Math 231 Calculus 1 Fall 20 Sample Final

(1) Differentiate the following functions. Do not simplify your answers.

(a) $-2x^5 + 2\sqrt[3]{x^4} + \tan(3x)$

(b) $f(x) = \dfrac{x^2 - x}{\ln(2x+1)}$

(c) $f(x) = e^{-3x}\cos(2-3x)$

(d) $f(x) = \sqrt[4]{e^{-\sin(3x)} + 2}$

(2) Evaluate the following integrals.

(a) $\displaystyle\int \frac{1}{x^2} + 3\cos(x) - e^x \, dx$

(b) $\displaystyle\int \frac{(1-2x)^2}{\sqrt{x^3}} \, dx$

(c) $\displaystyle\int_0^{\pi/4} \cos(4x)\sin^3(4x) \, dx$

(d) $\displaystyle\int \frac{1}{1+4x^2} \, dx$

(3) Note: the possible answers for limits are a number, $+\infty$, $-\infty$ or "does not exist" (DNE). Justify your answers.

(a) Find $\displaystyle\lim_{x\to 3} \frac{x^2 - x - 6}{x-3}$.

(b) Find $\displaystyle\lim_{x\to 0} \frac{1-e^{3x}}{\cos(5x)}$.

(c) Find $\displaystyle\lim_{x\to 0+} x^{\sin(x)}$.

(d) Find $\displaystyle\lim_{x\to 0} \frac{1}{x^2} - \frac{1}{\sin^2(x)}$.

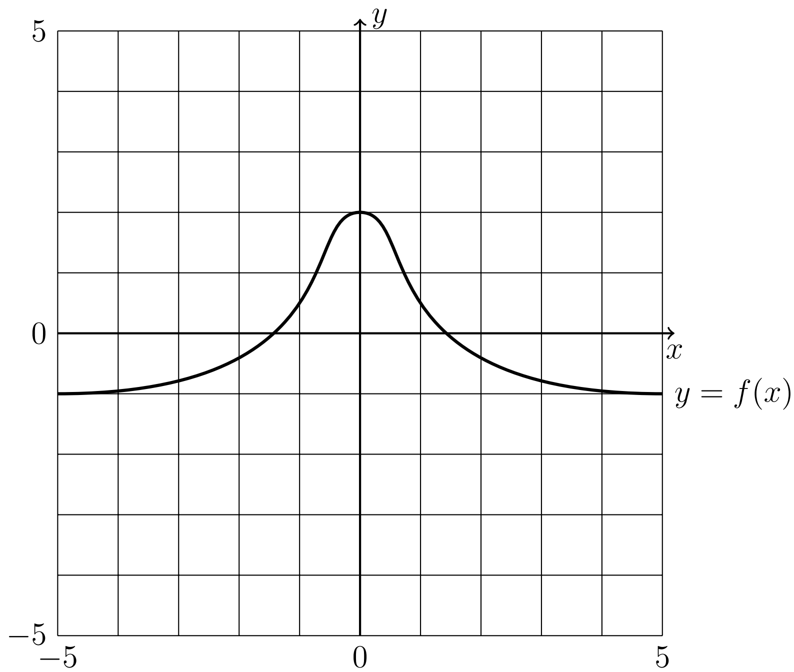
(4) Consider $f(x) = x^3 - 12x$.

(a) Find the derivative of $f(x)$, and find the critical points for $f(x)$.

(b) Give the interval(s) for which f is increasing.

(c) Give the intervals for which f is concave up, and for which it is concave down.
(d) Decide which critical points are maxima, minima, or neither.
(e) Sketch the graph of $f(x)$.

(5) Consider the function $f(x)$ defined by the following graph.



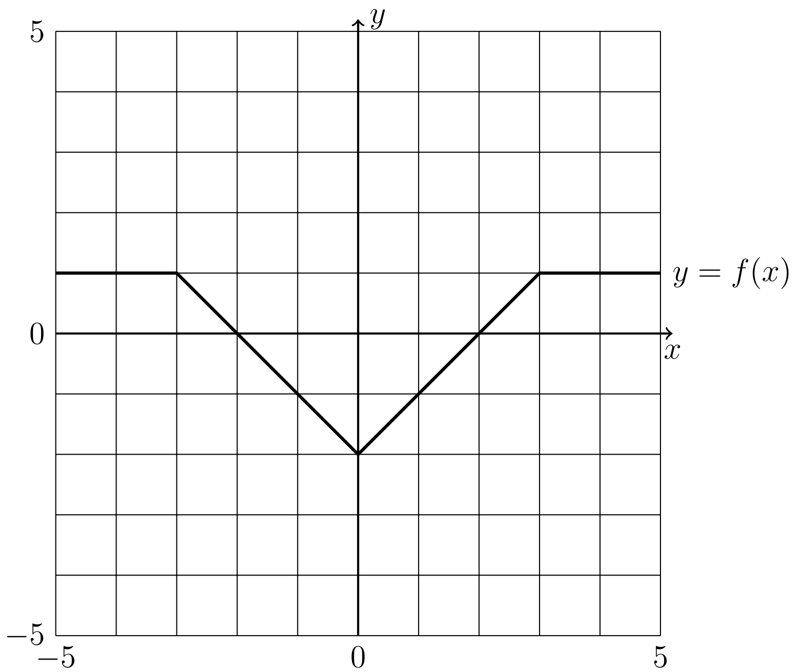
(a) Label all regions where $f(x) < 0$.
(b) Label all regions where $f'(x) > 0$.
(c) Sketch a graph of $f'(x)$ on the figure.

(6) Consider $f(x) = \dfrac{2}{x+2}$.
(a) Sketch the graph of $f(x)$ showing any asymptotes.
(b) Find the slope of the tangent line at $x = -1$, and write down the equation for the tangent line.
(c) Sketch the tangent line at $x = -1$ on your graph.

(7) Let $f(x) = x^2 - x$. Find the derivative *using the limit definition of the derivate.* Show all your work.

(8) Use implicit differentiation to find the tangent line to the curve given by the equation $x^3y - xy^4 + 4x = 10$ at the point $(-1, 2)$.

(9) Sketch the graph of $\int_{-5}^{x} f(t)dt$, where $f(x)$ is shown below.



(10) A region in the plane is bounded by the x-axis, the graph $y = 8 - x^2$, and the lines $x = -2$ and $x = 2$.
  (a) Sketch the region (shading it in) and label the boundaries.
  (b) Find the area of the region.

(11) You blow up a spherical balloon at the rate of $8\text{in}^3/\text{s}$. How fast is the surface area growing when $r = 6\text{in}$? (The volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the surface area is $A = 4\pi r^2$.)

(12) Use linear approximation to estimate $\sqrt[3]{120}$. Use you calculator to find the exact value, and find the absolute and percentage errors.

(13) You wish to build a running track in the shape of a rectangle with two semi-circular ends. If the running rack should have length 1200m, what shape minimizes the area?