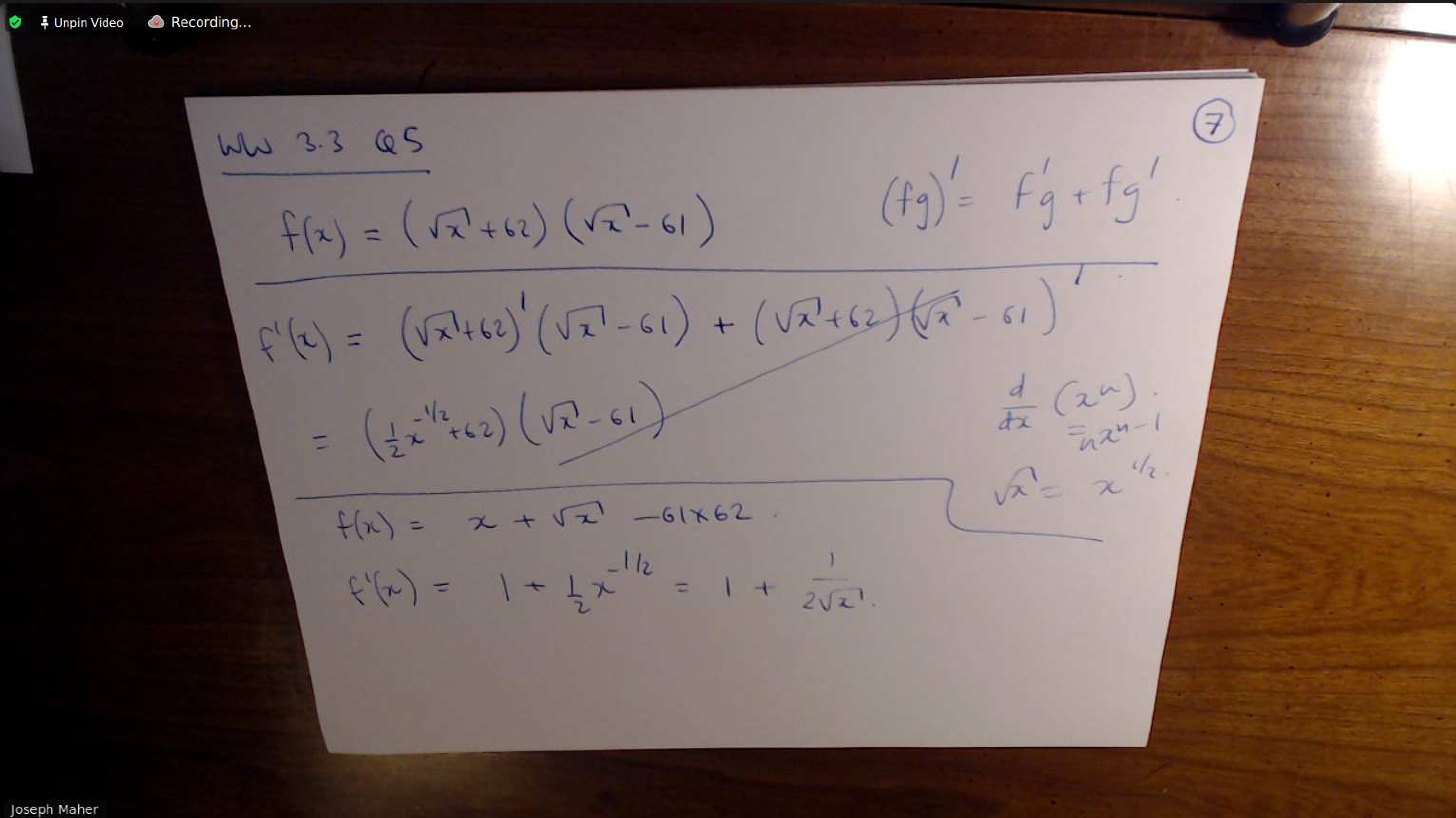
$$= \left(\tfrac{1}{2}x^{-1/2}+62\right)\cancel{(\sqrt{x}-61)}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\sqrt{x} = x^{1/2}.$$

$$f(x) = x + \sqrt{x} - 61\times 62.$$

$$f'(x) = 1 + \tfrac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}.$$



⑦

WW 3.3 Q5

$$f(x) = (\sqrt{x}+62)(\sqrt{x}-61) \qquad (fg)' = f'g + fg'.$$

$$f'(x) = (\sqrt{x}+62)'(\sqrt{x}-61) + (\sqrt{x}+62)(\sqrt{x}-61)'.$$

$$= \left(\tfrac{1}{2}x^{-1/2}+62\right)(\sqrt{x}-61)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\sqrt{x} = x^{1/2}$$

