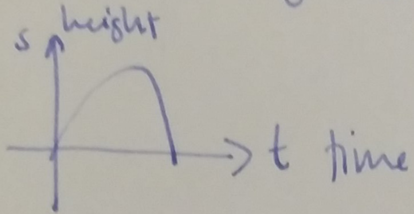
Motion under gravity



$s_0 = s(0) = $ height at $t=0$

$v_0 = v(0) = $ velocity at $t=0$

$$s''(t) = v'(t) = a(t) = -g = \begin{array}{l} 9.8\,m/s^2 \\ 32\,ft/s^2 \end{array}$$

$$s'(t) = v(t) = -gt + v_0$$

$$s(t) = -\tfrac{1}{2}gt^2 + v_0 t + s_0$$

Q: what is the max height?

A: when $s'(t) = v(t) = 0$: $v_0 - gt = 0 \Rightarrow t = \frac{v_0}{g}$ so

$$s\left(\frac{v_0}{g}\right) = \cancel{s\left(g\left(\frac{v_0}{g}\right)^2\right)} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\frac{v_0}{g} + s_0 = s_0 + \frac{1}{2}\frac{v_0^2}{g}$$

Example throw a stone upwards at 10m/s from a height of 2m what is max height?

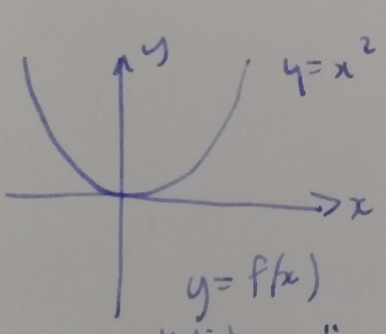
$s(t) = 2 + 10t - \frac{1}{2}gt^2$

$v(t) = 10 - gt$

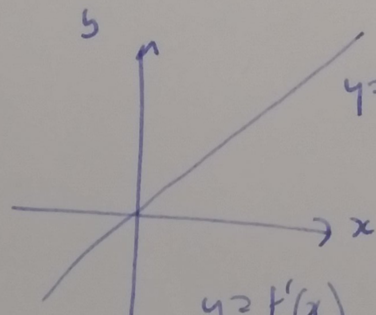
$v(t) = 0 \quad t = \frac{10}{g} \approx 1 \quad s(1) = 2 + 10 - 5 = 7m$

Q: if I can throw a stone 10m high, how fast can I throw it?

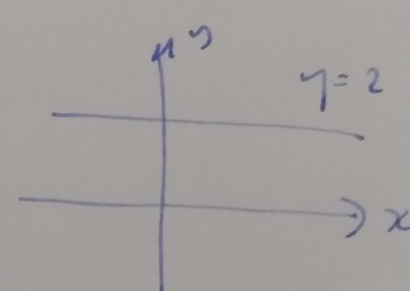
§3.5 Higher derivatives



$y=f(x)$
"distance"



$y=f'(x)$
slope of $f(x)$
"velocity"



$y=f''(x)$
slope of $f'(x)$
"acceleration"

