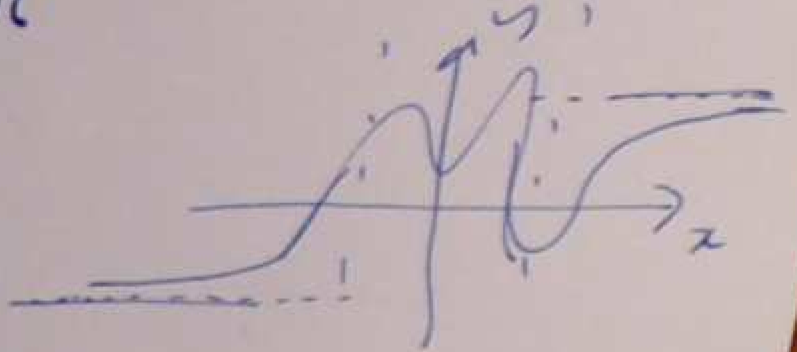
WW 2.7 Q6

$$f(x) = \frac{4x^{1/3}}{(64x^2+5)^{1/6}}$$

$$\lim_{x\to+\infty} f(x)$$

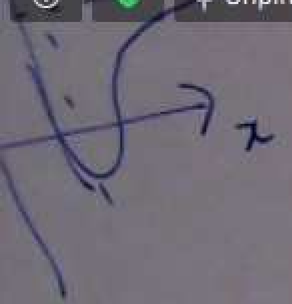
$$\lim_{x\to-\infty} f(x).$$



$$\lim_{x\to+\infty} \frac{4x^{1/3}}{(64x^2+5)^{1/6}} \longrightarrow L$$

$$\lim_{x\to\infty} \frac{4x^{1/3} \,/ x^{1/3}}{(64x^2+5)^{1/6} \,/ x^{1/3}} \qquad x^{1/3} = (x^2)^{1/6}.$$

$$\lim_{x\to\infty} \frac{4}{\quad}$$



$x^{1/3} = (x^2)^{1/6}.$

2

$$\frac{(64x^2+5)^{1/6}}{x^{1/3}} = \frac{(64x^2+5)^{1/6}}{(x^2)^{1/6}}$$

$$= \left(\frac{64x^2+5}{x^2}\right)^{1/6} = \left(64+\frac{5}{x^2}\right)^{1/6}.$$

$$\lim_{x\to+\infty} \frac{4}{\left(64+\frac{5}{x^2}\right)^{1/6}} = \frac{4}{(64)^{1/6}} = \frac{4}{2} = 2.$$

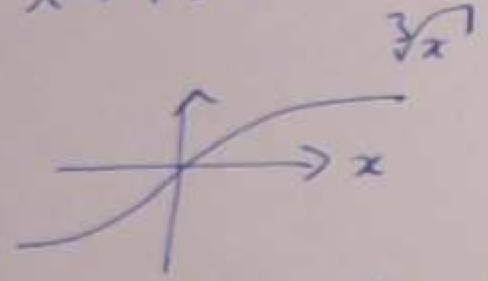
$2^6 = 4^3 = 4\times16$
$= 64.$

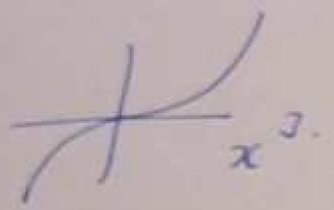
$$\lim_{x\to\infty} \frac{5}{x^2} = 0$$

$$\lim_{x\to-\infty} \frac{4x^{1/3}}{(64x^2+5)^{1/6}} \equiv \lim_{x\to-\infty} f(x)$$

$$= \lim_{x\to+\infty} f(-x)$$

$$\lim_{x\to+\infty} \frac{4(-x)^{1/3}}{\left(64(-x)^2+5\right)^{1/6}}$$





$$\lim_{x\to+\infty} \frac{-4x^{1/3}}{(64x^2+5)^{1/6}}$$

$$-\underbrace{\lim_{x\to\infty} \frac{4x^{1/3}}{(64x^2+5)^{1/6}}}_{2} = -2$$

WW3.1 Q6

$$\lim_{h\to 0} \frac{(6+h)^4 - 1296}{h} \quad (*)$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\boxed{\begin{array}{l}\text{try} \\ x = 6 \\ f(x) = x^4\end{array}}$$

