Thm <u>Quotient rule</u> (assume f, g differentiable at x, $g(x) \neq 0$)

then $\left(\frac{f}{g}\right)'(x) = \frac{g f' - f g'}{g^2} = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$

<u>Example</u> ① $\frac{d}{dx}\left(\frac{x}{x+1}\right) = \frac{(x+1)(x)' - x(x+1)'}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

② $\frac{d}{dt}\left(\frac{e^t}{e^t+t}\right) = \frac{(e^t+t)(e^t)' - (e^t)(e^t+t)'}{(e^t+t)^2} = \frac{(e^t+t)e^t - e^t(e^t+1)}{(e^t+t)^2}$

$= \frac{te^t - e^t}{(e^t+t)^2}$

③ battery power  R resistance

r internal resistance

power $P = \frac{v^2 R}{(r+R)^2}$

<u>Q</u>: when does the battery give maximal power?

<u>A</u>: when $\frac{dP}{dR} = 0$ $\quad P = \frac{v^2 R}{(r+R)^2}$ $\quad P(R)$, v, r constant

$\frac{dP}{dR} = \frac{(r+R)^2 (v^2 R)' - v^2 R((r+R)^2)'}{(r+R)^4} = \frac{(r+R)^2 v^2 - v^2 R(r^2 + 2rR + R^2)'}{(r+R)^4}$

$= \frac{v^2\left[(r+R)^2 - R(2r+2R)\right]}{(r+R)^4} = \frac{v^2(r+R)(r+R-2R)}{(r+R)^4} = \frac{v^2(r-R)}{(r+R)^4} = 0$

i.e. $R = r$.

<u>§3.4 Rates of change</u>

recall: average rate of change $= \frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$

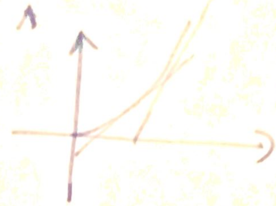
(instantaneous) rate of change $f'(x) = \lim_{x \to x_0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

<u>observation</u>: if Δx is small, we can use average rate of change to approximate actual rate of change, and vice versa.

Example area of circle is πr^2

calculate rate of change of area wrt radius $\frac{dA}{dr} = 2\pi r$

e.g. $\left.\frac{dA}{dr}\right|_{r=2} = 4\pi$ $\quad \left.\frac{dA}{dr}\right|_{r=5} = 10\pi$



area of ring when $r=4 \simeq 4\pi\Delta r$

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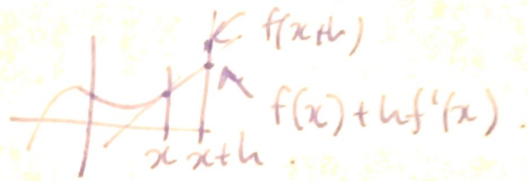
for small h, $f'(x_0) \approx \frac{f(x_0+h)-f(x)}{h} \leftrightarrow f(x_0+h) \approx f(x_0) + hf'(x)$

this is the linear approximation to f at x



Example stopping distance in feet given by

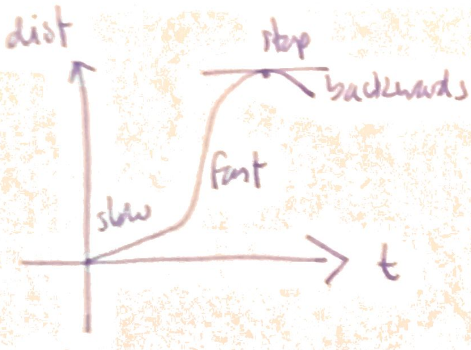
$F(s) = 1.1s + 0.05s^2$ (s speed in mph) calculate stopping distance

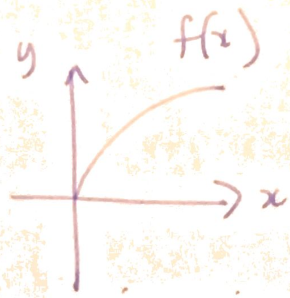
when $s=30$. $F(30) = 3.3 + \frac{900}{20} = 48.3$

find $F'(30)$. $F'(s) = 1.1 + 0.1s$ $\quad F'(30) = 4.1$ ft/mph.

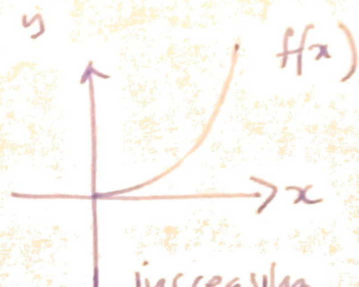
estimate stopping distance at $s=31$: $F(31) \approx F(30) + 1.F'(30) = 48.3 + 4.1$
$= 52.4$.

On the interpretation of graphs





increasing
$f'(x) > 0$
rate of change decreasing



increasing
$f'(x) > 0$
rate of change increasing