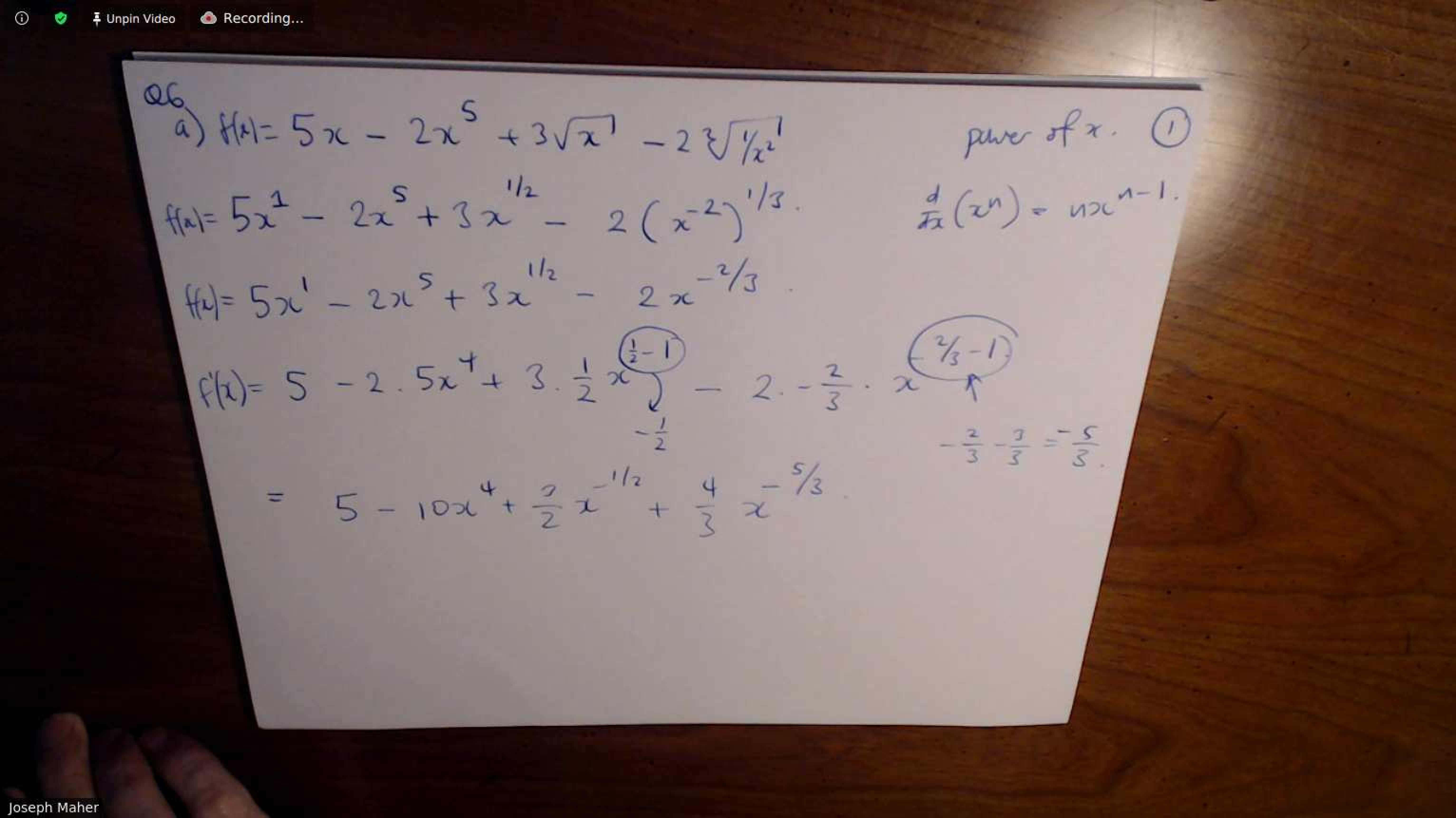


Q6

a) $f(x) = 5x - 2x^5 + 3\sqrt{x} - 2\sqrt[3]{1/x^2}$ power of x. ①

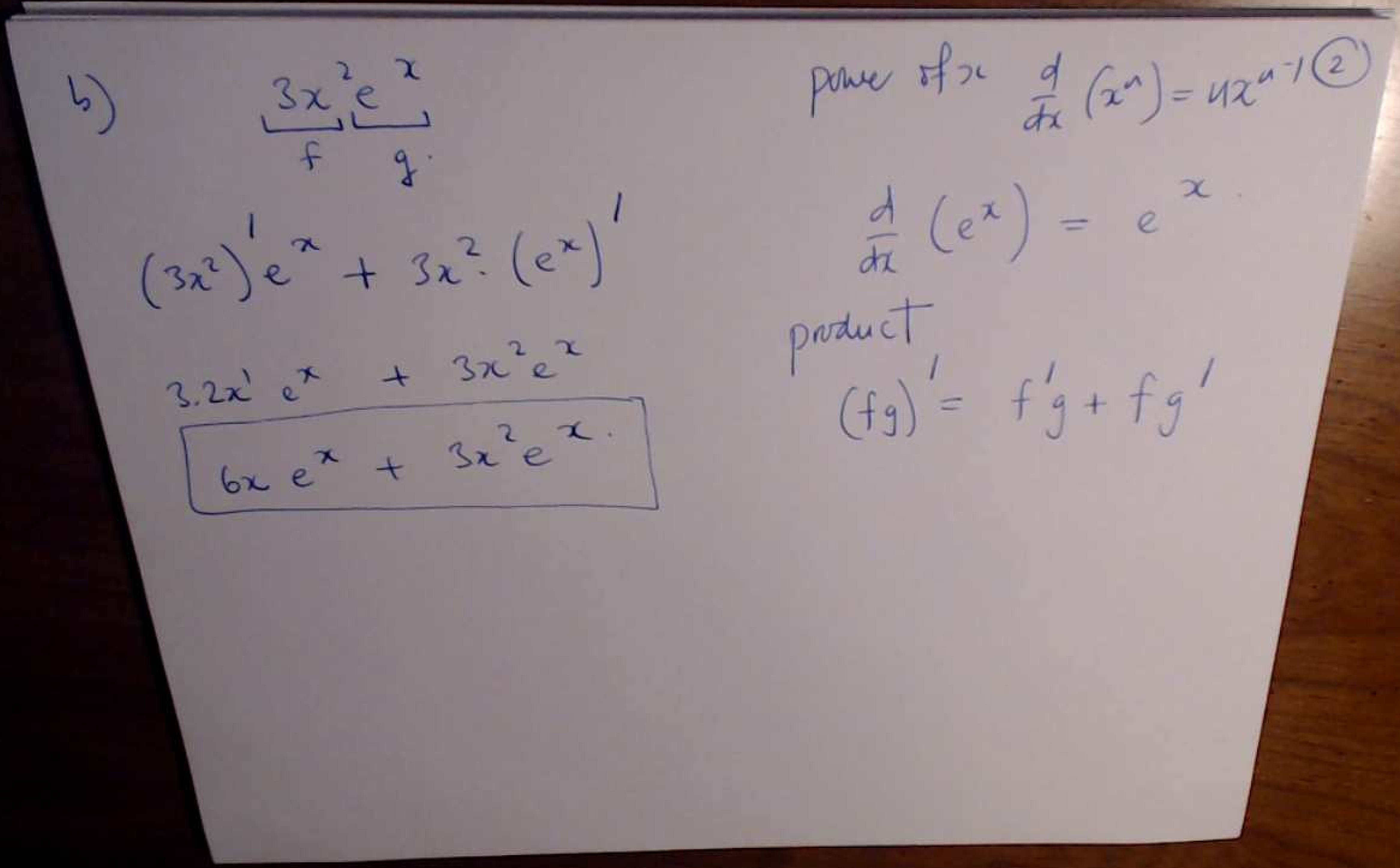
$f(x) = 5x^1 - 2x^5 + 3x^{1/2} - 2(x^{-2})^{1/3}$. $\frac{d}{dx}(x^n) = nx^{n-1}$.