$$f(x) = x + \sqrt{x} - 61 \times 62.$$

$$f'(x) = 1 + \tfrac{1}{2}x^{-1/2} = 1 + \frac{1}{2\sqrt{x}}.$$

WW 3.3 Q9

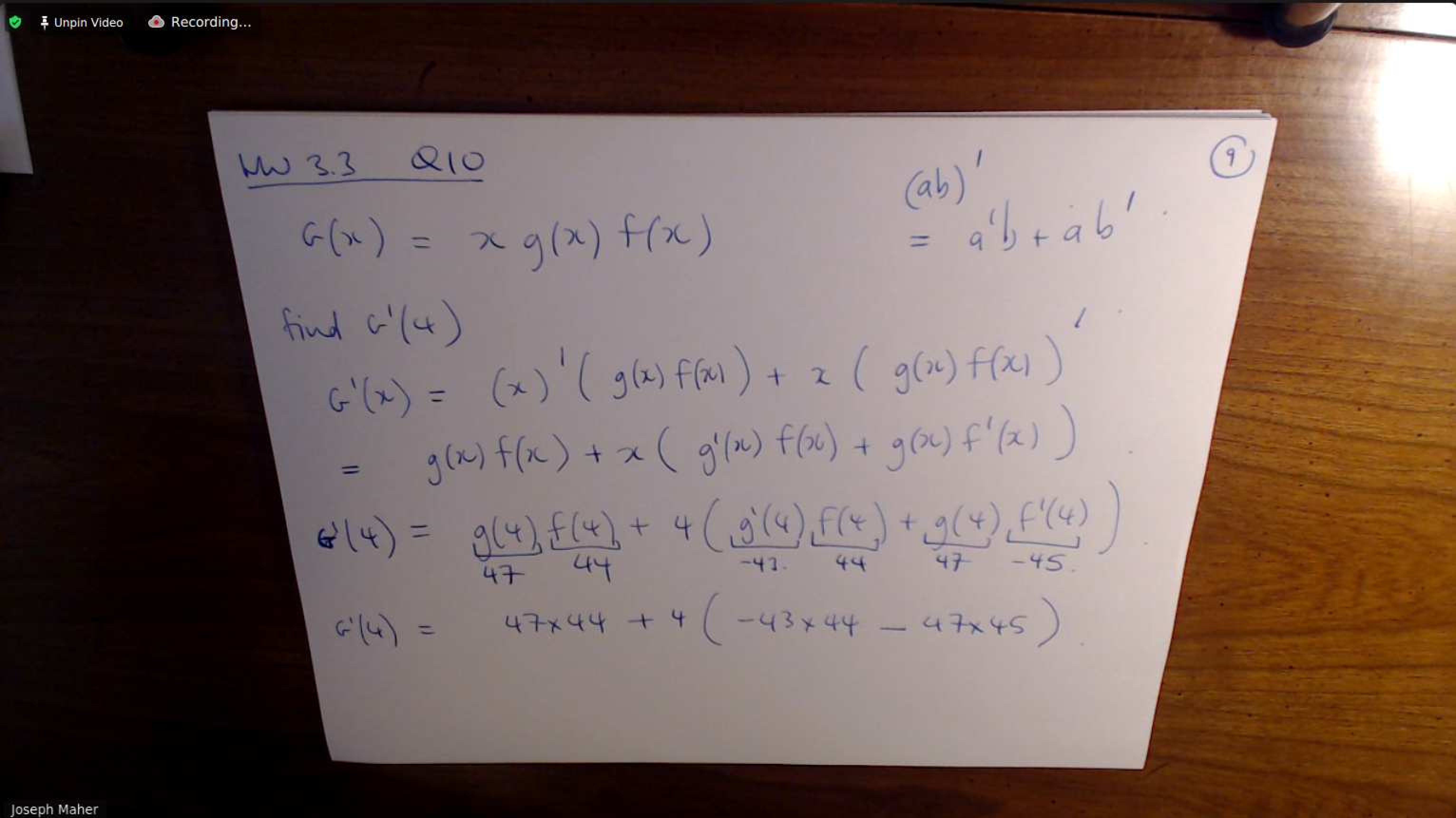
⑧

$$f(x) = 7x\,R(x) + 7Q(x) \qquad (fg)' = f'g + fg'$$

$$f'(x) = (7x)'\,R(x) + 7x\,(R(x))' + 7Q'(x).$$

$$f'(x) = 7R(x) + 7x\,\underbrace{R'(x)}_{P(x)} + 7\,\underbrace{Q'(x)}_{-R(x)}.$$

