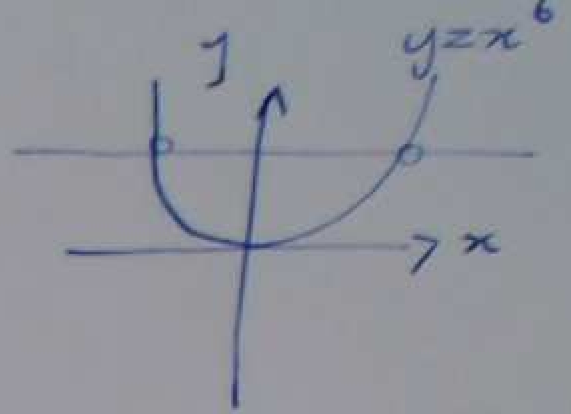
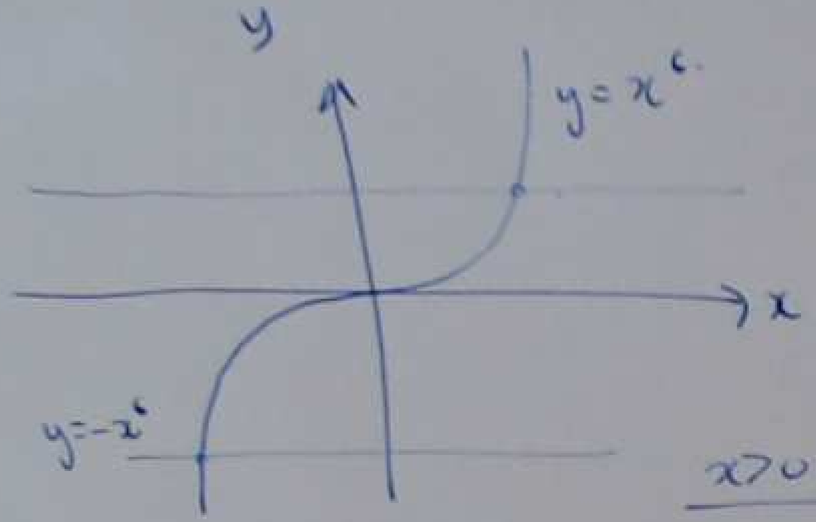
WW1.5.Q3

$$g(x) = \begin{cases} -x^6 & x < 0 \\ x^6 & x \geq 0 \end{cases}$$





$$\boxed{g^{-1}(x) = \begin{cases} x^{1/6} & x > 0 \\ (-x)^{1/6} & x \leq 0 \end{cases}}$$

$x > 0$

$y = x^6$

$y^{1/6} = x$

$x \leq 0$

$y = -x^6$

$-y = x^6$

$(-y)^{1/6} = x$

Lesson01 1.2 Review Linear Quad Trig Funct: Problem 2

This set is **visible to**

(1 point) **Rogawski_4E/1_Precalculus_Review/1.2_Linear_and_Quadratic_Functions/1.2.14.pg**

Find the equation of the line with the given description:

Passes through $(-1, 16)$ and $(62, 25)$.

(Use symbolic notation and fractions where needed.)

$y =$ [ ] help (fractions)

Solution:

Edit3

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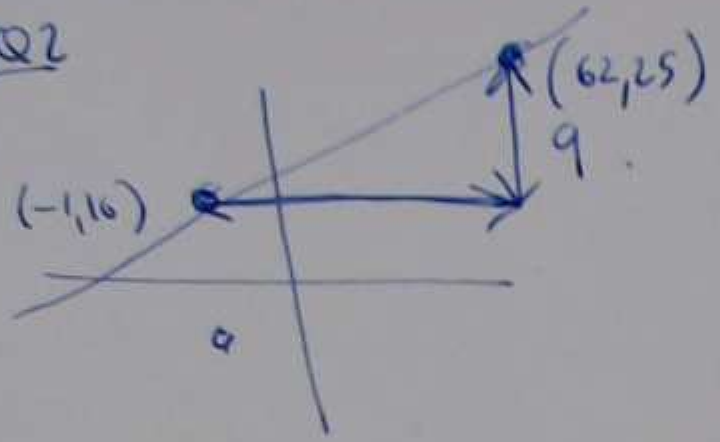
WW1.2 Q2