$f(x) = 5x^1 - 2x^5 + 3x^{1/2} - 2x^{-2/3}$.

$f'(x) = 5 - 2.5x^4 + 3.\frac{1}{2}x^{\boxed{\frac{1}{2}-1}} - 2.-\frac{2}{3}.x^{\boxed{-2/3-1}}$

$-\frac{1}{2}$ $-\frac{2}{3}-\frac{3}{3} = -\frac{5}{3}$.

$= 5 - 10x^4 + \frac{3}{2}x^{-1/2} + \frac{4}{3}x^{-5/3}$.



b) $\underbrace{3x^2}_{f}\underbrace{e^x}_{g}.$

$(3x^2)'e^x + 3x^2 \cdot (e^x)'$

$3.2x^1 e^x + 3x^2 e^x$

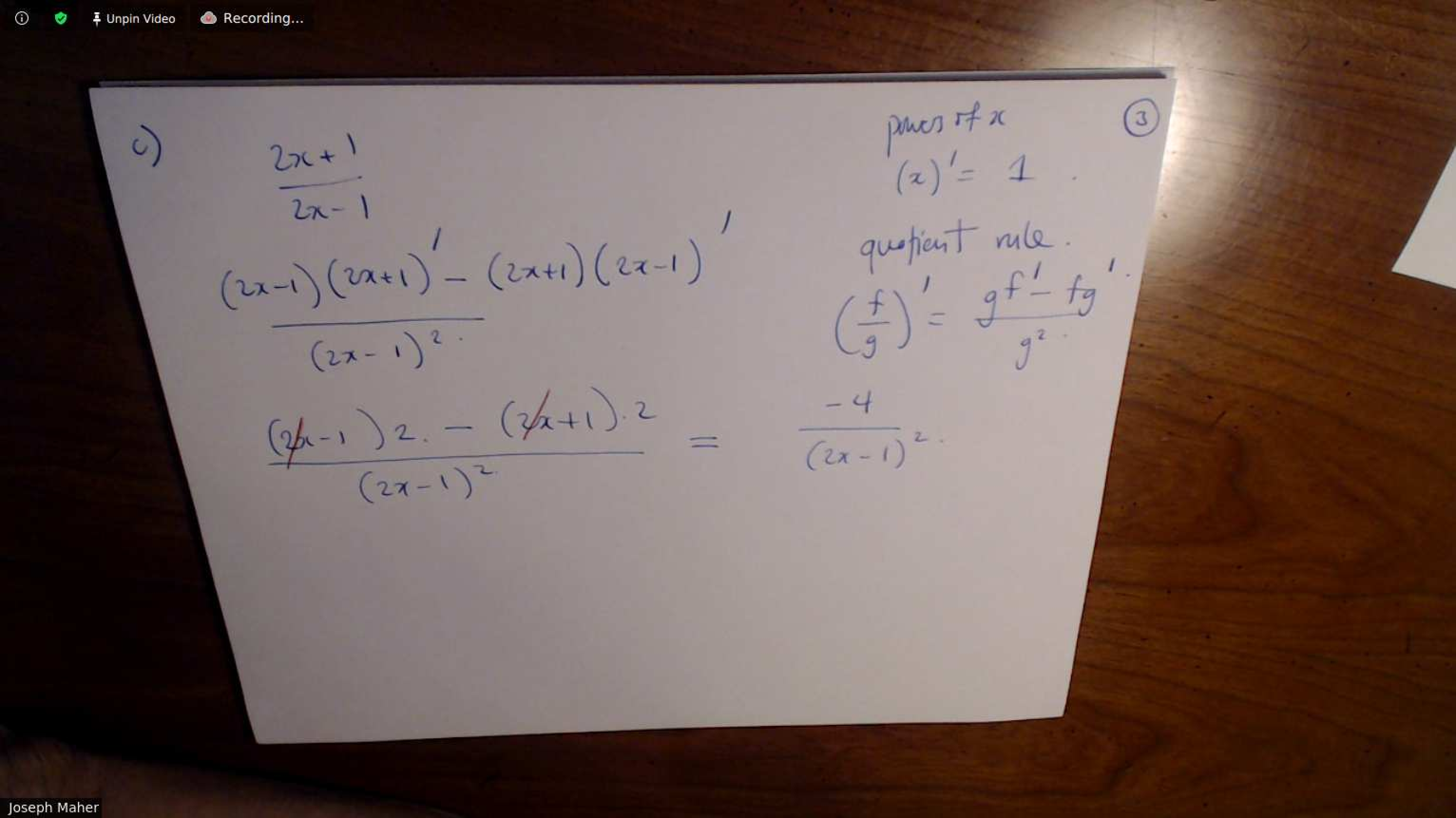
$\boxed{6x e^x + 3x^2 e^x.}$

power of x $\frac{d}{dx}(x^n) = nx^{n-1}$ ②

$\frac{d}{dx}(e^x) = e^x.$

product

$(fg)' = f'g + fg'$



③

powers of x

$(x)' = 1$.

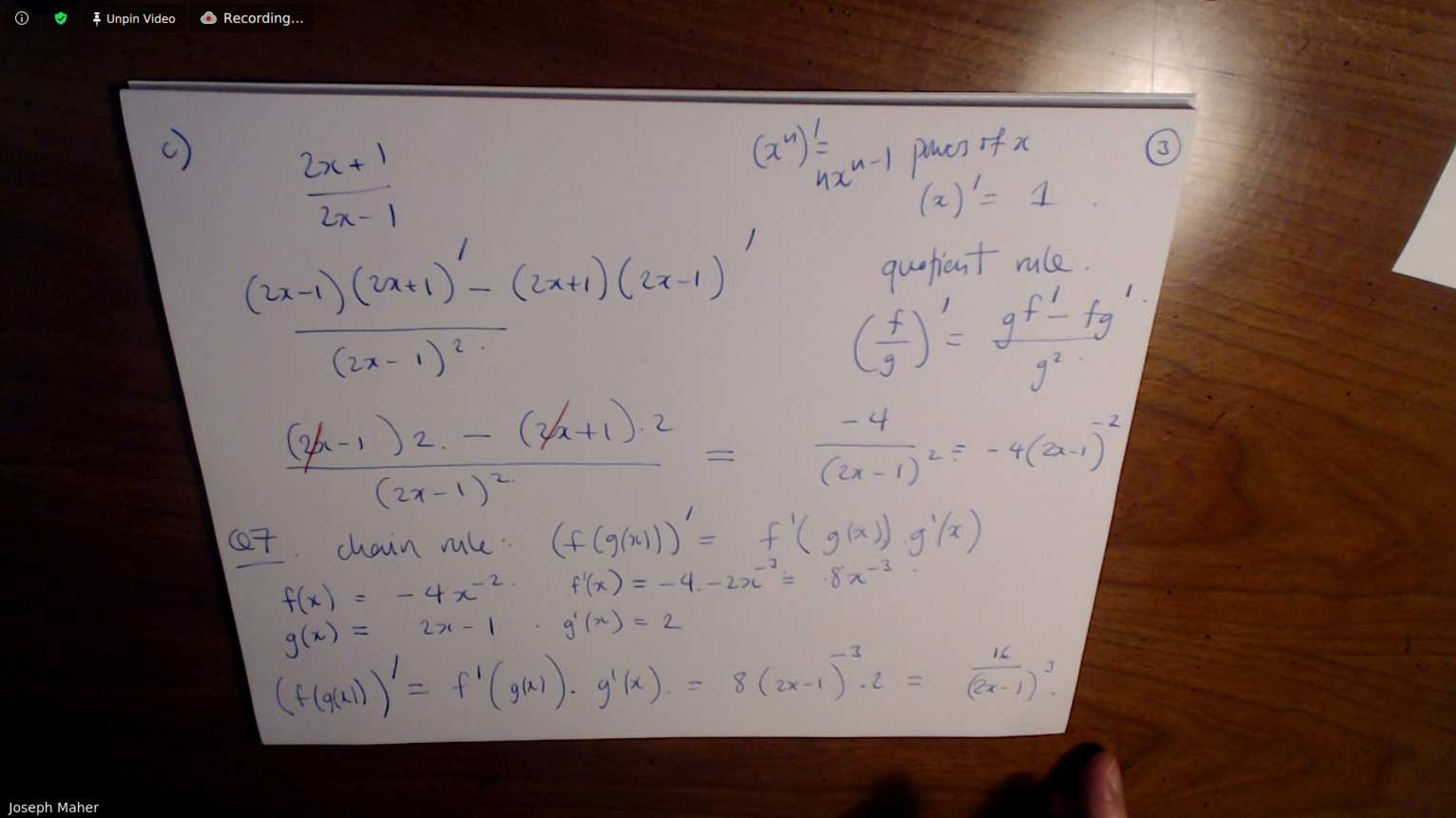
quotient rule.