$$f'(x) = \cancel{7R(x)} + 7x\,P(x) - \cancel{7R(x)} = 7x\,P(x).$$



HW 3.3 Q10

⑨

$$(ab)' = a'b + ab'$$

$$G(x) = x\,g(x)\,f(x)$$

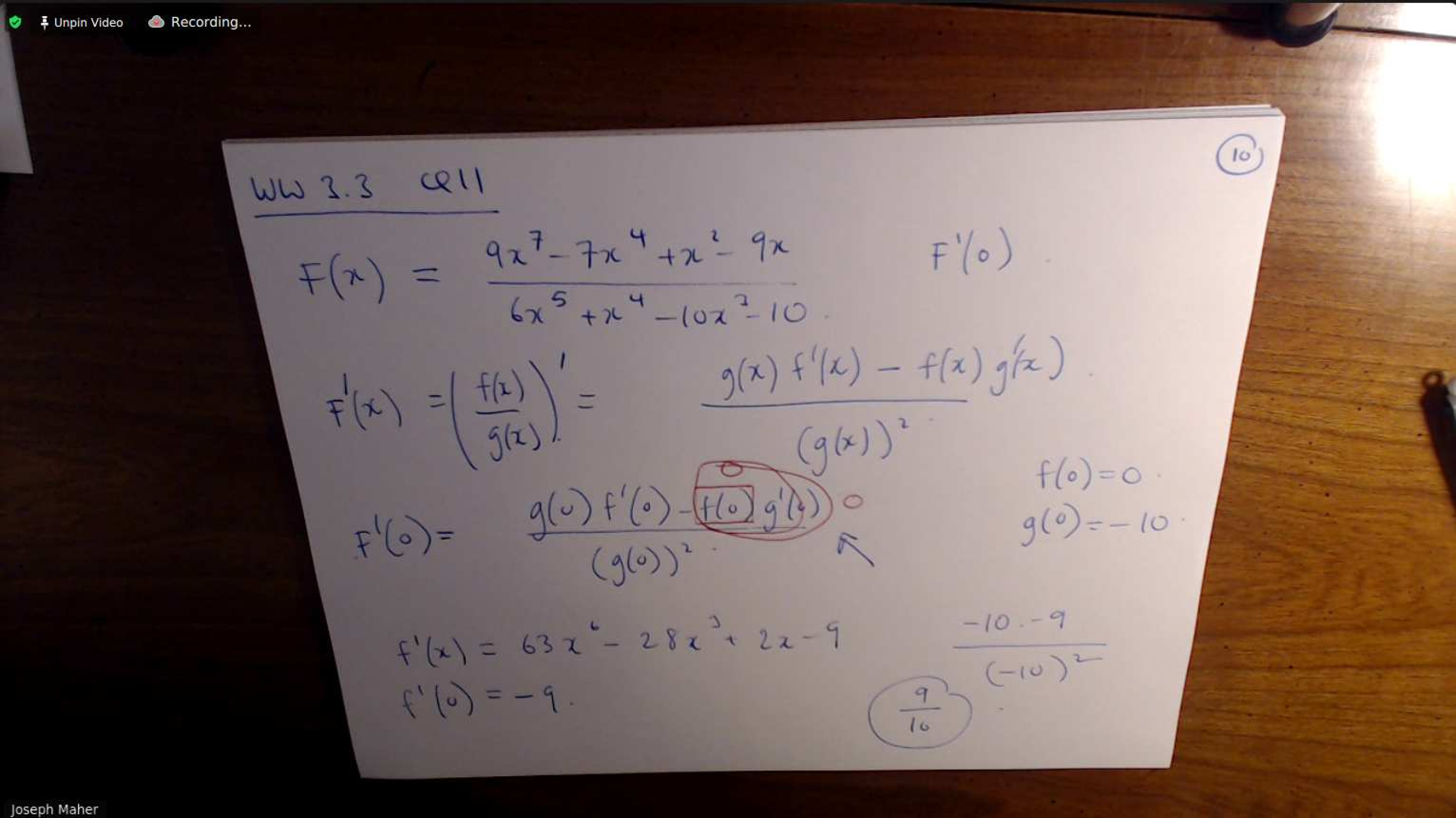
find $G'(4)$

$$G'(x) = (x)'\big(g(x)f(x)\big) + x\big(g(x)f(x)\big)'$$

$$= g(x)f(x) + x\big(g'(x)f(x) + g(x)f'(x)\big)$$

$$G'(4) = \underbrace{g(4)}_{47}\underbrace{f(4)}_{44} + 4\Big(\underbrace{g'(4)}_{-43}\underbrace{f(4)}_{44} + \underbrace{g(4)}_{47}\underbrace{f'(4)}_{-45}\Big)$$

$$G'(4) = 47 \times 44 + 4(-43 \times 44 - 47 \times 45)$$



WW 3.3 Q11

$$F(x) = \frac{9x^7 - 7x^4 + x^2 - 9x}{6x^5 + x^4 - 10x^2 - 10}$$

$F'(0)$

$$F'(x) = \left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$$

$$F'(0) = \frac{g(0)\,f'(0) - \overset{0}{\boxed{f(0)}}\,g'(0)}{(g(0))^2} \quad 0$$

↖

$f(0) = 0$

$g(0) = -10$

$f'(x) = 63x^6 - 28x^3 + 2x - 9$

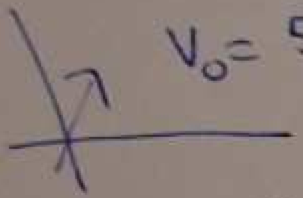
$f'(0) = -9$

$$\frac{-10 \cdot -9}{(-10)^2}$$

$$\frac{9}{10}$$

HW3.4 Q2

$\downarrow -9.8 m/s^2$



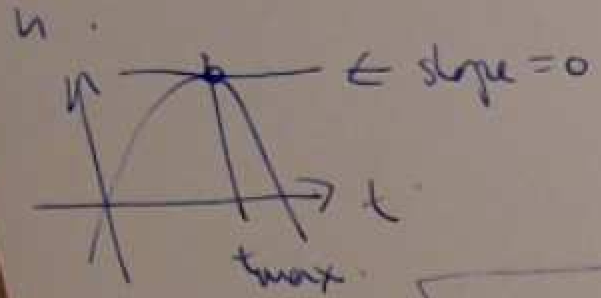
$V_0 = 52$ m/s max height?