slope $m = \dfrac{\Delta y}{\Delta x} = \dfrac{9}{63} = \dfrac{1}{7}$

$$y - y_1 = m(x - x_1)$$

$$y - 16 = \frac{1}{7}(x - (-1))$$

$$y = \frac{1}{7}(x+1) + 16$$

$$y = \frac{1}{7}x + \left(16 + \frac{1}{7}\right)$$

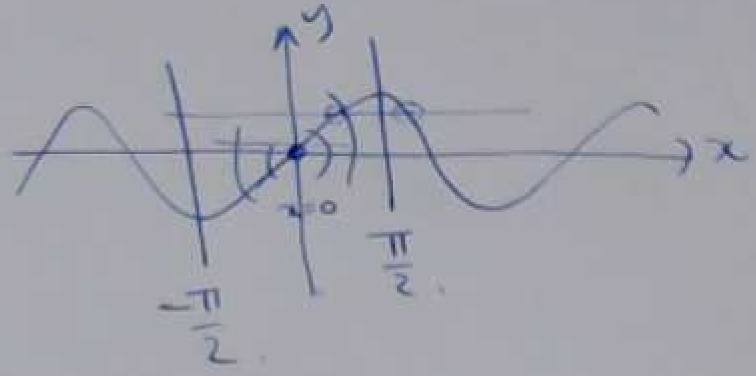
HW1.5 Q12

$$\ln\left(\sqrt{e}\cdot e^{23/7}\right) \qquad \sqrt{e} = e^{1/2}.$$

$$\ln\left(e^{1/2}\cdot e^{23/7}\right) = \ln\left(e^{1/2+23/7}\right) = \ln\left(e^{\frac{7+46}{14}}\right).$$

$$= \ln\left(e^{53/14}\right) = \frac{53}{14}$$

Q1



$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$\boxed{y = mx + b}$ 6

$$x + cy = 5 \iff cy = -x + 5$$

$$y = -\frac{1}{c}x + 5.$$

- slope -6
- thru $(-8, -2)$
- horizontal?
- vertical?

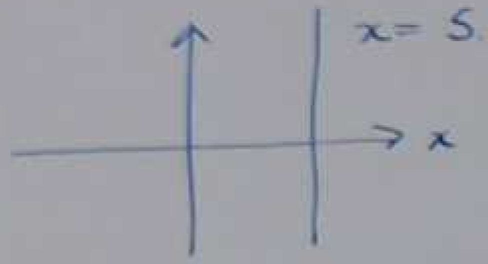
$$-\frac{1}{c} = -6. \iff c = 1/6 \checkmark.$$

$$\left.\begin{array}{l} x = -8 \\ y - 2 \end{array}\right\} \quad -8 + c\cdot(-2) = 5.$$

$$-2c = 13.$$

$$c = -\frac{13}{2}. \quad \checkmark.$$

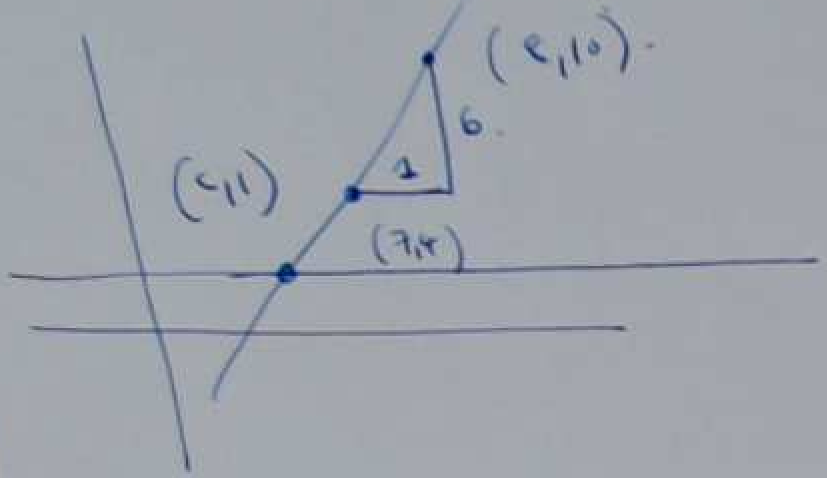
- horizontal? no. $-\frac{1}{c} = 0$ ← no solutions.
- vertical? yes $c = 0$ $x = 5.$