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}.$$

c) $\frac{2x+1}{2x-1}$

$$\frac{(2x-1)(2x+1)' - (2x+1)(2x-1)'}{(2x-1)^2}.$$

$$\frac{(\cancel{2x}-1)2. - (\cancel{2x}+1)\cdot 2}{(2x-1)^2} = \frac{-4}{(2x-1)^2}.$$



③

c) $\frac{2x+1}{2x-1}$

$(x^n)' = nx^{n-1}$ powers of x

$(x)' = 1$

$$\frac{(2x-1)(2x+1)' - (2x+1)(2x-1)'}{(2x-1)^2}$$

quotient rule.

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

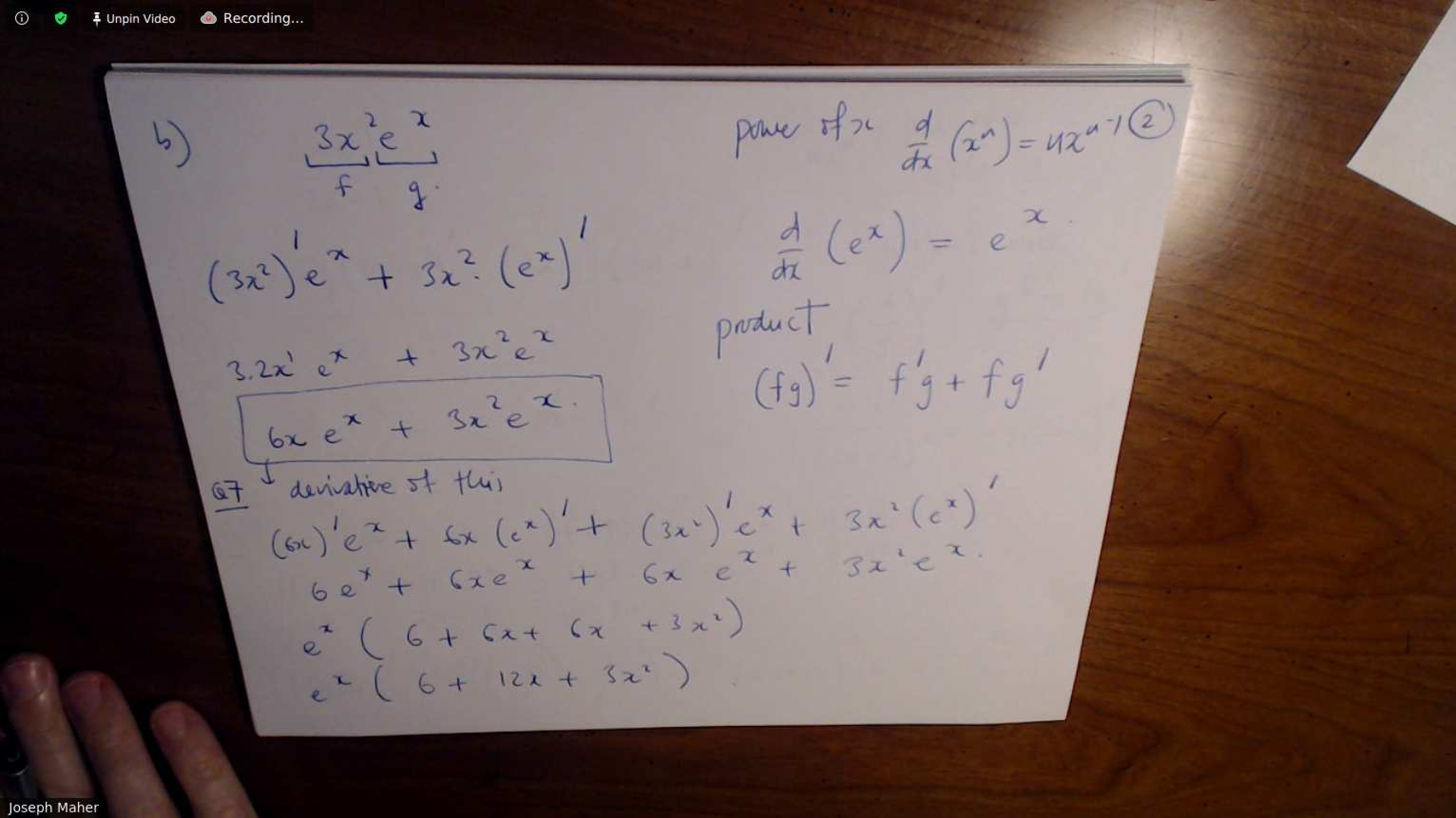
$$\frac{(\cancel{2}x-1)2 - (\cancel{2}x+1)\cdot 2}{(2x-1)^2} = \frac{-4}{(2x-1)^2} = -4(2x-1)^{-2}$$

Q7 chain rule: $(f(g(x)))' = f'(g(x))\,g'(x)$

$f(x) = -4x^{-2}$. $f'(x) = -4.-2x^{-3} = 8x^{-3}$.