$-9.8 \frac{1}{2} t^2 + 52t + 0 \leftarrow h_0$

$h(t)$

$h'(t) = v(t)$ $-9.8t + \boxed{V_0} = 52$

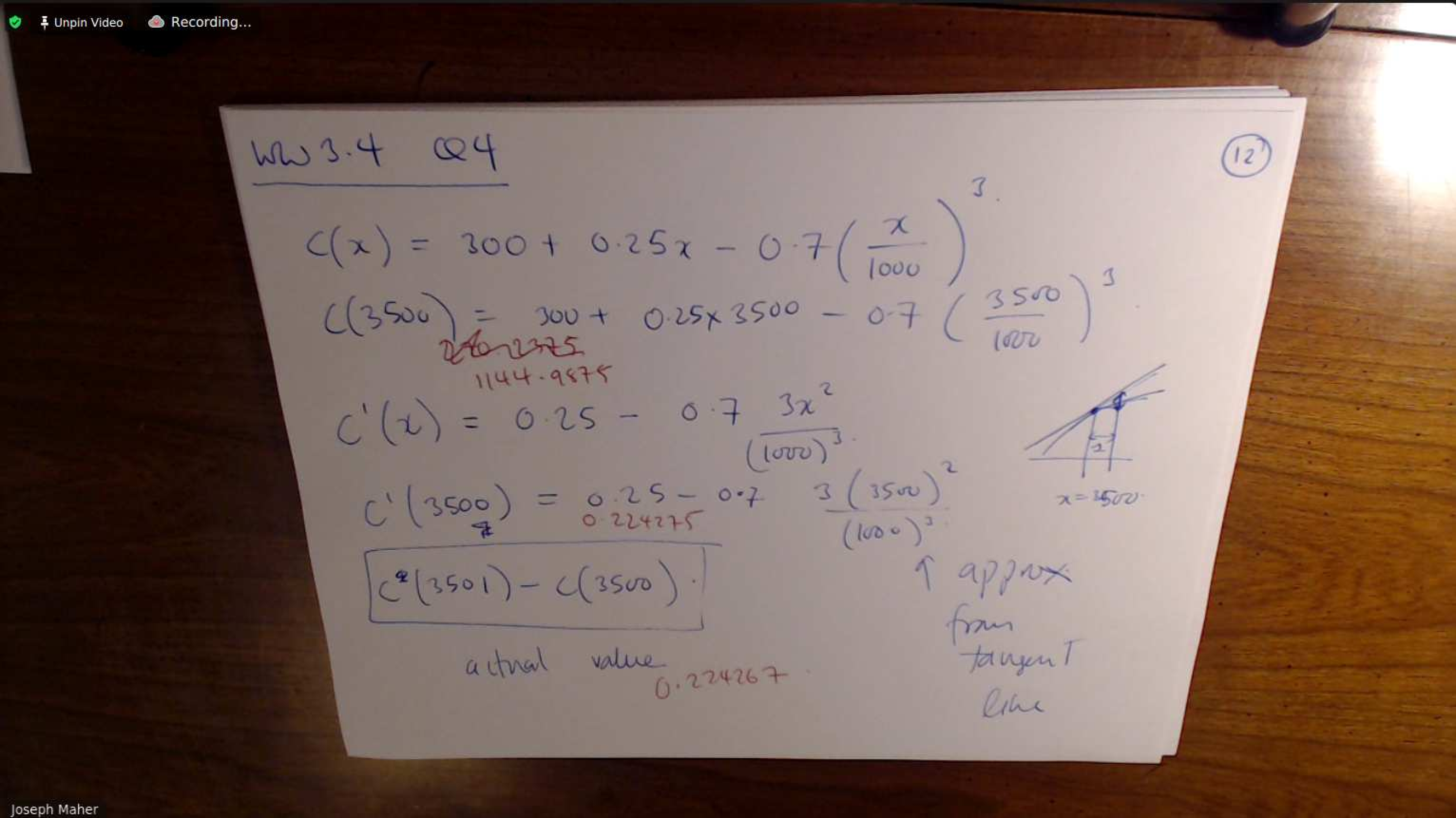
$h''(t) = v'(t) = a(t) = -9.8 = g$



$h'(t) = -9.8t + 52 = 0$

$= t_{max} = \frac{52}{9.8}$

$$\boxed{h(t_{max}) = -9.8 \cdot \frac{1}{2} \left(\frac{52}{9.8}\right)^2 + 52 \cdot \frac{52}{9.8}}$$



WW 3.4 Q4

$$C(x) = 300 + 0.25x - 0.7\left(\frac{x}{1000}\right)^3.$$

$$C(3500) = 300 + 0.25 \times 3500 - 0.7\left(\frac{3500}{1000}\right)^3.$$

~~240.2375~~

1144.9875

$$C'(x) = 0.25 - 0.7\,\frac{3x^2}{(1000)^3}.$$

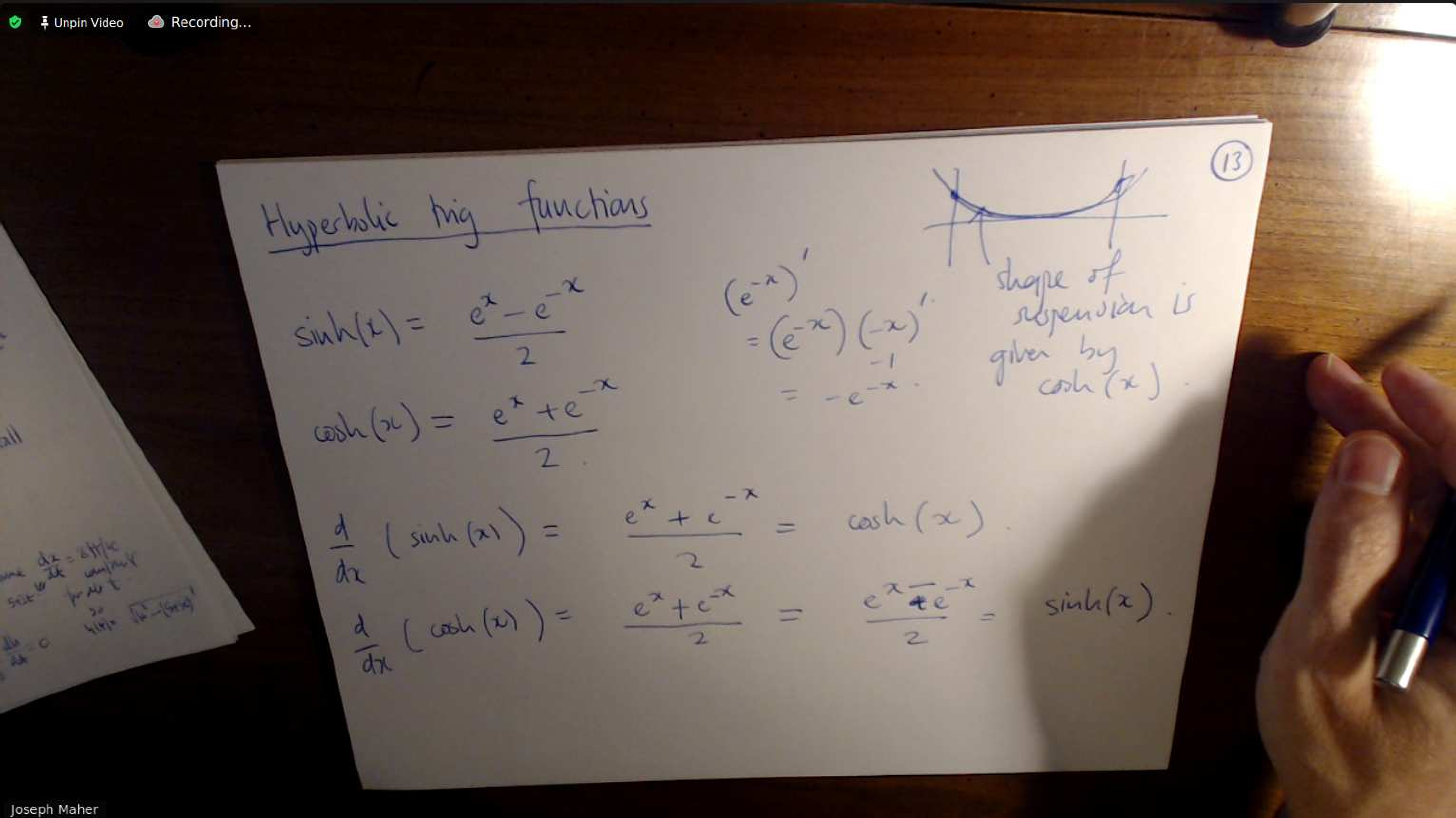
$$C'(3500) = 0.25 - 0.7\;\frac{3(3500)^2}{(1000)^3}$$