✓

$$f'(6) = \lim_{h\to 0} \frac{f(6+h) - f(6)}{h}$$

$f(x) = x^4$

$$= \lim_{h\to 0} \frac{(6+h)^4 - 6^4}{h} = (*)$$

④

Joseph Maher

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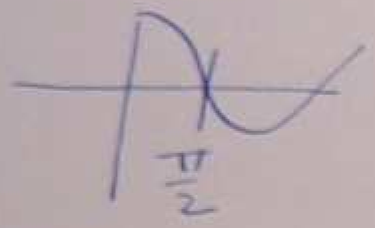
WW 3.1 Q8

$$\lim_{h \to 0} \frac{\cos(\frac{\pi}{2}+h)}{h}.$$

try

$x = \frac{\pi}{2}$

$f(x) = \cos(x)$ ✓.



$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\cos(\frac{\pi}{2}+h) - \cos(\frac{\pi}{2})}{h}.$$

$$= \lim_{h \to 0} \frac{\cos(\frac{\pi}{2}+h)}{h}$$

WW3.1 Q9

$$\lim_{h\to 0} \frac{6^{4+h} - 1296}{h}$$

try

$x = \cancel{8}\ 4$.

$f(x) = 6^x$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{6^{4+h} - 6^4}{h}$$

recall

$f(x)$	$f'(x)$	
x^n	nx^{n-1}	
e^x	e^x	
$\sin(x)$	$\cos(x)$	sneak preview — do these soon
$\cos(x)$	$-\sin(x)$	
$k f(x)$	$k f'(x)$	linearity rules
$f(x)+g(x)$	$f'(x)+g'(x)$	
$f(x)\,g(x)$	$f'(x)\,g(x) + f(x)\,g'(x)$	product rule
$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{(g(x))^2}$	quotient rule

Thm Quotient rule (assume f, g differentiable at x, $g(x) \neq 0$).

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2} \quad \square$$

Example ① $\displaystyle \frac{d}{dx}\left(\frac{\overset{f(x)}{x}}{\underset{g(x)}{x+1}}\right) = \frac{(x+1)(x)' - (x)(x+1)'}{(x+1)^2}$

$$= \frac{\cancel{x}+1-\cancel{x}}{(x+1)^2} = \frac{1}{(x+1)^2}.$$

(9)

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}.$$

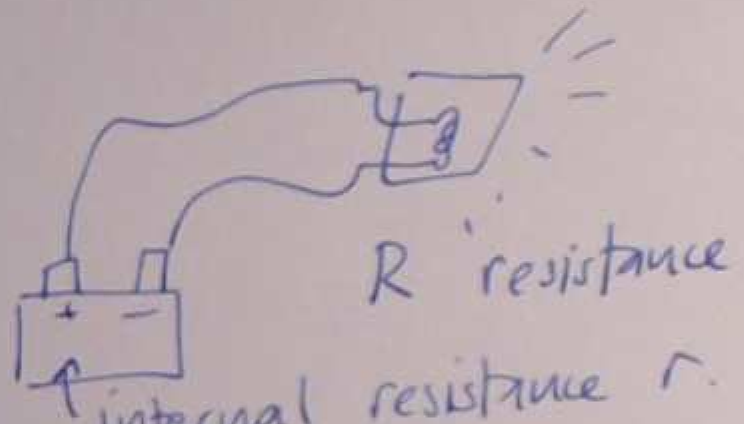
(i)

$$\frac{d}{dt}\left(\frac{e^t}{e^t+t}\right) = \frac{(e^t+t)(e^t)' - e^t(e^t+t)'}{(e^t+t)^2}$$

$$= \frac{(e^t+t)(e^t) - e^t(e^t+1)}{(e^t+t)^2}.$$

$$= \frac{\cancel{e^{2t}} + te^t - \cancel{e^{2t}} - e^t}{(e^t+t)^2} = \frac{e^t(t-1)}{(e^t+t)^2}.$$