$g(x) = 2x-1$. $g'(x) = 2$

$$(f(g(x)))' = f'(g(x)).\, g'(x) = 8(2x-1)^{-3}.2 = \frac{16}{(2x-1)^3}.$$



b) $\underbrace{3x^2}_{f}\underbrace{e^x}_{g}.$

power of x $\frac{d}{dx}(x^n) = nx^{n-1}$ ②

$(3x^2)'e^x + 3x^2\cdot(e^x)'$

$\frac{d}{dx}(e^x) = e^x.$

product

$(fg)' = f'g + fg'$

$3.2x^1 e^x + 3x^2 e^x$

$\boxed{6x e^x + 3x^2 e^x.}$

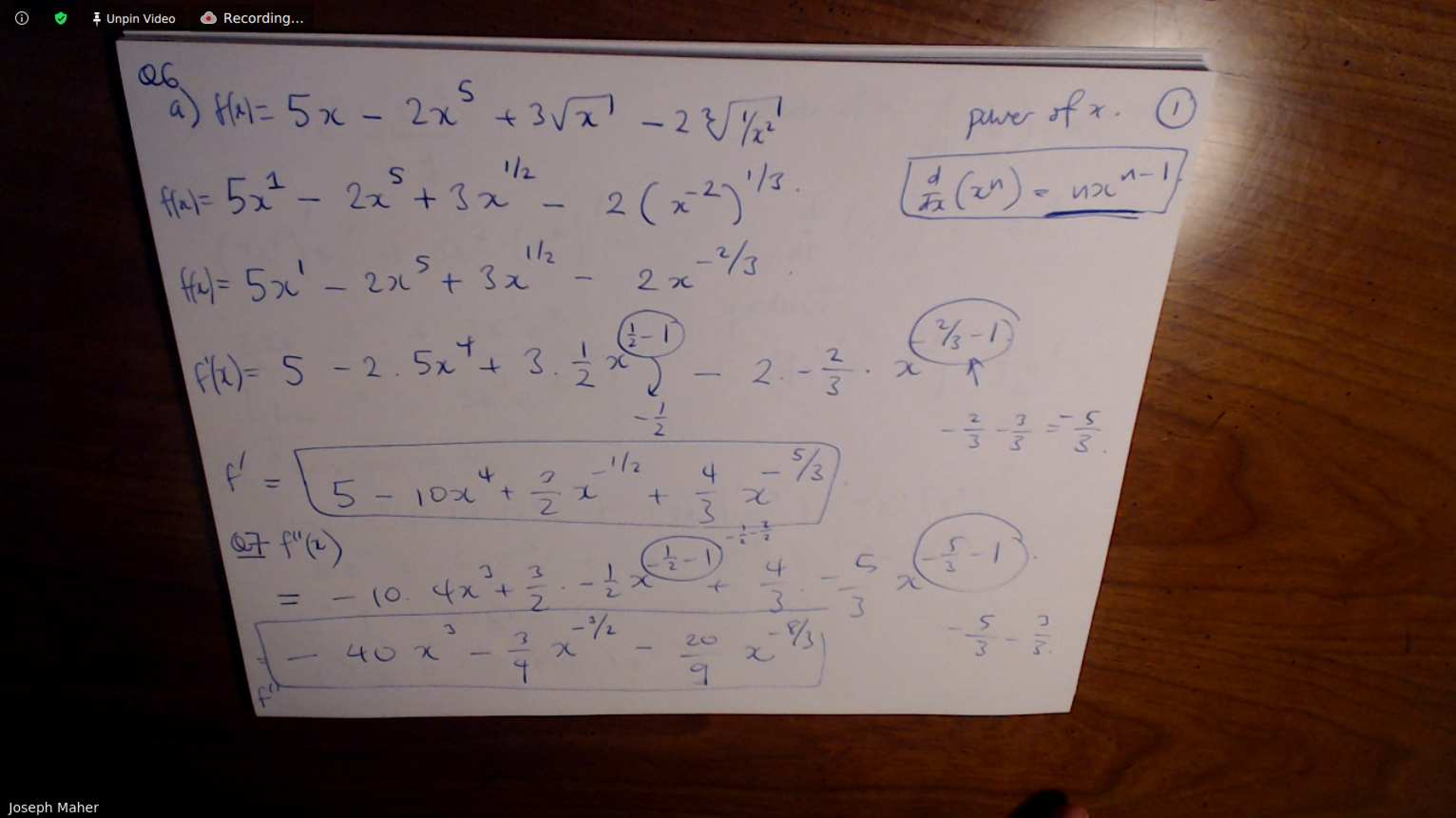
Q7 derivative of this

$(6x)'e^x + 6x(e^x)' + (3x^2)'e^x + 3x^2(e^x)'$

$6e^x + 6xe^x + 6x e^x + 3x^2 e^x.$

$e^x(6 + 6x + 6x + 3x^2)$

$e^x(6 + 12x + 3x^2)$



Q6

a) $f(x) = 5x - 2x^5 + 3\sqrt{x}^{1} - 2\sqrt[3]{1/x^2}^{1}$ power of x. ①

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

$f(x) = 5x^1 - 2x^5 + 3x^{1/2} - 2(x^{-2})^{1/3}.$

$f(x) = 5x^1 - 2x^5 + 3x^{1/2} - 2x^{-2/3}.$

$f'(x) = 5 - 2 \cdot 5x^4 + 3 \cdot \frac{1}{2} x^{(\frac{1}{2}-1)} - 2 \cdot -\frac{2}{3} \cdot x^{(-\frac{2}{3}-1)}$

$-\frac{1}{2}$

$-\frac{2}{3} - \frac{3}{3} = -\frac{5}{3}.$

$f' = \boxed{5 - 10x^4 + \frac{3}{2} x^{-1/2} + \frac{4}{3} x^{-5/3}}$

Q7 $f''(x)$

$-\frac{1}{2} - \frac{2}{2}$

$= -10 \cdot 4x^3 + \frac{3}{2} \cdot -\frac{1}{2} x^{(-\frac{1}{2}-1)} + \frac{4}{3} \cdot -\frac{5}{3} x^{(-\frac{5}{3}-1)}.$