1.2 Q6

$$\boxed{(7,4) \quad (8,10)} \qquad \overset{x\ \ y}{(c,1)}$$

$$m = \frac{\Delta y}{\Delta x} = 6$$



$$y - y_1 = 6(x - x_1)$$

$$y - 4 = 6(x - 7)$$

plug in $y = 1$:

$$1 - 4 = 6(x - 7)$$

$$-3 = 6(x - 7)$$

$$-\tfrac{1}{2} = x - 7$$

$$x = 7 - \tfrac{1}{2} = 6\tfrac{1}{2} = \tfrac{13}{2}$$

$$\left(\tfrac{13}{2}, 1\right)$$

Last time: $\lim_{x \to c} f(x) = L$. $\leftarrow$ means $f(x)$ gets arbitrarily close to L as x gets close to c.

§2.3 Basic limit laws

<u>Example</u> $$\lim_{x \to 0} 2x + 2 = \underbrace{\lim_{x \to 0} 2x}_{0} + \underbrace{\lim_{x \to 0} 2}_{2} = 2.$$

<u>Thm</u> assume that $\lim_{x \to c} f(x)$, $\lim_{x \to c} g(x)$ exist, are finite.

Then

1) sum: $$\lim_{x \to c} (f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x).$$

2) constant multiples: $$\lim_{x \to c} k f(x) = k \lim_{x \to c} f(x).$$

3) products: $\lim\limits_{x\to c}\left(f(x)g(x)\right) = \left(\lim\limits_{x\to c} f(x)\right)\left(\lim\limits_{x\to c} g(x)\right)$

4) quotients: $\lim\limits_{x\to c}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$ as long as $\lim\limits_{x\to c} g(x) \neq 0$

<u>Warning</u> these rules do not work if either $\lim\limits_{x\to c} f(x)$ or $\lim\limits_{x\to c} g(x)$ DNE.

recall $\lim\limits_{x\to c} x = c$.

<u>Example</u>

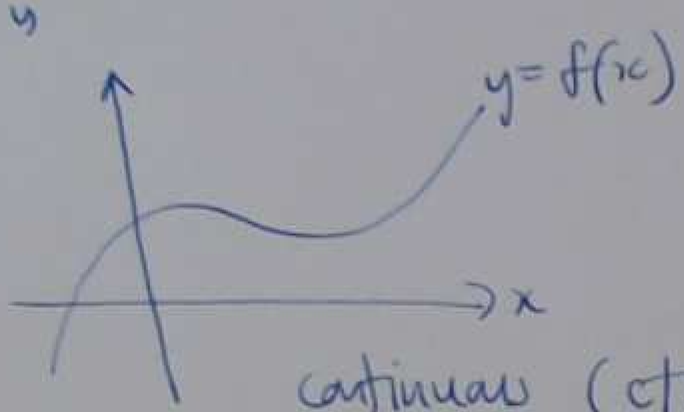
① $\lim\limits_{x\to 3} x^2 = \left(\lim\limits_{x\to 3} x\right)\left(\lim\limits_{x\to 3} x\right) = 3\times 3 = 9$.

② $\lim\limits_{t\to 2} \dfrac{t+5}{3t} = \dfrac{\lim\limits_{t\to 2} t+5}{\lim\limits_{t\to 2} 3t} = \dfrac{7}{6}$.

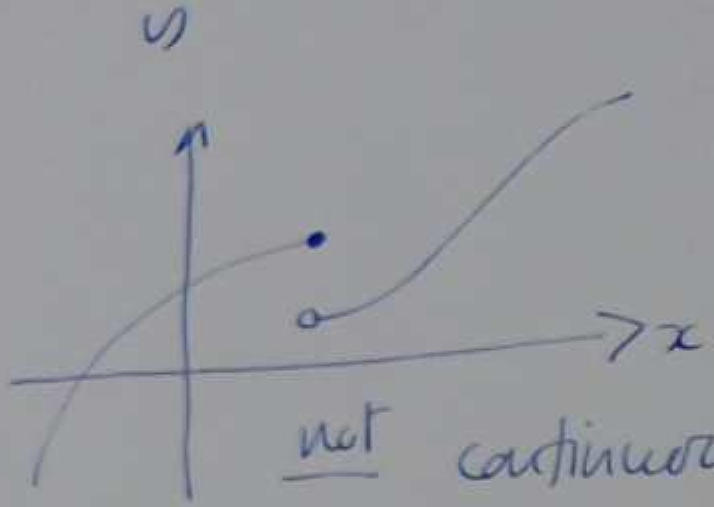
continuous /
continuous
$f(x)$ is not continuous at c
$f(x) = \begin{cases} 1 & x\neq 2 \\ 2 & x=2 \end{cases}$
not continuous at $x=2$