③ application battery power

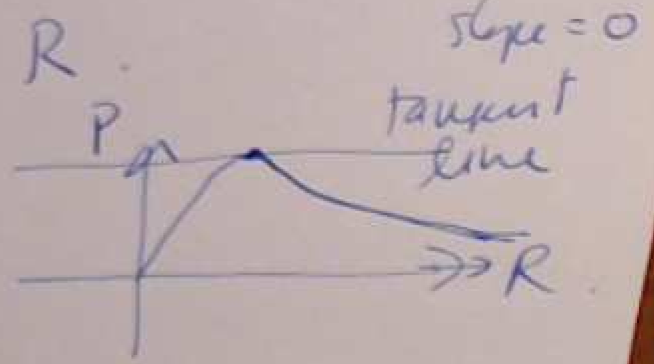


power $P = \dfrac{v^2 R}{(r+R)^2}$ v = voltage of the battery.

Q: when does the battery give out maximum power?

imagine: v, r fixed, can vary R.

A $\dfrac{dP}{dR} = 0$ $P(R) = \dfrac{v^2 R}{(r+R)^2}$.



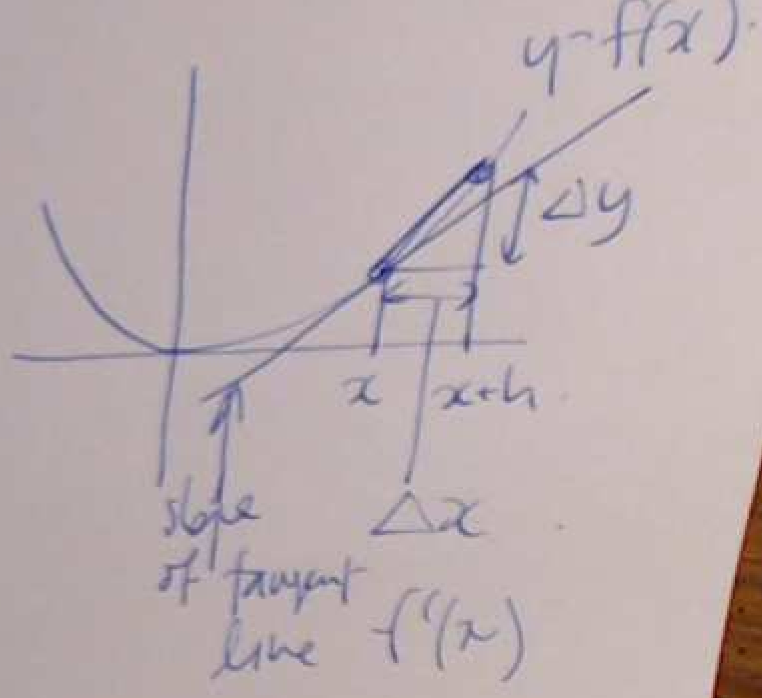
$$P(R) = \frac{v^2 R}{(r+R)^2} \qquad \text{solve } \frac{dP}{dR} = 0$$

$$P'(R) = \frac{(r+R)^2 (v^2 R)' - (v^2 R)\left((r+R)^2\right)'}{\left((r+R)^2\right)^2}$$

$$= \frac{(r+R)^2 v^2 - v^2 R \left(r^2 + 2rR + R^2\right)'}{(r+R)^4}$$

$$= \frac{(r+R)^2 v^2 - v^2 R (2r + 2R)}{(r+R)^4}$$

§3.4 Rates of change



recall: average rate of change

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}.$$

(actual)
(instantaneous) rate of change

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

observation if $\Delta x = h$ we can use the average rate of change to approximate the actual rate of change (and vice versa)

$A = \pi r^2$.