$-\frac{5}{3} - \frac{3}{3}$

$f'' = \boxed{-40x^3 - \frac{3}{4} x^{-3/2} - \frac{20}{9} x^{-8/3}}$

rules for differentiation

④

$$(x^n)' = nx^{n-1}$$

$$(e^x)' = e^x$$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

$$(f(g(x)))' = f'(g(x))\, g'(x).$$

$$(f+g)' = f' + g'$$

$$(kf)' = kf'$$

$$(\sin(x))' = \cos(x)$$

$$(\cos(x))' = -\sin(x)$$

$$(\tan(x))' = \sec^2(x)$$

$$(\cot(x))' = -\operatorname{cosec}^2(x)$$

$$(\sec(x))' = \sec x \tan x$$

$$(\operatorname{cosec}(x))' = -\operatorname{cosec} x \cot x.$$

⑤

Q8 a) $f(x) = \frac{2}{x-1}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

find $f'(2)$

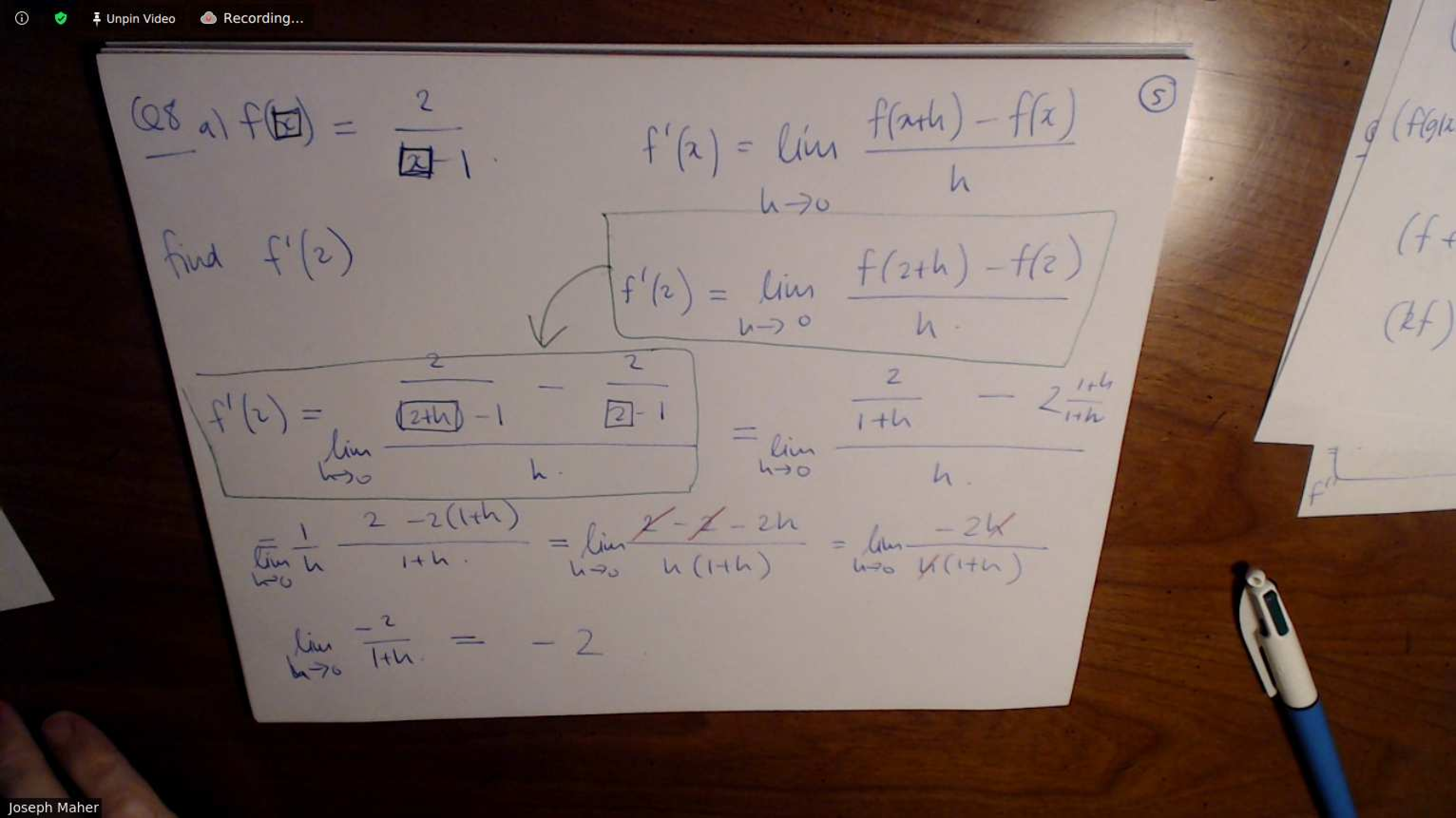
$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}.$$

$$f'(2) = \lim_{h \to 0} \frac{\frac{2}{(2+h)-1} - \frac{2}{2-1}}{h}. = \lim_{h \to 0} \frac{\frac{2}{1+h} - 2\frac{1+h}{1+h}}{h}.$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{2 - 2(1+h)}{1+h}. = \lim_{h \to 0} \frac{\cancel{2} - \cancel{2} - 2h}{h(1+h)} = \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)}$$

$$\lim_{h \to 0} \frac{-2}{1+h}. = -2$$

Joseph Maher



⑤

Q8 a) $f(\boxed{x}) = \frac{2}{\boxed{x}-1}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

find $f'(2)$

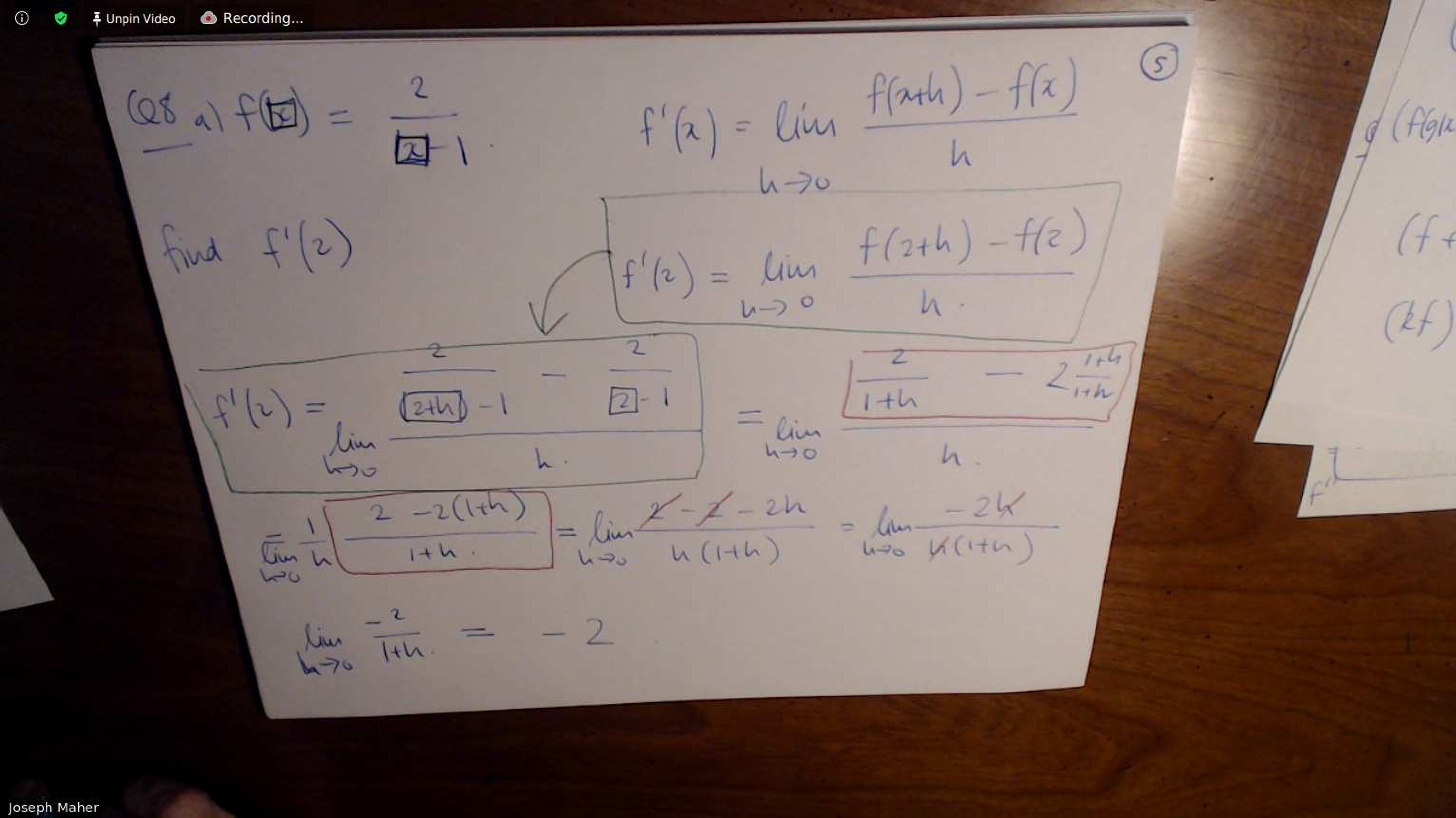
$$\boxed{f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}.}$$

$$\boxed{f'(2) = \lim_{h\to 0} \frac{\frac{2}{\boxed{2+h}-1} - \frac{2}{\boxed{2}-1}}{h}.}$$

$$= \lim_{h\to 0} \frac{\frac{2}{1+h} - 2\frac{1+h}{1+h}}{h}.$$

$$= \lim_{h\to 0} \frac{1}{h}\,\frac{2-2(1+h)}{1+h}. = \lim_{h\to 0} \frac{\cancel{2}-\cancel{2}-2h}{h(1+h)} = \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)}$$

$$\lim_{h\to 0} \frac{-2}{1+h} = -2$$



⑤

Q8 a) $f(\boxed{x}) = \dfrac{2}{\boxed{x}-1}.$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