§2.4 Limits and continuity

Example



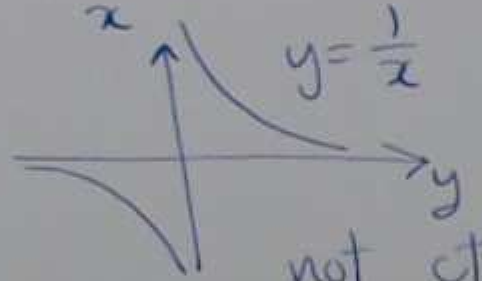
continuous (cts)



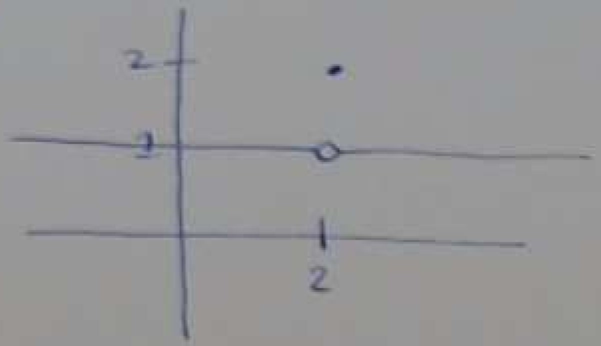
not continuous.

aka discontinuous

Examples

① 

not cts at $x = 0$

② 

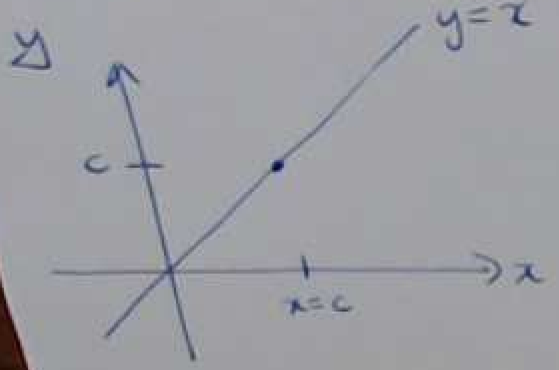
not cts at $x = 2$ $\longrightarrow$ $f(x) = \begin{cases} 1 & x \neq 2 \\ 2 & x = 2 \end{cases}$

Def$^{\underline{n}}$ we say $f(x)$ is continuous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c).$$

if the limit DNE / is infinite / not equal to $f(c)$ $\underline{\text{not}}$ cts at c.

Example show $f(x) = x$ is continuous



value at c: $f(c) = c$.