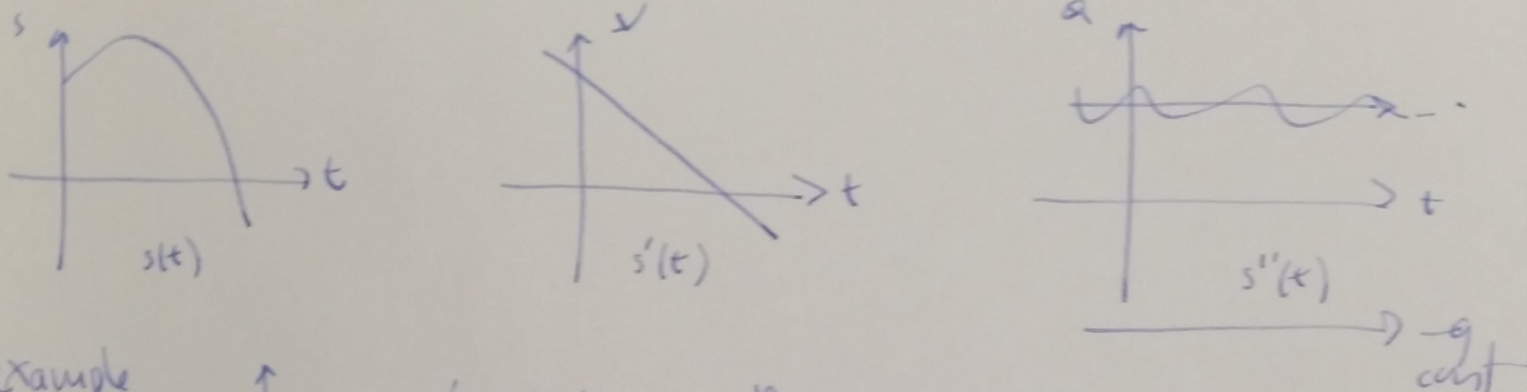
Example $f(x) = xe^x$

$f'(x) = xe^x + e^x$

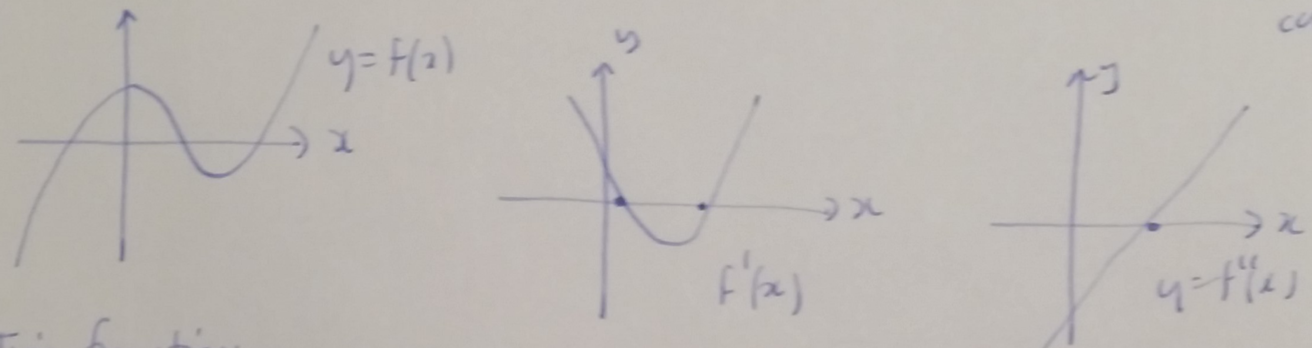
$f''(x) = xe^x + e^x + e^x = xe^x + 2e^x$

$f^{(3)}(x) = xe^x + e^x + 2e^x = xe^x + 3e^x$ etc.

Example acceleration due to gravity



Example



§3.6 Trig functions

Thm: $\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x$

Proof $(\sin x)$ $\frac{d}{dx}(\sin(x)) = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$

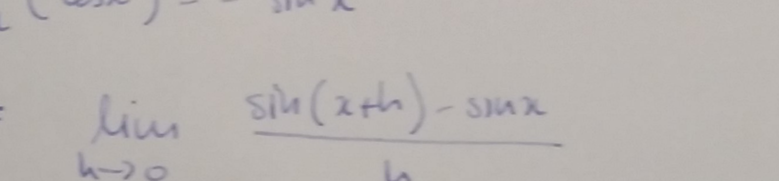
recall: $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$

$= \lim_{h\to 0} \sin x \frac{\cos h - 1}{h} + \cos \frac{\sin h}{h} = \sin x \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1}$

$= \cos x$. $\square$

Q: can this be right?



Example $f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$.

Thm $\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad$ $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$.

Proof ($\tan x$) $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x . \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x}$ $\square$

Example $\frac{d}{dx}(e^x \cos x) = e^x . -\sin x + e^x \cos x$.

§3.7 Chain rule

composition of functions $f(g(x)) = (f \circ g)(x)$

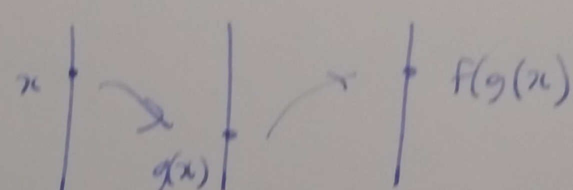
Examples e^{4x}, $\sin^2 x$, $e^{\sqrt{x}}$.

Thm Chain rule If f and g are differentiable, then $f \circ g$ is differentiable, and $(f(g(x)))' = f'(g(x))\, g'(x)$

mnemonic: $[f(g(x))]' =$ outside'(inside) . inside'.

note $f(g(x))$ $\quad \mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$

$$x \mapsto g(x) \mapsto f(g(x))$$



$\mathbb{R} \xrightarrow{g} \mathbb{R} \xrightarrow{f} \mathbb{R}$

scaling factor $g'(x)$ $\quad$ scaling factor $f'(g(x))$

Examples $e^{4x} = f(g(x))$ $\quad f(x) = e^x \quad f'(x) = e^x$

$g(x) = 4x \quad g'(x) = 4$

so $(e^{4x})' = f'(g(x)) . g'(x) = e^{4x} . 4$.