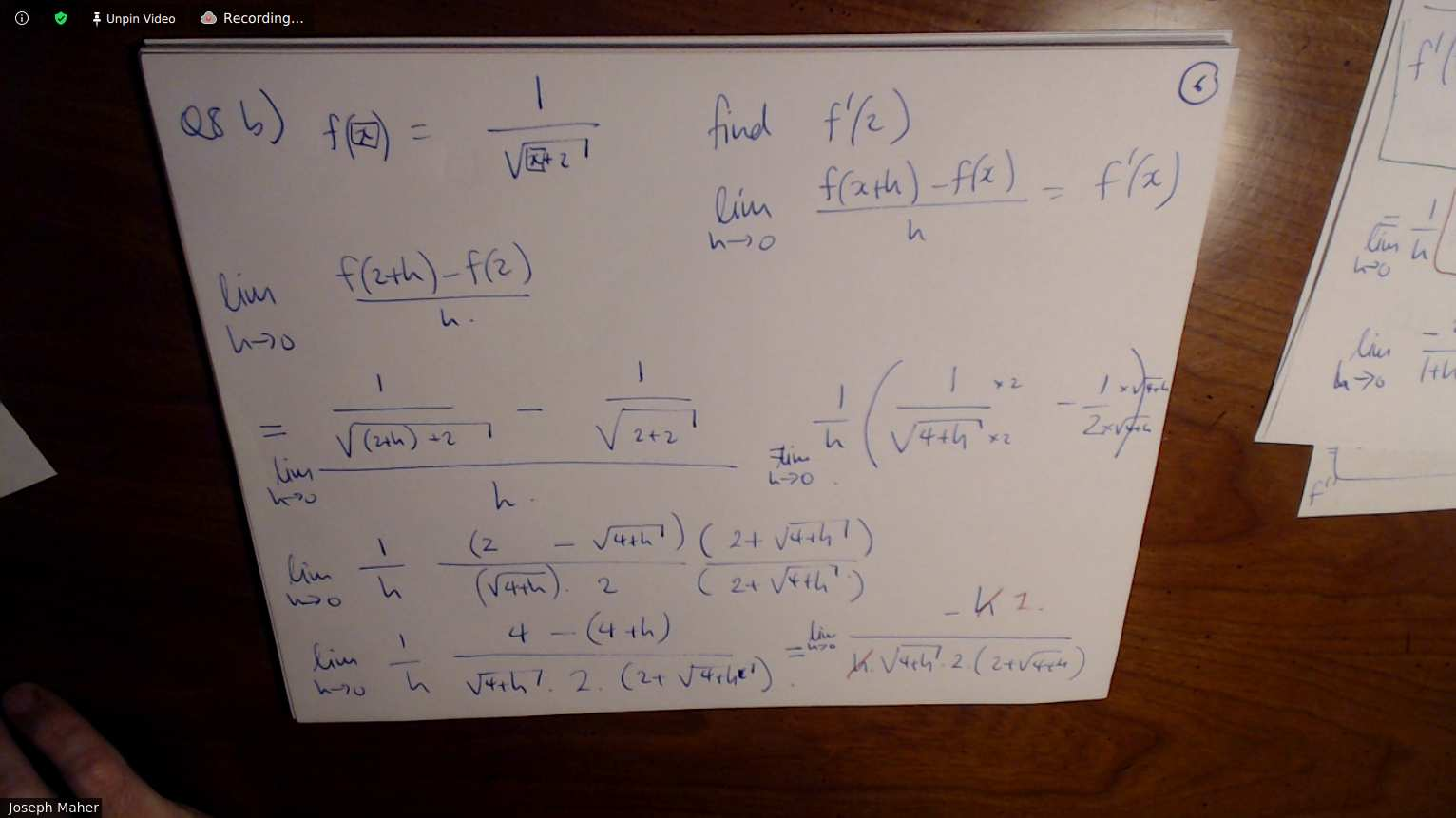
find $f'(2)$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}.$$

$$f'(2) = \lim_{h\to 0} \frac{\dfrac{2}{\boxed{2+h}-1} - \dfrac{2}{\boxed{2}-1}}{h}. = \lim_{h\to 0} \frac{\dfrac{2}{1+h} - 2\dfrac{1+h}{1+h}}{h}.$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2-2(1+h)}{1+h}.\right) = \lim_{h\to 0} \frac{\cancel{2}-\cancel{2}-2h}{h(1+h)} = \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)}$$

$$\lim_{h\to 0} \frac{-2}{1+h}. = -2$$



Q8 b) $f(\boxed{x}) = \dfrac{1}{\sqrt{\boxed{x}+2}}$ find $f'(2)$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x)$$

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}.$$

$$= \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{(2+h)+2}} - \dfrac{1}{\sqrt{2+2}}}{h}.$$

$$\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{\sqrt{4+h}}\,\substack{\times 2\\ \times 2} - \frac{1}{2}\,\substack{\times\sqrt{4+h}\\ \times\sqrt{4+h}}\right).$$

$$\lim_{h\to 0} \frac{1}{h}\ \frac{(2 - \sqrt{4+h})}{(\sqrt{4+h}).\ 2}\ \frac{(2+\sqrt{4+h})}{(2+\sqrt{4+h})}$$

$$\lim_{h\to 0} \frac{1}{h}\ \frac{4-(4+h)}{\sqrt{4+h}.\ 2.\ (2+\sqrt{4+h})}. = \lim_{h\to 0} \frac{-\cancel{h}\,1.}{\cancel{h}.\sqrt{4+h}.\,2.\,(2+\sqrt{4+h})}$$

⑦

$$\lim_{h\to 0} \frac{-1}{\sqrt{4+h}\cdot 2\cdot\left(2+\sqrt{4+h}\right)} =$$

$$= \frac{-1}{\sqrt{4}\cdot 2\left(2+\sqrt{4}\right)} = \frac{-1}{2\times 2(2+2)} = \frac{-1}{16}.$$