limit at c: $\lim_{x \to c} f(x)$

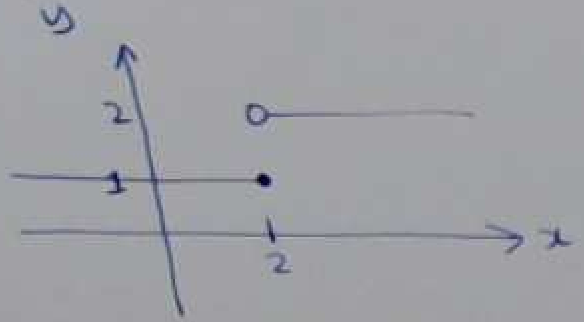
$$= \lim_{x \to c} x = c.$$

so $f(x) = x$ cts at c. so $f(x) = x$ cts

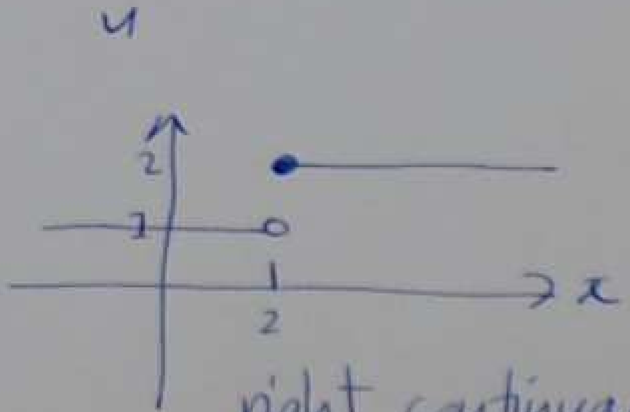
Defn $f(x)$ is left continuous at $x=c$ if $\lim_{x\to c^-} f(x) = f(c)$

$f(x)$ is right continuous at $x=c$ if $\lim_{x\to c^+} f(x) = f(c)$

Examples



left continuous



right continuous.

If at least one of the right or left limits is $\pm\infty$ then we say f has an infinite discontinuity.

Building continuous functions

Thm 0: $f(x) = k$, $f(x) = x$ are continuous.

Thm 1: suppose that $f(x)$, $g(x)$ are both cts at $x = c$. Then the following are cts at $x = c$:

1) $f(x) + g(x)$
2) $kf(x)$ k constant
3) $f(x)g(x)$
4) $\dfrac{f(x)}{g(x)}$ if $g(c) \neq 0$.

Proof: this follows directly from the limit

check 1) $f(x)$ cts at c means $\lim_{x \to c} f(x) = f(c)$

$g(x)$ " " $\lim_{x \to c} g(x) = g(c)$

so $\lim_{x \to c} \big(f(x) + g(x) \big) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

$= f(c) + g(c)$ ✓

other similar □

$= (f+g)(c)$

...these and
...e sometimes
...entary functions.

..., with inverse

...d $x=c$, and $g(x)$
... at $x=c$
$f(x) = \dfrac{2^x + \sin(x)}{\sqrt{x^2+x+1}}$
... $x=1$
... $\dfrac{x^2}{\sin(x)}$ cts?

Thm 2 Polynomials are continuous.

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0.$$

$a_k \in \mathbb{R}$ real numbers, ~~[illegible]~~.

examples: (1) $x^3 + x^2 + x + 7$.

$$\underbrace{a_4}_{0} x^4 + \underbrace{a_3}_{1} x^3 + \underbrace{a_2}_{1} x^2$$

(2) $\underbrace{x^6 + 1}$.

$$\underbrace{a_6}_{1} x^6 + \underbrace{a_5 x^5 + \cdots + a_1 x}_{a_5 = a_4 = \cdots 0} + \underbrace{a_0}_{1}$$

(3) 1 (4) 0 (5) $mx+b$.

polynomial.

not polynomial: $\dfrac{8}{x^3} + 1$ $\quad \dfrac{5}{x} + 1$ $\quad \dfrac{1}{x}$

not defined for $x=0$

Rational functions $\frac{p(x)}{q(x)}$ are c/s

except where $q(x)=0$. $\square$.

Examples $\frac{8}{x^3}+1 = \frac{8+x^3}{x^3}$. $\frac{5}{x}+1 = \frac{5+x}{x}$.

$\frac{1}{x}$.

not rational function: $\sqrt{x}$ $\sin(x)$.

e^x

$\log(x)$.

Q: how do you know these are not rational functions?

these and
re sometimes
entary functions.

, with inverse

at $x=c$, and $g(x)$
at $x=c$

$f(x) = \frac{2^x+\sin(x)}{\sqrt{x^2+x+1}}$,
at $x=1$

$\frac{x^2}{\sin(x)}$ c/s?

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Joseph Maher

Thm 26 Rational functions $\frac{p(x)}{q(x)}$ are cts except where $q(x)=0$. $\square$

Examples

$$\frac{8}{x^3}+1 = \frac{8+x^3}{x^3}. \qquad \frac{5}{x}+1 = \frac{5+x}{x}.$$

$$\frac{1}{x}.$$

not rational function: $\sqrt{x}$, $\sin(x)$, e^x, $\log(x)$.

Q: how do you know these are not rational functions?