0.224275

↑ approx from tangent line

$$\boxed{C(3501) - C(3500).}$$

actual value 0.224267.



Hyperbolic trig functions

shape of suspension is given by $\cosh(x)$.

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}.$$

$$(e^{-x})' = (e^{-x})\underset{-1}{(-x)'} = -e^{-x}.$$

$$\frac{d}{dx}(\sinh(x)) = \frac{e^x + e^{-x}}{2} = \cosh(x).$$

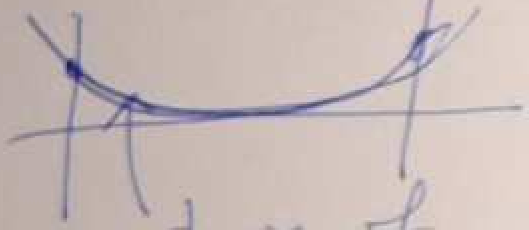
$$\frac{d}{dx}(\cosh(x)) = \frac{e^x + e^{-x}}{2} = \frac{e^x - e^{-x}}{2} = \sinh(x).$$

Hyperbolic trig functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}.$$

$$(e^{-x})' = (e^{-x})\underbrace{(-x)'}_{-1} = -e^{-x}.$$



shape of suspension is given by $\cosh(x)$.

$$\frac{d}{dx}(\sinh(x)) = \frac{e^x + c^{-x}}{2} = \cosh(x).$$

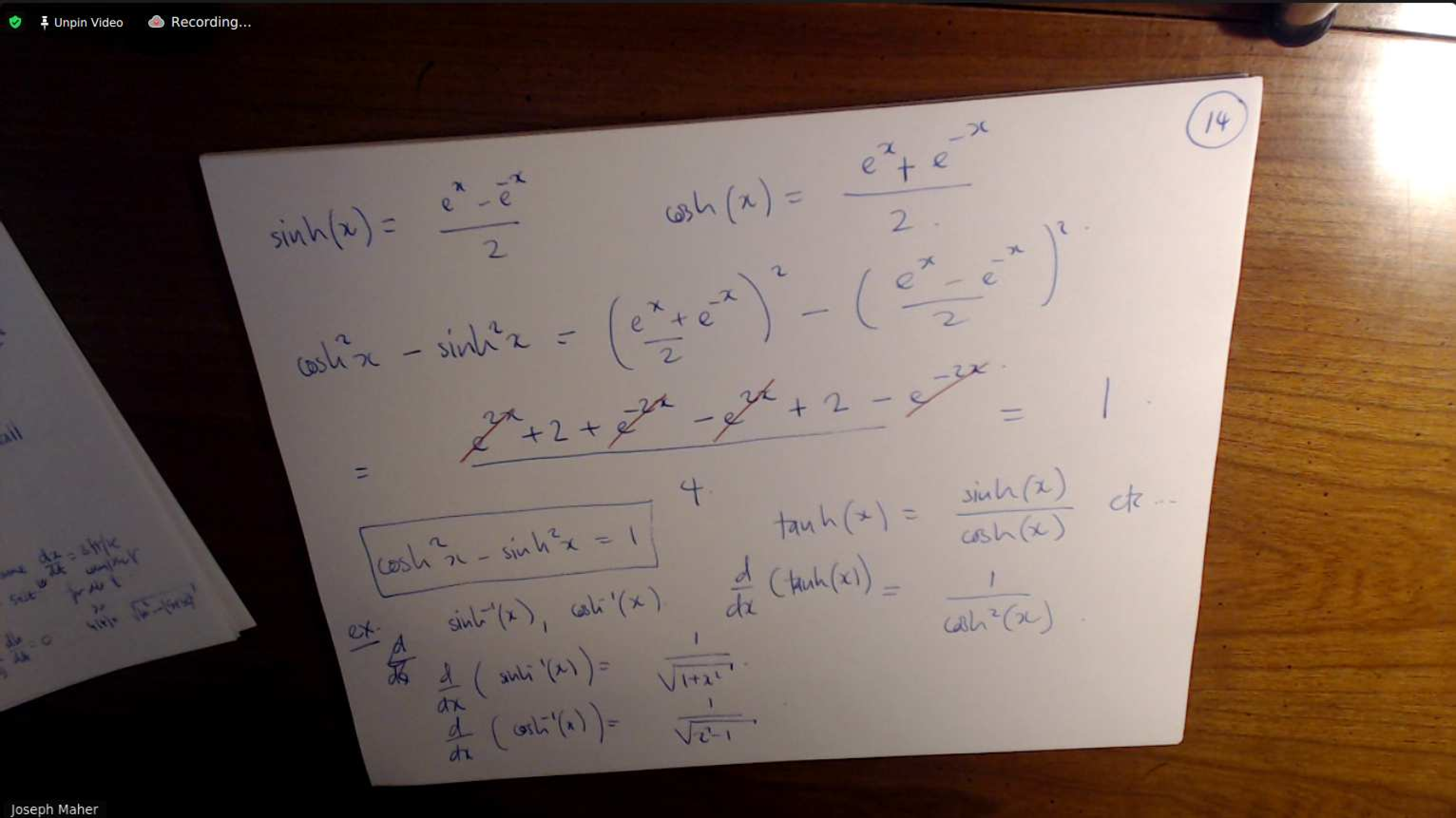
$$\frac{d}{dx}(\cosh(x)) = \frac{e^x + c^{-x}}{2} = \frac{e^x \overset{-}{\cancel{+}} e^{-x}}{2} = \sinh(x).$$