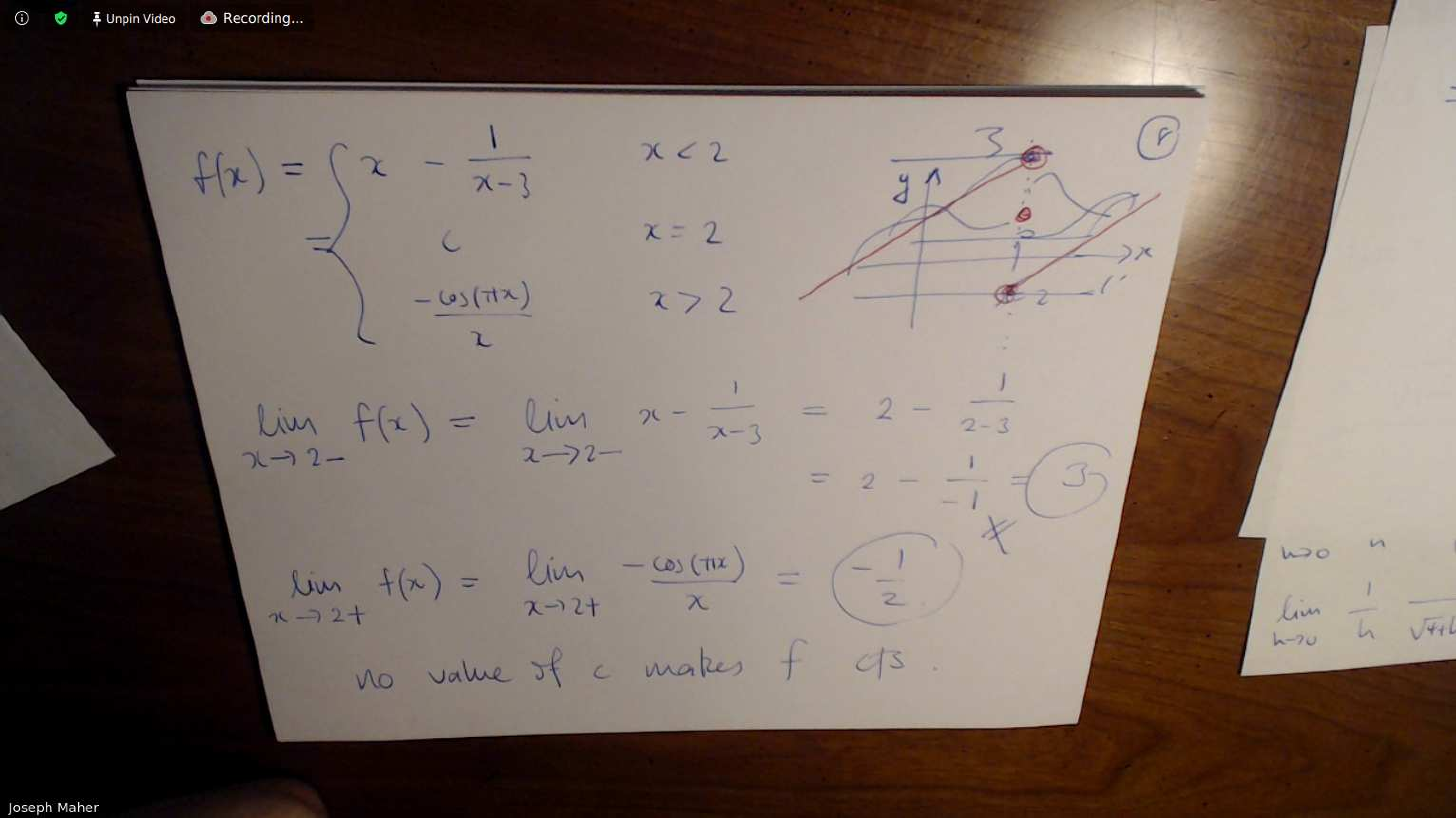
Q8 b) f

$$\lim_{h\to 0} \frac{f(2+}{}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{(2+h)+2}}}{h}$$

$$\lim_{h\to 0} \frac{1}{h} \frac{(2}{(\sqrt{4+h})}$$

$$\lim_{h\to 0} \frac{1}{h} \frac{4-}{\sqrt{4+h}\cdot 2.}$$



⑧

$$f(x) = \begin{cases} x - \frac{1}{x-3} & x < 2 \\ c & x = 2 \\ -\frac{\cos(\pi x)}{x} & x > 2 \end{cases}$$

$$\lim_{x \to 2-} f(x) = \lim_{x \to 2-} x - \frac{1}{x-3} = 2 - \frac{1}{2-3}$$

$$= 2 - \frac{1}{-1} = \boxed{3}$$

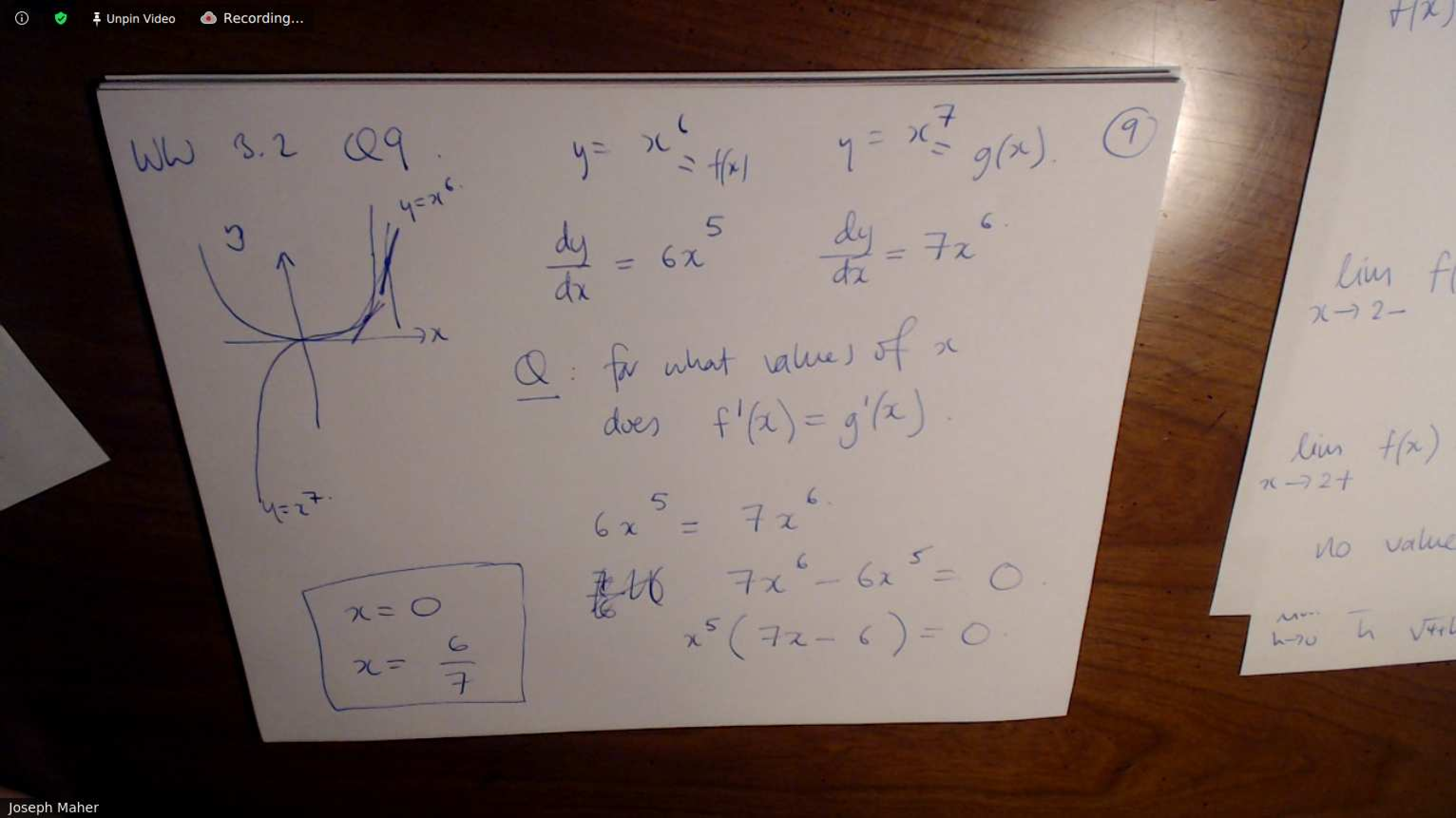
$$\neq$$

$$\lim_{x \to 2+} f(x) = \lim_{x \to 2+} -\frac{\cos(\pi x)}{x} = \boxed{-\frac{1}{2}}$$

no value of c makes f cts.