Proof $f(x) = x$ cts. $f(x) \cdot f(x) = x \cdot x = x^2$ cts.

so x^n cts, sums, constant multiples $\Rightarrow$ $a_n x^n + \dots + a_0$ is cts.

quotiens $\dfrac{p(x)}{q(x)}$ cts where $q(x) \neq 0$. $\square$

useful facts

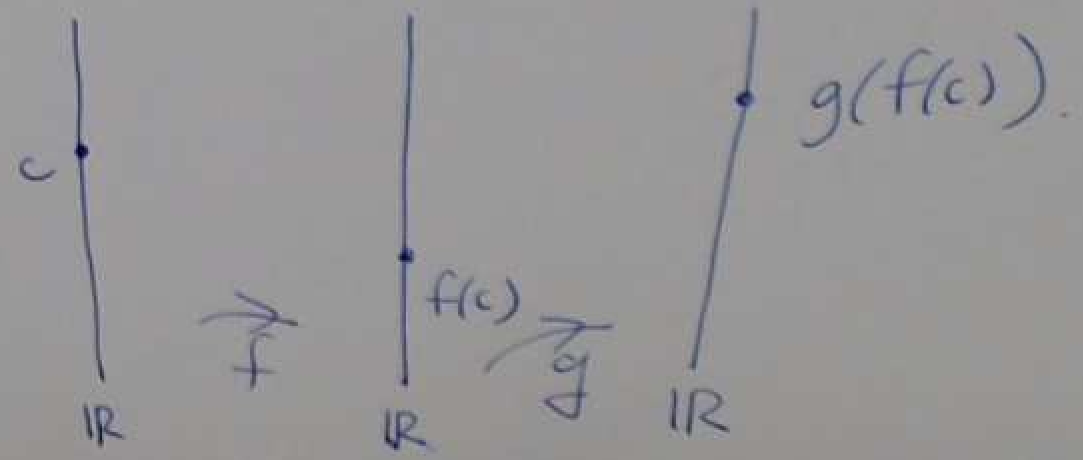
Thm 3 $\sin(x)$, $\cos(x)$ are cts.
b^x is cts.
$\log_b(x)$ is cts.
$x^{1/n}$ is cts.

combinations of these with polynomials are sometimes called elementary functions

$$\frac{\sqrt{x^3} + \log(x^2 + x + 1)}{\sin(x)}$$

<u>Thm 4</u> (inverse functions) If $f : A \to B$ is cts
$A \subseteq \mathbb{R}$, $B \subseteq \mathbb{R}$
with $f^{-1} : B \to A$
inverse then f^{-1} is cts.

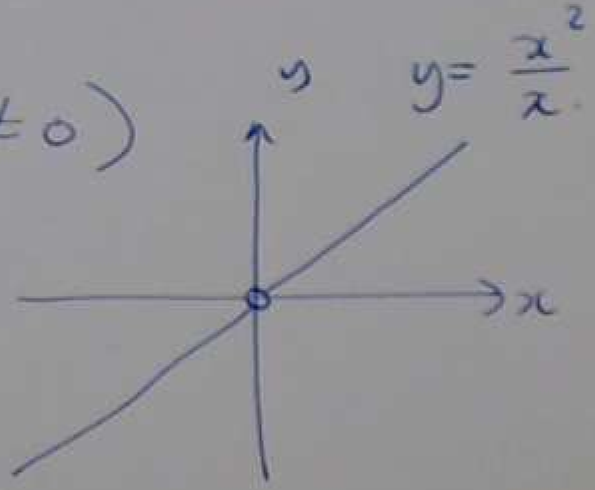
<u>Thm 5</u> (composition) If $f(x)$ is continuous at $x = c$ and $g(x)$ is cts at $x = f(c)$ then $g(f(x))$ is cts at $x = c$.

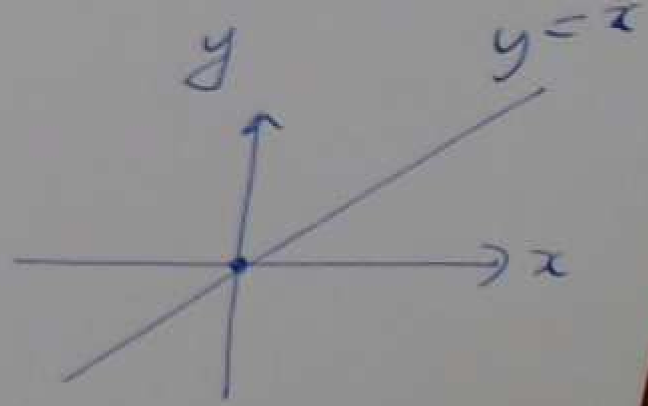


Example $\dfrac{x^2}{x}$ undefined at $x=0$ $\quad \left\lfloor \dfrac{0}{0} \right\rfloor$ indeterminate form.

