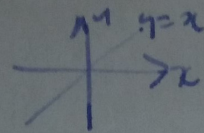
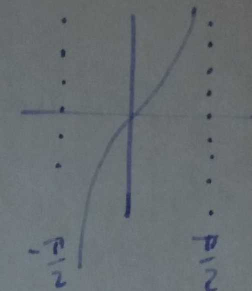
<u>warning</u> some functions do not have any max or min.

<u>Examples</u> $f: \mathbb{R} \to \mathbb{R}$, $x \mapsto x$



$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $x \mapsto \tan(x)$



<u>Thm</u> If $f(x)$ is continuous on a <u>closed bounded interval</u> $[a,b]$, then $f(x)$ has an absolute max and an absolute min.

<u>Defn</u> $f(x)$ has a <u>local max</u> at $x=c$ if there is a small ~~nbd~~ interval containing c s.t. $f(c)$ is a maximum on this interval $\cap$

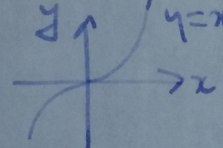
$f(x)$ has a <u>local min</u> at $x=c$ if there is a small interval containing c s.t. $f(c)$ is a minimum on this interval $\cup$

<u>Defn</u> We say that c is a critical point if $f'(c)=0$ (or is undefined)

<u>Thm</u> If c is a local max or min, then c is a critical point.

<u>warning</u>: $f'(c)=0 \not\Rightarrow$ local max or min.

<u>Example</u> $y = x^3$



$f'(x) = 3x^2$ $\quad f'(0)=0$

but $x=0$ <u>not</u> local max or min

How to find the absolute max or min of a differentiable function on a closed interval $[a,b]$: ① find critical points, evaluate function there. ② evaluate function at endpoints

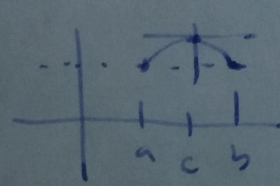
<u>Example</u> ① find abs max of $2x^3 - 15x^2 + 24x + 7$ on $[0,3]$

② $x^2 - 8$ on $[1,4]$

③ $\cos(x)\sin(x)$ on $[0,\pi]$.

<u>Thm</u> (Rolle's Thm) If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) if $f(a)=f(b)$ then there is a $c \in (a,b)$ s.t. $f'(c)=0$.

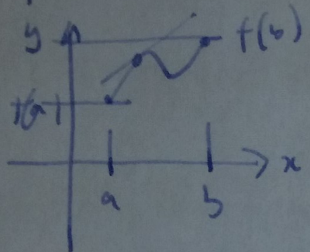
<u>Proof</u>



- If there is a $c \in (a,b)$ which is a local max/min then $f'(c)=0$ ✓
- if no local max/min, then $f(x) = \text{const} = f(a) = f(b)$ so $f'(c)=0$ $\forall c \in (a,b)$ $\square$.

§4.3 First derivative test

Thm (Mean value theorem)(MVT) suppose f is cts on $[a,b]$ and differentiable on (a,b), then there is a $c \in (a,b)$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$, i.e. there is a point where the slope is equal to the average rate of change.



Corollary If $f(x)$ is differentiable, and $f'(x)=0$, then $f(x)=c$ const.

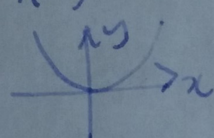
Proof (of corollary) suppose there is a,b with $f(a)\neq f(b)$. Then $\exists c \in (a,b)$ with $f'(c)= \frac{f(b)-f(a)}{b-a} \neq 0$ ⨳ □.

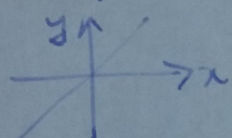
Monotonicity suppose f is differentiable on (a,b).

If $f'(x) > 0$ for all $x \in (a,b)$ then f is increasing on (a,b)

$f'(x) < 0$ decreasing

Example ① $f(x) = x^2$ $\quad f'(x) = 2x$ $\quad$ increasing on $(0,\infty)$, decreasing on $(-\infty,0)$



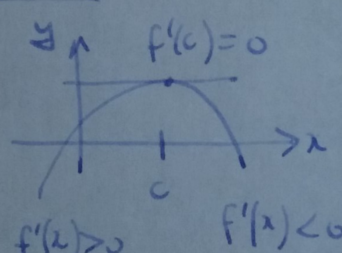


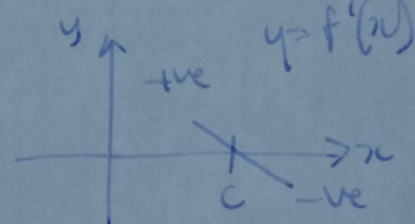
② $f(x) = x^2-2x-3$ $\quad f'(x)>0$ where $2x-2>0$

$f'(x) = 2x-2$ $\quad x>1$

First derivative test

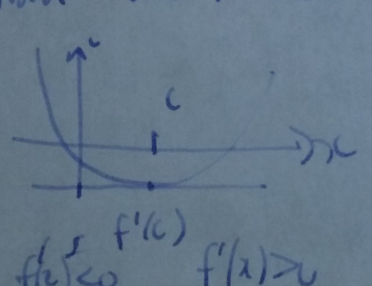
Local max:

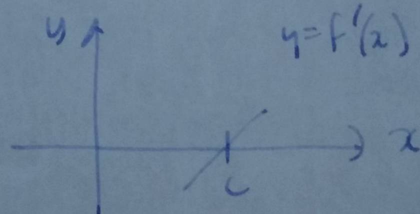




$f'(x)$ goes from +ve to −ve ⇒ local max.

Local min:





$f'(x)$ goes from −ve to +ve ⇒ local min.