Example (7.11) two 3rd grade classes - one new reading activity for 1, give reading test.

Group	n	$\bar{x}$	s
treatment	21	51.48	11.01
control	23	41.52	17.15

• find confidence interval for difference in means: $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1^2} + \frac{s_2^2}{n_2^2}}$.

$= \quad 9.96 \pm 4.31\, t^*$ $\quad t^*$ from t_{20}

t^*	1.725	2.086	2.197
C	0.9	0.95	0.96

95% confidence level $t^* = 2.086$. gives $(1.0, 18.9)$.

• significance test: H_0: means are the same $\mu_1 = \mu_2$.

two sample t-statistic is $\dfrac{(\bar{x}_1 - \bar{x}_2) - \overset{0}{(\mu_1 - \mu_2)}}{\sqrt{s_1^2/n_1^2 + s_2^2/n_2^2}} = \dfrac{9.96}{4.31} = 2.31.$ (~~not~~ significant!)

8.1 Inference for proportions
(for categorical data)

choose an SRS of size n from a large population with proportion p of success (unknown!)

the sample proportion is $\hat{p} = \dfrac{X}{n}$ $\quad X = $ # successes.

standard error of $\hat{p}$ is $SE_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

the margin of error is $m = z^* SE_{\hat{p}}$ for confidence level C, z^* corresponding to $(-z^*, z^*)$ has area C in $N(0,1)$.

the approximate level C confidence interval for p is $\hat{p} \pm m = \hat{p} \pm z^* SE_{\hat{p}} = \hat{p} \pm \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$.

Example (8.1) ask 1896 experts if they like robots, 910 say yes.

sample size $n = 1896$, $X = 910$. Find 95% confidence interval for proportion who like robots.

$\hat{p} = \dfrac{X}{n} = \dfrac{910}{1896} = 0.47996.$ $\quad SE_{\hat{p}} = \sqrt{\dfrac{0.47996(1-0.47996)}{1896}} = 0.011474.$

confidence interval is $\hat{p} \pm m$ $\quad m = \underset{1.96}{z^*} SE_{\hat{p}} = 0.480 \pm 0.022$

48% ± 2%.

Significance test for a single proportion

SRS of size n from (large) pop with unknown proportion of successes p.

null hypothesis $H_0: p = p_0$

compute z-statistic $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$.

alternative hypothesis: $H_a: p \neq p_0$ (two sided)  $2P(Z \geq |z|)$.

$p > p_0$ (one-sided)  $P(Z \geq z)$

$p < p_0$ "  $P(Z \leq z)$.

(valid if np_0 and $n(1-p_0)$ both ≥ 10).

Example (85) testing sunblocks.

$n = 20$: your sunblock better for $n = 13$ } is yours significantly better?
their sunblock $n = 7$

$H_0: p = 0.5$
$H_a\ p \neq 0.5$

$\hat{p} = \dfrac{X}{n} = \dfrac{13}{20} = 0.65$

test statistic $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{0.65 - 0.5}{\sqrt{\dfrac{(0.5)(0.5)}{20}}} \approx 1.34$ $P(Z < 1.34) = 0.9099$

symmetric test 

P-value is $(1 - 0.9099) \times 2 = 0.18$
not significant.

Choosing the sample size: p^* guess for proportion
z^* standard normal critical value for confidence interval.

recall margin of error is $m = z^* SE_{\hat{p}} = z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$.

so $n = \left(\dfrac{z^*}{m}\right)^2 \underbrace{p^*(1-p^*)}_{\leq 1/4} \leq \dfrac{1}{4}\left(\dfrac{z^*}{m}\right)^2$.

§8.2 Comparing two proportions

population	pop. proportion	sample size	# of successes	sample proportion
1	p_1	n_1	X_1	$\hat{p}_1 = X_1/n_1$
2	p_2	n_2	X_2	$\hat{p}_2 = X_2/n_2$

difference between proportions $D = \hat{p}_1 - \hat{p}_2$ ~ Normal for large samples.

Confidence interval for comparing two proportions

$D = \hat{p}_1 - \hat{p}_2$

standard error for D is $$SE_D = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

margin of error at confidence level C is: $m = z^* SE_D$

z^* is critical value for $N(0,1)$ at confidence level C.

approximate confidence interval is $D \pm m$. or $(D-m, D+m)$

[~~$n_1, n_2 \geq 10$~~ $X_1, X_2, n_1 - X_1, n_2 - X_2$ all ≥ 10].

Example (8.11) Instagram:

	n	X	$\hat{p} = X/n$
Women	537	328	0.6108
Men	532	234	0.4398
Total	1069	562	0.5257.

$D = \hat{p}_1 - \hat{p}_2 = 0.6108 - 0.4398 = 0.1710$

$$SE_D = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} = \sqrt{\frac{0.6108(1-0.6108)}{537} + \frac{(0.4398)(1-0.4398)}{532}}$$

$= 0.0301$

95% confidence interval, $z^* = 1.96$ $m = z^* SE_D = (1.96)(0.0301)$
$= 0.0590$

95% confidence interval is $D \pm m = 0.1710 \pm 0.0590$

$(0.112, 0.230)$.

Significance test for difference in proportions

pop	pop par.	sample size	sample successes	sample proportion
1	p_1	n_1	X_1	$\hat{p}_1$
2	p_2	n_2	X_2	$\hat{p}_2$

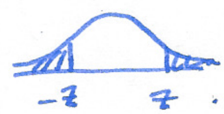
null hypothesis $H_0 : p_1 = p_2$

pooled standard error.

z statistic $z = \dfrac{\hat{p}_1 - \hat{p}_2}{SE_{D_p}}$, where $SE_{D_p} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$.

where $\hat{p} = \dfrac{X_1 + X_2}{n_1 + n_2}$ pooled estimate

P-values : $H_a : p_1 \neq p_2$ p-value is $2\mathbb{P}(Z \geq |z|)$



(want # success, # failures in each sample ≥ 5).

Example (8.14) Instagram use:

pop	n	X	$\hat{p} = X/n$
women	537	328	0.6108
men	532	234	0.4398
Total	1069	562	0.5257

$H_0 : p_1 = p_2$

$H_a : p_1 \neq p_2$

pooled estimate $\hat{p} = \dfrac{328 + 234}{532 + 537} = \dfrac{562}{1069} = 0.5257$

$$SE_{D_p} = \sqrt{(0.5257)(1-0.5257)\left(\frac{1}{537} + \frac{1}{532}\right)} = 0.03055$$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{SE_{D_p}} = \frac{0.6108 - 0.4398}{0.03055} = 5.60$$

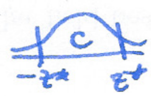
P-value $2\,\text{pnorm}(-5.60)$

$= 0.0004$.

significant.

Sample size for desired margin of error

choose confidence level $C \rightsquigarrow z^*$ critical value



Fact can choose $n = \left(\frac{1}{2}\right)\left(\frac{z^*}{m}\right)^2$ $\left[n = \left(\frac{z^*}{m}\right)^2 \left(\underbrace{p_1^*(1-p_1^*) + p_2^*(1-p_2^*)}_{\leq \frac{1}{2}} \right) \right]$