$(e^x)' = e^x$

$(\sinh(x))'$
$\cosh(x)$
$(\cosh(x))'$
$\sinh(x)$

$\sin(x)$
$\cos(x)$
$-\sin(x)$
$-\cos(x)$
$\sin(x)$

Q3 periodic?



$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

$$\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2.$$

$$= \frac{\cancel{e^{2x}} + 2 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2 - \cancel{e^{-2x}}}{4} = 1.$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ etc...

$\frac{d}{dx}(\tanh(x)) = \frac{1}{\cosh^2(x)}$.

$\sinh^{-1}(x)$, $\cosh^{-1}(x)$

ex. $\cancel{\frac{d}{dx}}$

$\frac{d}{dx}(\sinh^{-1}(x)) = \frac{1}{\sqrt{1+x^2}}$.

$\frac{d}{dx}(\cosh^{-1}(x)) = \frac{1}{\sqrt{x^2-1}}$.

§ 3.10 Related rates.

Example blowing up a balloon

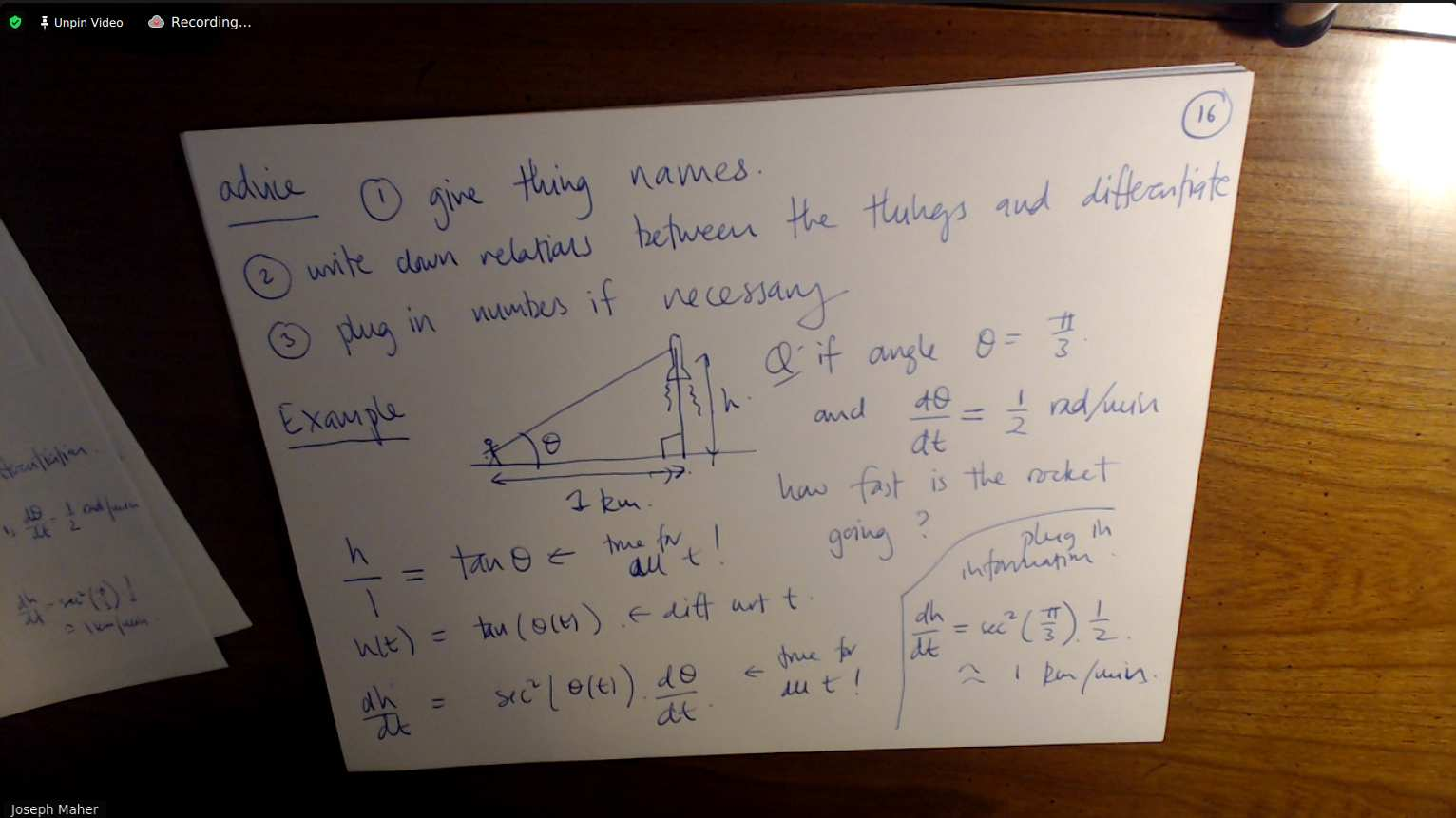
$$V = \frac{4}{3}\pi r^3.$$

$$V(t) = \frac{4}{3}\pi (r(t))^3. \leftarrow$$ differentiate wrt t

