# SMT1 Solutions

**Q1** a) 2 b) 1 c) DNE d) −1 e) −1 f) −3

**Q2** a) $\lim_{x \to -2} \frac{x^2 - x - 12}{x+3} = \frac{4+2-12}{1} = -6$

b) $\lim_{x \to \infty} \frac{\sqrt{5+2x^2}}{x-4} = \lim_{x \to \infty} \frac{\sqrt{2+5/x^2}}{1-4/x} = \sqrt{2}$

c) $\lim_{x \to 0} \frac{\sin 3x}{7x} = \lim_{\theta \to 0} \frac{\sin\theta}{7\theta/3} = \frac{3}{7}$ $\quad \theta = 3x$

d) $\lim_{x \to 3+} \frac{1}{\sqrt{x^2-9}} - \frac{1}{\sqrt{x-3}} = \lim_{x \to 3+} \frac{(1-\sqrt{x+3})}{\sqrt{\underbrace{x^2-9}_{(x-3)(x+3)}}} \frac{(1+\sqrt{x+3})}{(1+\sqrt{x+3})}$.

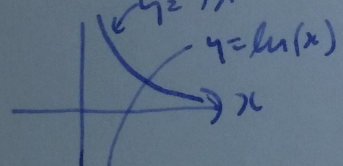
$= \lim_{x \to 3+} \frac{1-x-3}{\sqrt{x^2-9}\,(1+\sqrt{x+3})} = -\infty$.

**Q3** $\lim_{x \to 2-} x - \frac{1}{x-3} = -2 - \frac{1}{-2-3} = -1.8$

$\lim_{x \to 2+} \frac{-\cos(\pi x)}{x} = -\frac{1}{2} \neq -1.8$ so no value of $f(2)=c$ works.

**Q4** $S = 4\pi r^2$ $\quad \frac{S(6)-S(5)}{6-5} = 4\pi(36-25) = 44\pi$

**Q5**




consider $f(x) = \frac{1}{x} - \ln(x)$ $\quad f(1) = 1 - 0 = 1 > 0$

$f(e) = \frac{1}{e} - 1 < 0$

so there is a solution in $[1, e]$.

**Q6** a) $5 - 10x^4 + \frac{3}{2}x^{-1/2} - 2 \cdot \frac{-2}{3} x^{-5/3}$

b) $6xe^x + 3x^2e^x$ $\quad$ c) $\frac{(2x-1)\cdot 2 - (2x+1)\cdot 2}{(2x-1)^2} = \frac{-4}{(2x-1)^2}$

**Q7** a) $-40x^3 - \frac{3}{4}x^{-3/2} - \frac{20}{9}x^{-8/3}$ $\quad$ c) $8(2x-1)^{-3} \cdot 2$.

b) $6e^x + 6xe^x + 6xe^x + 3x^2e^x = (6+12x+3x^2)e^x$

Q8 a) $$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{2}{2+h-1} - \frac{2}{2-1}}{h} = \lim_{h\to 0} \frac{2-2(2+h-1)}{h(2+h-1)}$$

$$= \lim_{h\to 0} \frac{-2h}{h(2+h-1)} = \lim_{h\to 0} \frac{-2}{2+h-1} = -2$$

b) $$\lim_{h\to 0} \frac{\frac{1}{\sqrt{2+h+2}} - \frac{1}{\sqrt{2+2}}}{h} = \lim_{h\to 0} \frac{\left(1-\frac{\sqrt{4+h}}{2}\right)}{h\sqrt{4+h}} \cdot \frac{\left(1+\frac{\sqrt{4+h}}{2}\right)}{\left(1+\frac{\sqrt{4+h}}{2}\right)}$$

$$= \lim_{h\to 0} \frac{1-\frac{4+h}{4}}{h\sqrt{4+h}\left(1+\frac{\sqrt{4+h}}{2}\right)} = \lim_{h\to 0} \frac{-h/4}{h\sqrt{4+h}\left(1+\frac{\sqrt{4+h}}{2}\right)} = \lim_{h\to 0} \frac{-1/4}{\sqrt{4+h}\left(1+\frac{\sqrt{4+h}}{2}\right)}$$

$$= \frac{-1/4}{\sqrt{4}\left(1+\sqrt{4}/2\right)} = \frac{-1/4}{2(1+1)} = \frac{-1}{16}.$$