<u>Example</u> area of circle is πr^2.

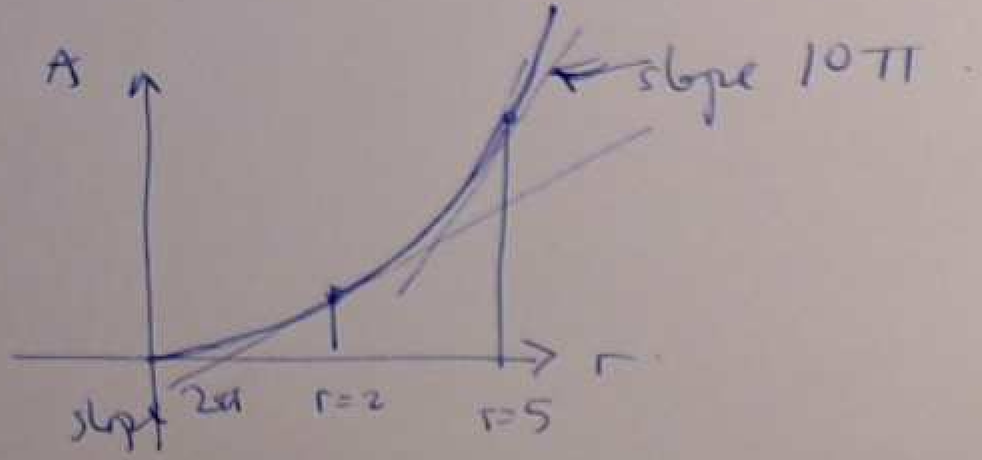
calculate the rate of change of area wrt radius.

$$\frac{dA}{dr} = \pi 2r = 2\pi r$$

<u>r=2</u>

$$\underbrace{\left.\frac{dA}{dr}\right|_{r=2}}_{A'(2)} = 4\pi.$$

$$\left.\frac{dA}{dr}\right|_{r=5} = 10\pi$$



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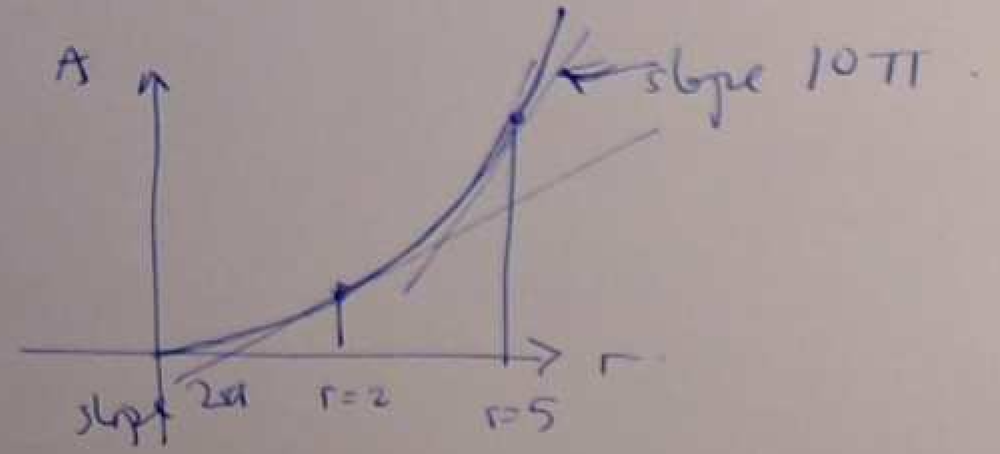
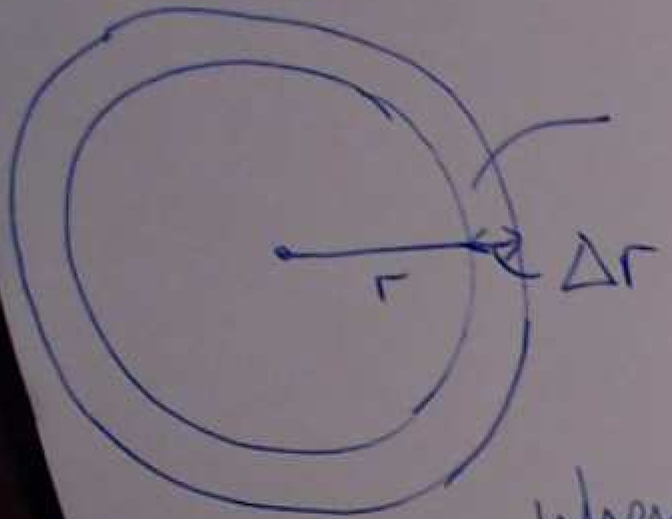


