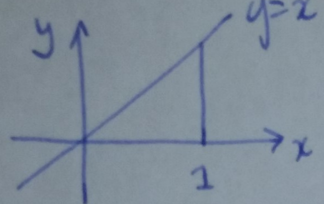
$$\approx \frac{\text{area of rectangle } f(t) \times h}{h} = f(t)$$

i.e. $\int_a^t f(x)\,dx$ is an antiderivative for $f(x)$, so $\int_a^t f(x)\,dx = F(t)+c$

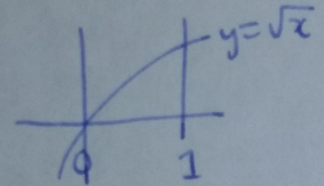
what is the constant? $t=a$: $\int_a^a f(x)\,dx = 0 = F(a)+c \Rightarrow c = -F(a)$

so $\int_a^t f(x)\,dx = F(t) - F(a)$ $\square$.

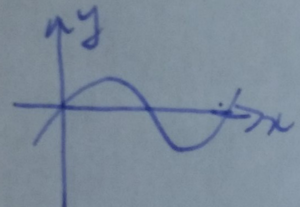
<u>Examples</u>



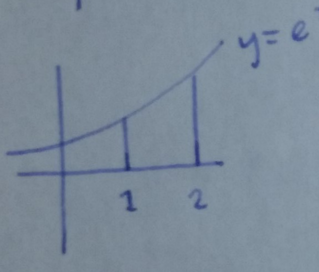
$$\int_0^1 x\,dx = \left[\tfrac{1}{2}x^2\right]_0^1 = \tfrac{1}{2} - 0 = \tfrac{1}{2}$$



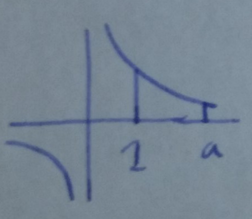
$$\int_0^1 x^{1/2}\,dx = \left[\tfrac{2}{3}x^{3/2}\right]_0^1 = \tfrac{2}{3} - 0 = \tfrac{2}{3}$$



$$\int_0^\pi \sin(x)\,dx = \left[-\cos(x)\right]_0^\pi = -\cos(\pi) + \cos(0) = -(-1)+1 = 2$$



$$\int_1^2 e^x\,dx = \left[e^x\right]_1^2 = e^2 - e$$



$$\int_1^a \frac{1}{x}\,dx = \left[\ln|x|\right]_1^a = \ln(a) - \ln(1) = \ln(a)$$

<u>Observations</u>

① choice of antiderivative doesn't matter, let $F(x)$, $F(x)+c$ be antiderivatives for $f(x)$

Then $\int_a^b f(x)\,dx = F(b) - F(a)$

$$= F(b)+c - (F(a)+c) = F(b) - F(a)$$

② $\int_a^t f(x)dx$ is a function of t! x is called a dummy variable

i.e $\int_a^t f(x)dx = \int_a^t f(y)dy$, if you want a function of x, use $\int_a^x f(t)dt$

§5.4 Fundamental theorem of calculus II

Thm (FTC ②) Let $f(x)$ be a cts function on $[a,b]$, then $A(x) = \int_a^x f(t)dt$ is an antiderivative for $f(x)$, i.e. $A'(x) = f(x) = \frac{dA}{dx}$, i.e. $\frac{d}{dx}\int_a^x f(t)dt = f(x)$.

Furthermore, $A(a) = 0$.

§5.6 Substitution / change of variable

"reverse chain rule for integration".

recall: chain rule: $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$

substitution / change of variable

$\int f(x)dx$, $x(u)$

$$\boxed{\int_{x=a}^{x=b} f(x)dx = \int_{u=x^{-1}(a)}^{u=x^{-1}(b)} f(x(u))\frac{dx}{du}\,du}$$

remember: three things to change: function, differential, limits.

mnemonic: $du = \frac{du}{dx}\cdot dx$ (cancelling fractions)

why does this work?

$\int f(x)dx$, $x(u)$ set $F(x) = \int f(x)dx$, so $F'(x) = f(x)$

$\int f(x)dx = F(x) \underset{\text{wrt } u}{\overset{\text{diff}}{\rightsquigarrow}} \frac{d}{du}(F(x(u))) = F'(x(u))\cdot x'(u)$

$\underset{\text{wrt}}{\overset{\text{integrate}}{\rightsquigarrow}} \int F'(x(u))x'(u)\,du = \int f(x(u))\frac{dx}{du}\,du$ $\square$.

useful fact $\boxed{\frac{dx}{du} = 1/\frac{du}{dx}}$