$h \to 0 \quad h$

$\lim_{h \to 0} \frac{1}{h} \quad \sqrt{4+}$



WW 3.2 Q9.

$y = x^6 = f(x)$ $y = x^7 = g(x)$. ⑨

$\frac{dy}{dx} = 6x^5$ $\frac{dy}{dx} = 7x^6$.

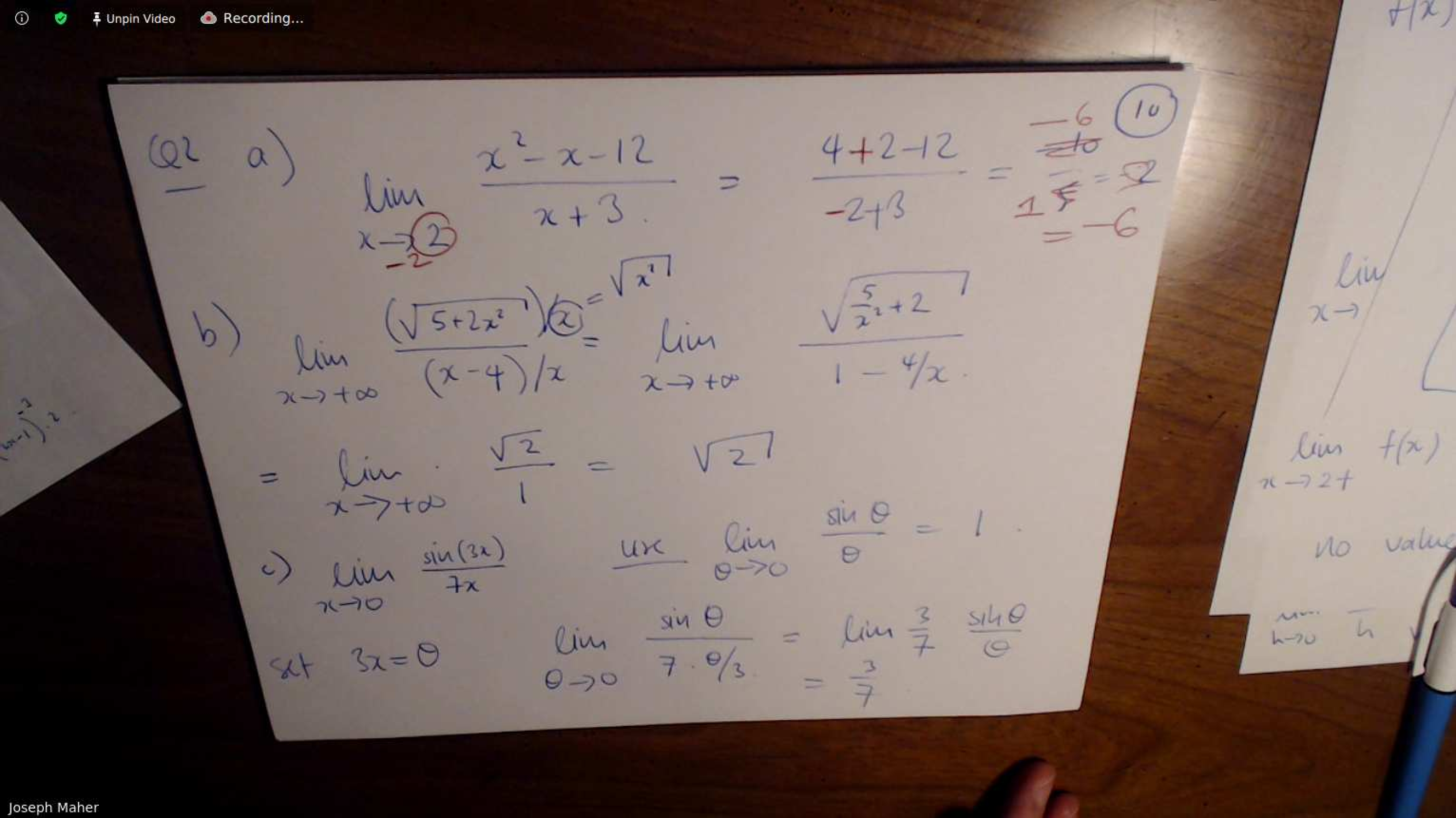
Q: for what values of x does $f'(x) = g'(x)$.

$6x^5 = 7x^6$.

~~7 16~~ $7x^6 - 6x^5 = 0$.

$x^5(7x - 6) = 0$.

$$\boxed{\begin{array}{l} x = 0 \\ x = \frac{6}{7} \end{array}}$$



Q2 a) $\lim_{\substack{x\to 2\\ -2}} \frac{x^2-x-12}{x+3} = \frac{4+2-12}{-2+3} = \frac{-6 \;\; \cancel{-10}}{1\;\cancel{5}} = \cancel{2}$ ⑩

$= -6$

b) $\lim_{x\to+\infty} \frac{(\sqrt{5+2x^2})/x}{(x-4)/x}$ $x = \sqrt{x^2}$ $= \lim_{x\to+\infty} \frac{\sqrt{\frac{5}{x^2}+2}}{1-4/x}$

$= \lim_{x\to+\infty} \frac{\sqrt{2}}{1} = \sqrt{2}$

c) $\lim_{x\to 0} \frac{\sin(3x)}{7x}$ use $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$.

set $3x = \theta$ $\lim_{\theta\to 0} \frac{\sin\theta}{7\cdot\theta/3} = \lim \frac{3}{7}\frac{\sin\theta}{\theta}$

$= \frac{3}{7}$

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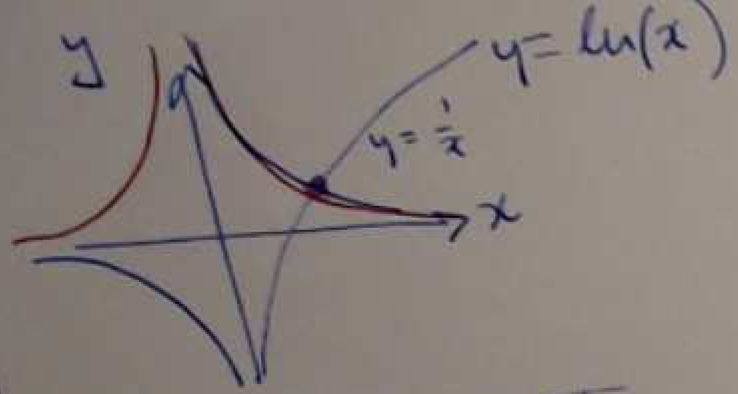
Q5 show $\ln(x) = \frac{1}{x^2}$ has a solution.

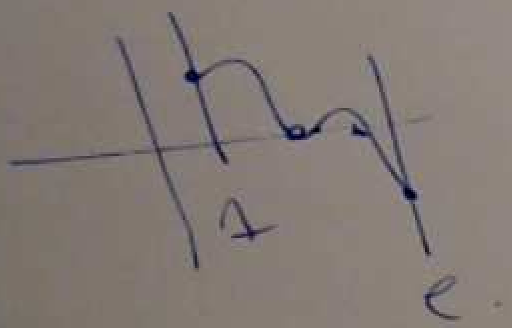


solve $\ln(x) = \frac{1}{x^2}$.

same as

solve $f(x) = \frac{1}{x^2} - \ln(x) = 0$.

IVT.



$f(1) = 1 - 0 = 1 > 0$

$f(e) = \frac{1}{\underset{\approx 2.7}{e^2}} - 1 < 0$.

IVT ⇒ there is a solution in [1, e].

Joseph Maher