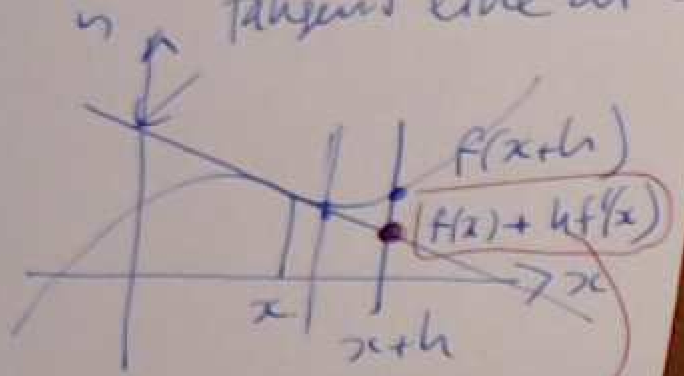


how big is this area?

area of this ring $\approx \left.\frac{dA}{dr}\right|_r \cdot \Delta r$.

when $r=2$ $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi$.

area of ring $\approx 4\pi \Delta r$.



<u>summary</u> for small $h / \Delta x$

$$f'(x) \simeq \frac{f(x+h)-f(x)}{h}.$$

$$hf'(x) \simeq f(x+h)-f(x)$$

$$\boxed{f(x+h) \simeq f(x) + hf'(x)}$$

this is the linear approx to f at x.

Example stopping distance in feet given by

$$F(s) = 1.1s + 0.05s^2$$

s speed in mph.

F = # of feet required to stop.

calculate stopping distance at $s = 30$ mph.

$$F(30) = 1.1 \times 30 + 0.05 \times 30^2 = 3.3 + \frac{\overset{45}{90\cancel{0}}}{\cancel{2}} = 48.3$$

find $F'(30)$: $F'(s) = 1.1 + 0.1s$.

$$F'(30) = 1.1 + 0.1 \times 30 = 4.1.$$

estimate stopping distance at $F(31) \approx F(30) + F'(30) \times 1 = 48.3 + 4.1$