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3(r(t))^2 \cdot \frac{dr}{dt} \quad \circledast$$

suppose $\frac{dV}{dt} = 1 \text{ ft}^3/\text{min}$. how fast is the radius growing when $r = 1$ ft?

$$1 = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}(1)^2 \frac{dr}{dt} = \frac{1}{4\pi} \text{ ft/min}$$



advice ① give thing names.

② write down relations between the things and differentiate

③ plug in numbers if necessary

Example

Q: if angle $\theta = \frac{\pi}{3}$ and $\frac{d\theta}{dt} = \frac{1}{2}$ rad/min

how fast is the rocket going?

$\frac{h}{1} = \tan\theta$ ← true for all t!

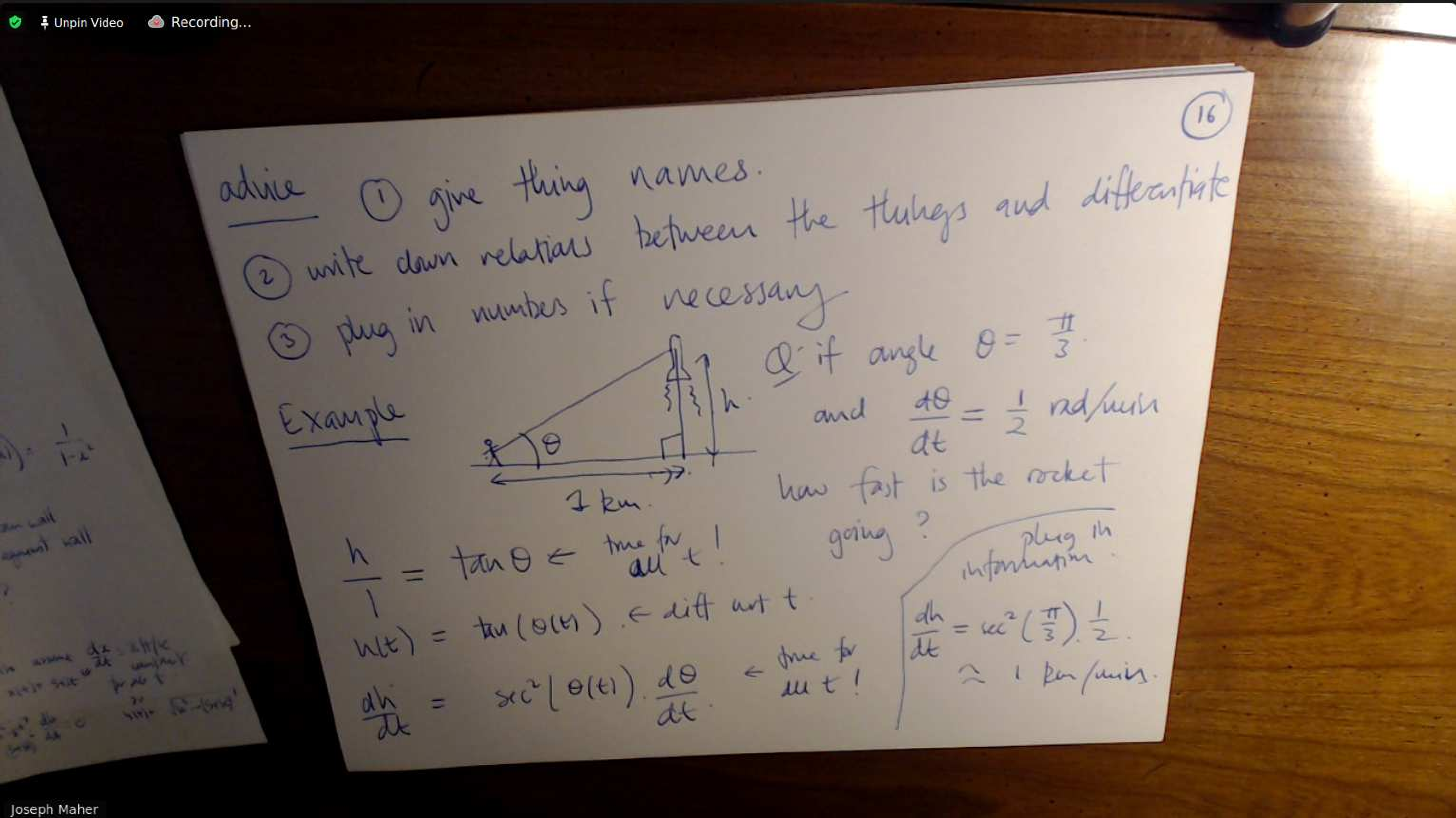
$h(t) = \tan(\theta(t))$ ← diff wrt t

$\frac{dh}{dt} = \sec^2(\theta(t)) \frac{d\theta}{dt}$ ← true for all t!

plug in information

$\frac{dh}{dt} = \sec^2(\frac{\pi}{3}) \frac{1}{2}$

≈ 1 km/min.



