

Examples ① $\int_{x=0}^{x=1} e^{-7x} dx$ set $u = -7x$
 $\frac{du}{dx} = -7$
 $\int_{u=0}^{u=-7} e^u \frac{dx}{du} du = \int_0^{-7} e^u \cdot -\frac{1}{7} du = -\frac{1}{7} [e^u]_0^{-7} = -\frac{1}{7} (e^{-7} - 1)$

② $\int_0^2 x^2 \sqrt{x^3+1} dx$, $\int_0^{\pi/4} \tan^2 x \sec^2 x dx$, $\int_0^1 \frac{x}{x^2+1} dx$

§5.8 More integrals

recall: $\frac{d}{dx} (\ln(x)) = \frac{1}{x}$ so $\int \frac{1}{x} dx = \ln|x| + c$

$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$ $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x) + c$

$\frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1+x^2}$ $\int \frac{1}{1+x^2} dx = \tan^{-1}(x) + c$

$\frac{d}{dx} (\sec^{-1}(x)) = \frac{1}{|x| \sqrt{x^2-1}}$ $\int \frac{1}{|x| \sqrt{x^2-1}} dx = \sec^{-1}(x) + c$

$\frac{d}{dx} (e^x) = e^x$ $\int e^x dx = e^x + c$

recall $b^x = e^{x \ln(b)}$ $\int b^x dx = \int e^{x \ln(b)} dx = \frac{1}{\ln(b)} e^{x \ln(b)} + c$
 (nb $u = x \ln(b)$).

example $\int \frac{1}{9+x^2} dx = \frac{1}{9} \int \frac{1}{1+x^2/9} dx = \frac{1}{9} \int \frac{1}{1+(x/3)^2} dx$ sub $u = x/3 \dots$

$\int \frac{1}{\sqrt{1-4x^2}} dx = \int \frac{1}{\sqrt{1-(2x)^2}} dx$ sub $u = 2x \dots$

examples $\int x(2x+1)^9 dx$, $\int \sqrt{4x-1} dx$, $\int \sin^2 x \cos x dx$