$= 52.4$

$= 48.3 + 4.1$
$= 52.4$

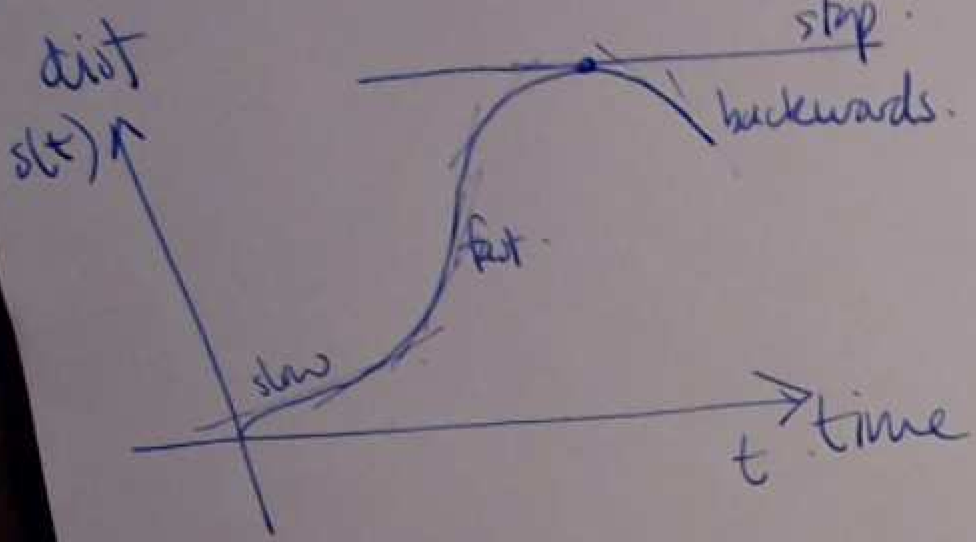
$f(x)$

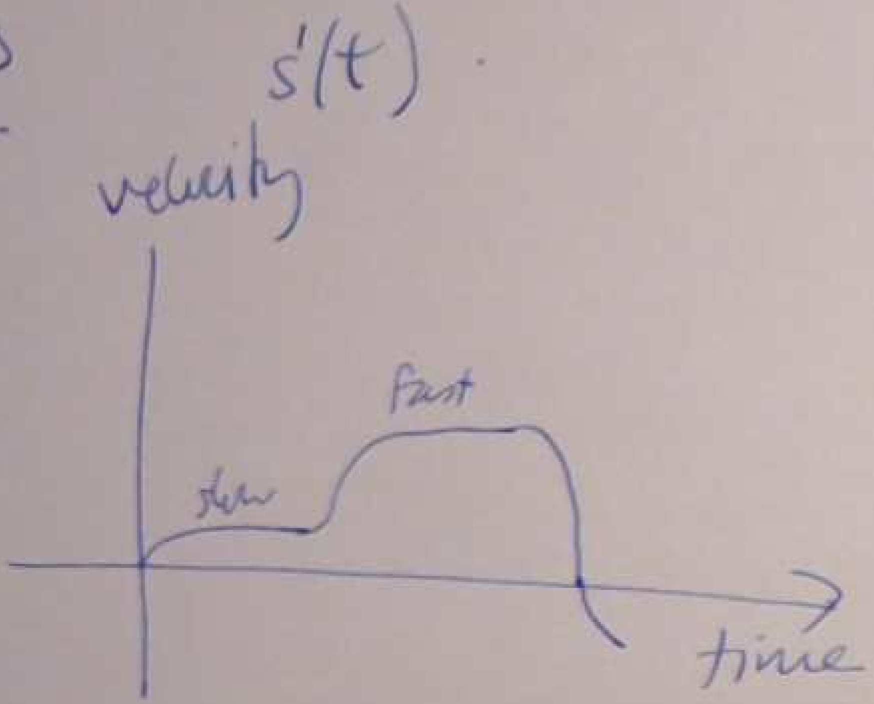
$\to x$

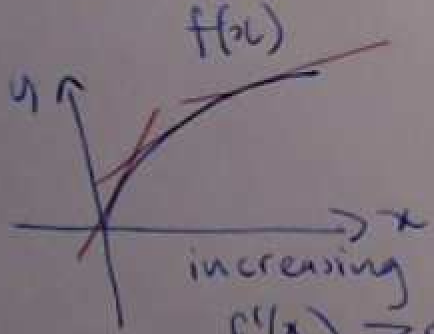
increasing
$f'(x) > 0$
rate of change increasing