$\displaystyle\lim_{x\to 0} \frac{x^2}{x}$ does not depend on value at $x=0$, just values close to 0

$\dfrac{x^2}{x} = x \quad (x \neq 0)$





$\displaystyle\lim_{x\to 0} \frac{x^2}{x}$

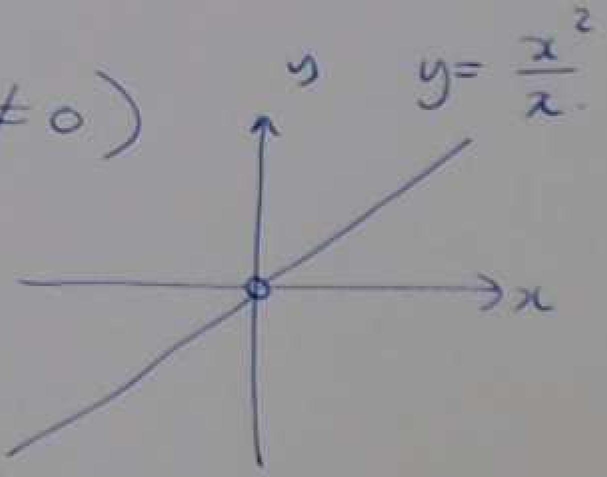
$\displaystyle = \lim_{x\to 0} x = 0.$

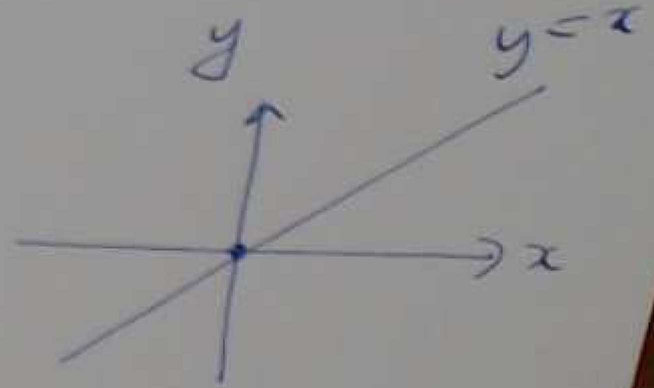
$\displaystyle\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}.$

Example $\dfrac{x^2}{x}$ undefined at $x=0$ $\boxed{\dfrac{0}{0}}$ indeterminate form.

$\displaystyle\lim_{x\to 0}\frac{x^2}{x}$ does not depend on value at $x=0$, just values close to 0

$\dfrac{x^2}{x} = x \quad (x \neq 0)$





$\displaystyle\lim_{x\to 0}\frac{x^2}{x}$

$= \displaystyle\lim_{x\to 0} x = 0.$

$\displaystyle\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{1}{\sqrt{x}+2} = \frac{1}{4}$

20

$$3\underbrace{\sin^{-1}\left(\tfrac{1}{2}\right)}_{\pi/6} - 2\underbrace{\cos^{-1}\left(\tfrac{1}{2}\right)}_{\pi/3} + \underbrace{\tan^{-1}(1)}_{\pi/4}.$$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$.
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$.
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

$$\sin\frac{\pi}{6} = \frac{1}{2}.$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$

$$\tan^{-1}(1) = \frac{\pi}{4}.$$

$$\cos\frac{\pi}{3} = \frac{1}{2}.$$

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

$$\underbrace{3\sin^{-1}\left(\tfrac{1}{2}\right)}_{\pi/6} - 2\underbrace{\cos^{-1}\left(\tfrac{1}{2}\right)}_{\pi/3} + \underbrace{\tan^{-1}(1)}_{\pi/4}.$$

(20)

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

$$\sin\frac{\pi}{6} = \frac{1}{2}.$$

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