Math 231 Calculus 1 Fall 20 Sample Midterm 2
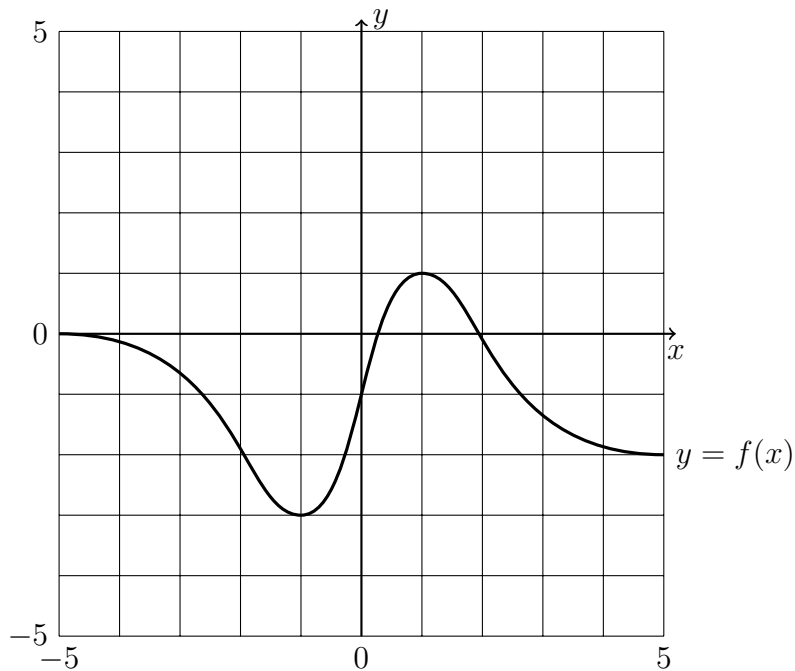
(1) Consider the function $f(x)$ defined by the following graph.

(a) Label all regions where $f'(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) Sketch a graph of $f'(x)$ on the figure.
(d) What is $\lim_{x\to\infty} f(x)$?
(e) What is $\lim_{x\to-\infty} f'(x)$?

(2) Find the derivatives of the following functions
(a) $x^5 e^{-3x^3}$
(b) $\dfrac{\sqrt{2x-1}}{3 - \tan(2x)}$
(c) x^{4x}
(d) $\ln(\sec(\sqrt{x}))$
(e) $\tan^{-1}(2/\sqrt[4]{x})$
(f) $\sin^{-1}(3 - 2x)$

(3) Find the second derivatives of the functions above.

(4) The graphs of the functions f and g are shown below.
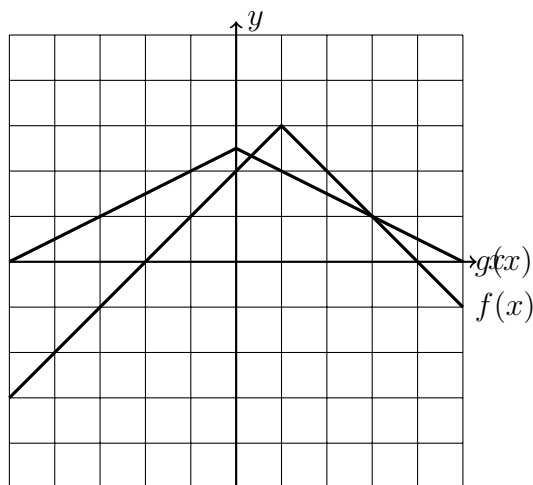
(a) Let $h(x) = f(x)g(x)$ Find $h'(3)$.
(b) Let $h(x) = f(g(x))$. Find $h'(-1)$.

(5) Use implicit differentiation to find the tangent line to the hyperbola $16x^2 - 3y^2 = 4$ at the point $(1, -2)$.

(6) Find $\frac{dy}{dx}$ for the implicit function $x^3y + x^2y^2 = \sin(xy)$.

(7) You inflate a spherical balloon at a rate of 20cm^3 per second. How fast is the area of the balloon increasing when the radius is 20cm?

(8) Use a linear approximation to estimate $\sqrt[3]{26}$. What is the percentage error?

(9) Find all the critical points for the function $f(x) = e^x(x^2 - x - 5)$. Use the first derivative test to identify them as local maxima or local minima.

(10) Find the absolute maximum and minimum of the function $f(x) = x^2 - 2x - 3$ on the interval $[-2, 2]$.