$f'(x)$

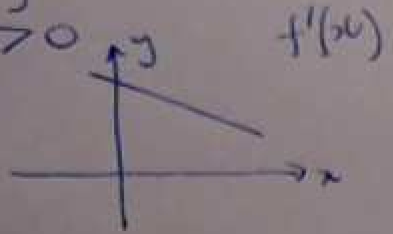
On the interpretation of graphs

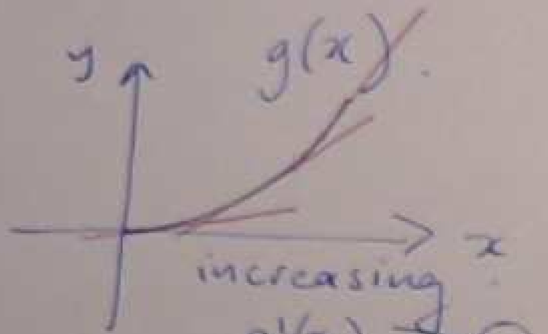




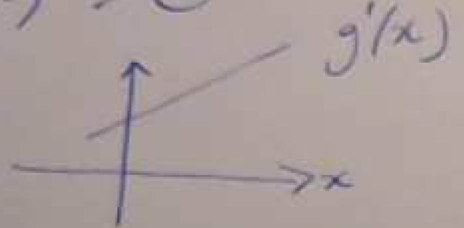


$f'(x) > 0$





$g'(x) > 0$

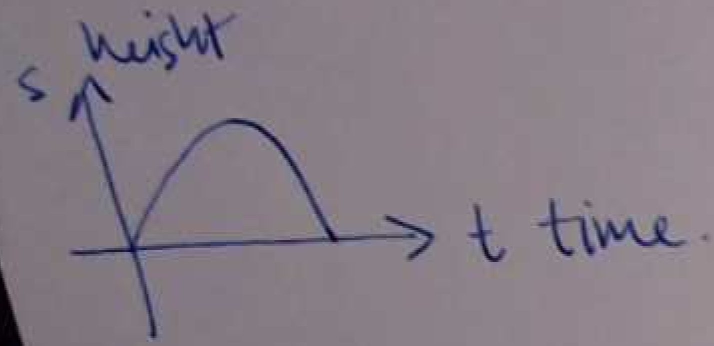


motion under gravity

$s(t)$ height at time t

$s'(t) = v(t)$ velocity at time t

$s''(t) = \cancel{a(t)}\ v'(t) = a(t)$ acceleration at time t.



$$s''(t) = v'(t) = a(t) = -g = \begin{matrix} 9.8\ m/s^2 \\ 32\ ft/s^2 \end{matrix}$$

$$s'(t) = v(t) = -gt + v_0$$

$v_0 = v(0) = s'(0)$

↑ velocity at time $t=0$

$$\boxed{s(t) = -\tfrac{1}{2} g t^2 + v_0 t + s_0}$$

↑ location at time t

starting at $s(0) = s_0$

with velocity $v(0) = s'(0) = v_0$.

$s_0 = s(0)$

location at time $t=0$.

Q: what is the max height?

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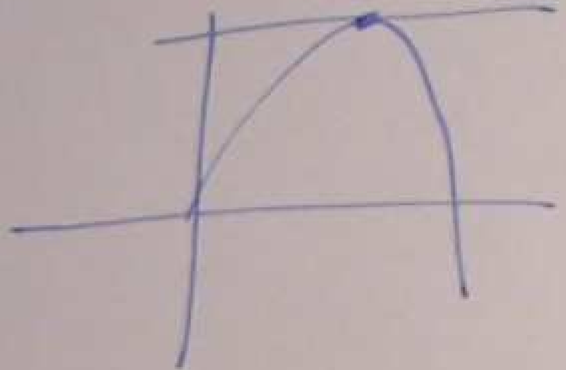
$$s(t) = -\frac{1}{2}g t^2 + v_0 t + s_0.$$



Q: max height?

A: $v(t) = s'(t) = 0$.

$$s'(t) = -gt + v_0 = v(t) = 0.$$

$$-gt + v_0 = 0 \qquad t = \frac{v_0}{g}.$$

when is max height? $t = \frac{v_0}{g}$

$$-\frac{1}{2}\frac{v_0^2}{g} + \frac{v_0^2}{g}$$

how high is max height? $s\left(\frac{v_0}{g}\right) = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\frac{v_0}{g} + s_0.$

$$= \frac{1}{2}\frac{v_0^2}{g} + s_0$$

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