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Example

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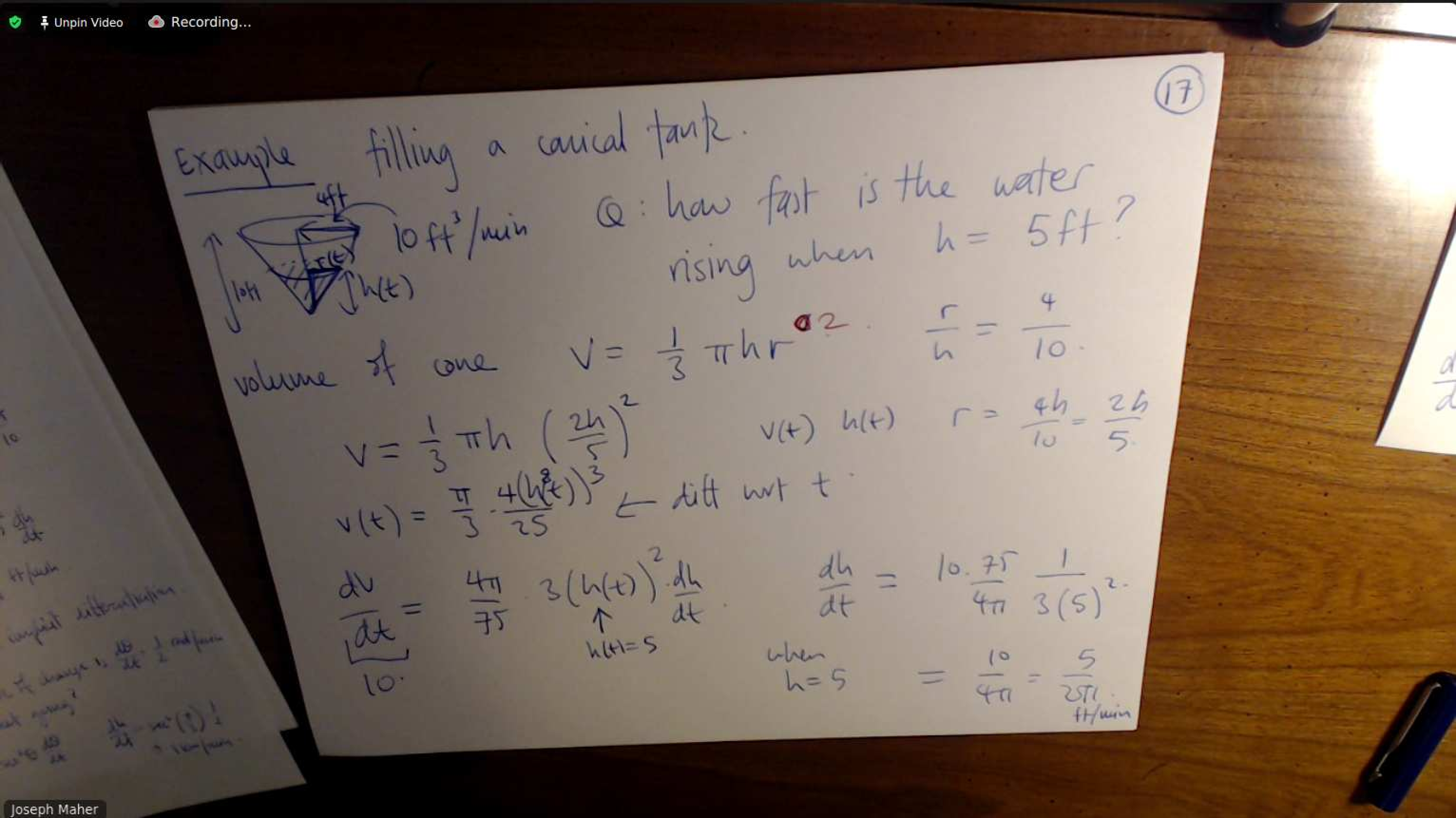
$\frac{dh}{dt} = \sec^2(\theta(t)) \frac{d\theta}{dt}$ ← true for all t!

plug in information

$\frac{dh}{dt} = \sec^2\left(\frac{\pi}{3}\right) \frac{1}{2}$

$\simeq 1$ km/min.



⑰

Example filling a conical tank.

Q: how fast is the water rising when $h = 5\text{ft}$?

volume of cone $V = \frac{1}{3}\pi h r^2$. $\frac{r}{h} = \frac{4}{10}$.

$$V = \frac{1}{3}\pi h \left(\frac{2h}{5}\right)^2 \qquad V(t) \quad h(t) \qquad r = \frac{4h}{10} = \frac{2h}{5}$$

$$V(t) = \frac{\pi}{3} \cdot \frac{4(h(t))^3}{25} \leftarrow \text{diff wrt } t$$

$$\underbrace{\frac{dV}{dt}}_{10} = \frac{4\pi}{75} \cdot 3(\underset{h(t)=5}{\underset{\uparrow}{h(t)}})^2 \cdot \frac{dh}{dt} \qquad \frac{dh}{dt} = 10 \cdot \frac{75}{4\pi} \frac{1}{3(5)^2}$$

$$\text{when } h=5 \quad = \frac{10}{4\pi} = \frac{5}{2\pi} \text{ ft/min}$$